# Theory of Structures - I

Chapter 6. Statically Determinate Arches [Part II of II]

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Chapter 7. Suspension Cable Systems [Part I of II]

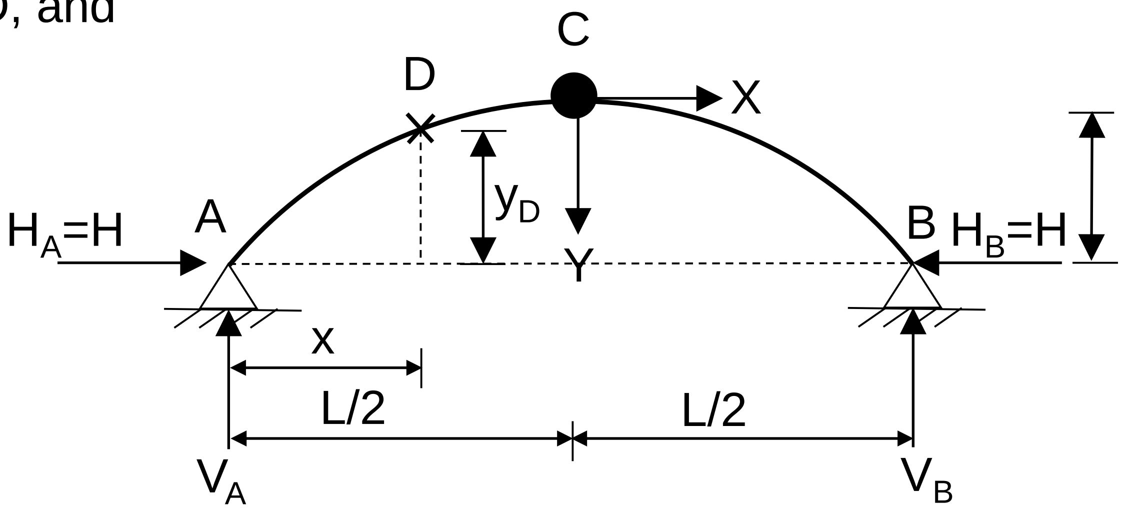
Lecturer: Dr. Sanjeema Bajracharya

# Contents

# 6.4 ILDs for reactions, bending moments, shearing forces, and normal forces in three-hinged arches

We determine the following parameters for a three-hinged arch using ILDs (Influence Line Diagrams):

- Horizontal thrust, H
- Bending moment at section D,
- Normal thrust at section D, and
- Radial shear at section D.

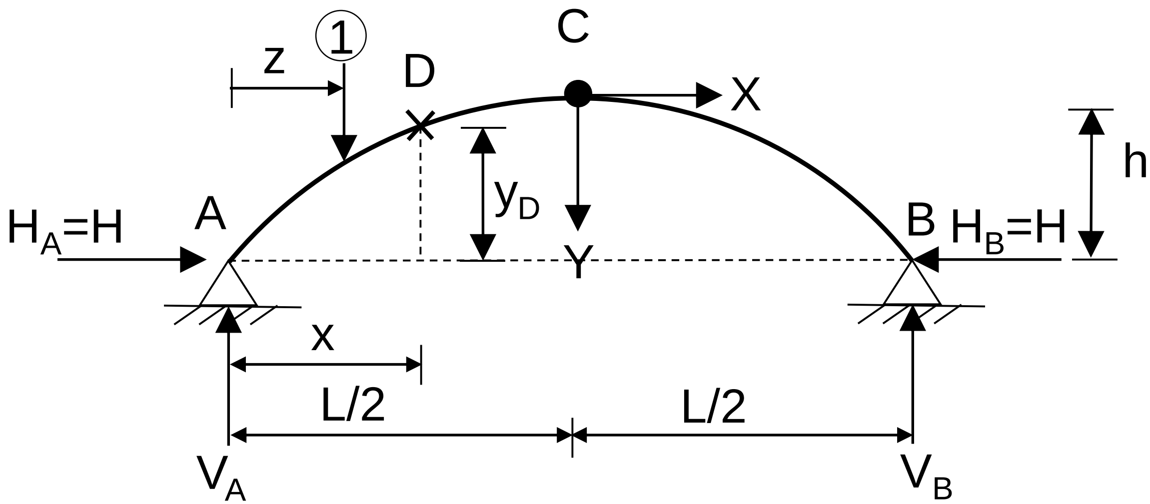



**Note:** Section D at a distance x from support A, and $y_D$ is the height of D from the springing level

Source: Bhavikatti, S. S. (2011). *Structural Analysis –I* (4th ed.).New Delhi: Vikas Publishing House.

# 6.4 ILDs for reactions, bending moments, shearing forces, and normal forces in three-hinged arches

Let us consider a parabolic, three-hinged arch of span L and height h; C is the crown, A and B are the supports. D is a section at a distance x from the left support and $y_D$ from the springing levels. Let z be the distance of unit load from support A.




Source: Bhavikatti, S. S. (2011). *Structural Analysis –I* (4th ed.).New Delhi: Vikas Publishing House.

# 6.4 ILDs for reactions, bending moments, shearing forces, and normal forces in three-hinged arches

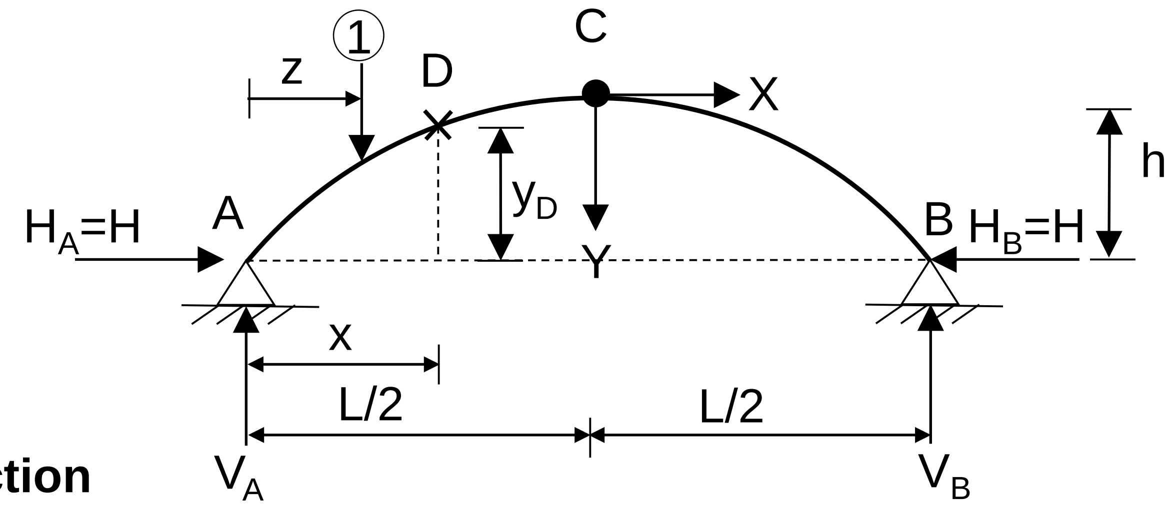
**[A] ILD for H**

Taking moment about A, we get

$\Sigma M_A = 0$

$Or, V_B * L - 1 * z = 0$

$Or, V_B = \frac{z}{L}$ **: Support reaction**




Source: Bhavikatti, S. S. (2011). *Structural Analysis –I* (4th ed.).New Delhi: Vikas Publishing House.

Also, we have,

$V_A + V_B = 1$

$Or, V_A = 1 - \frac{z}{L}$

$Or, V_A = \frac{L - z}{L}$ **: Support reaction**

# 6.4 ILDs for reactions, bending moments, shearing forces, and normal forces in three-hinged arches

**[A] ILD for H**

*(i) When unit load is in portion AC ($0 \le z \le L/2$)*

$$\Sigma M_{C\ (left)} = 0$$

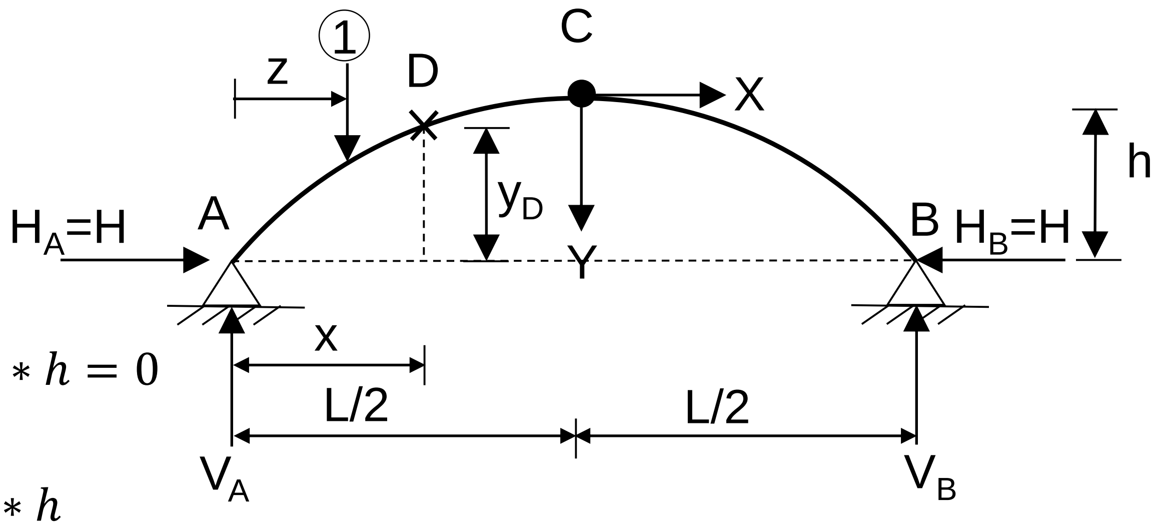
$$Or, -V_A * \frac{L}{2} + 1 * \left(\frac{L}{2} - z\right) + H * h = 0$$

$$Or, \frac{L-z}{L} * \frac{L}{2} = \left(\frac{L-2z}{2}\right) + H * h$$

$$Or, Hh = \frac{L}{2} - \frac{z}{L} * \frac{L}{2} - \frac{L}{2} + z$$

$$Or, Hh = -\frac{z}{2} + z = \frac{z}{2}$$

$$Or, H = \frac{z}{2h}$$




Source: Bhavikatti, S. S. (2011). *Structural Analysis –I* (4th ed.).New Delhi: Vikas Publishing House.

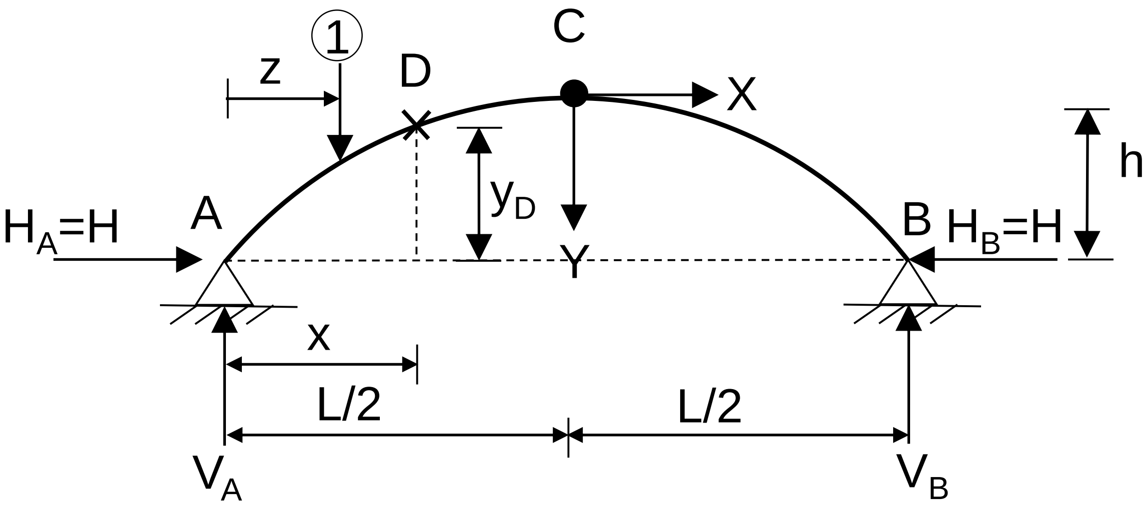
# 6.4 ILDs for reactions, bending moments, shearing forces, and normal forces in three-hinged arches

**[A] ILD for H**

*(i) When unit load is in portion AC ($0 \leq z \leq L/2$)*

$$Or, H = \frac{z}{2h}$$

Linear variation




Source: Bhavikatti, S. S. (2011). *Structural Analysis –I* (4th ed.).New Delhi: Vikas Publishing House.

Using boundary conditions,

| z=0 | H=0 |
|---|---|
| z=L/2 | H=L/4h |

# 6.4 ILDs for reactions, bending moments, shearing forces, and normal forces in three-hinged arches

**[A] ILD for H**

*(ii) When unit load is in portion CB (L/2 ≤ z ≤ L)*

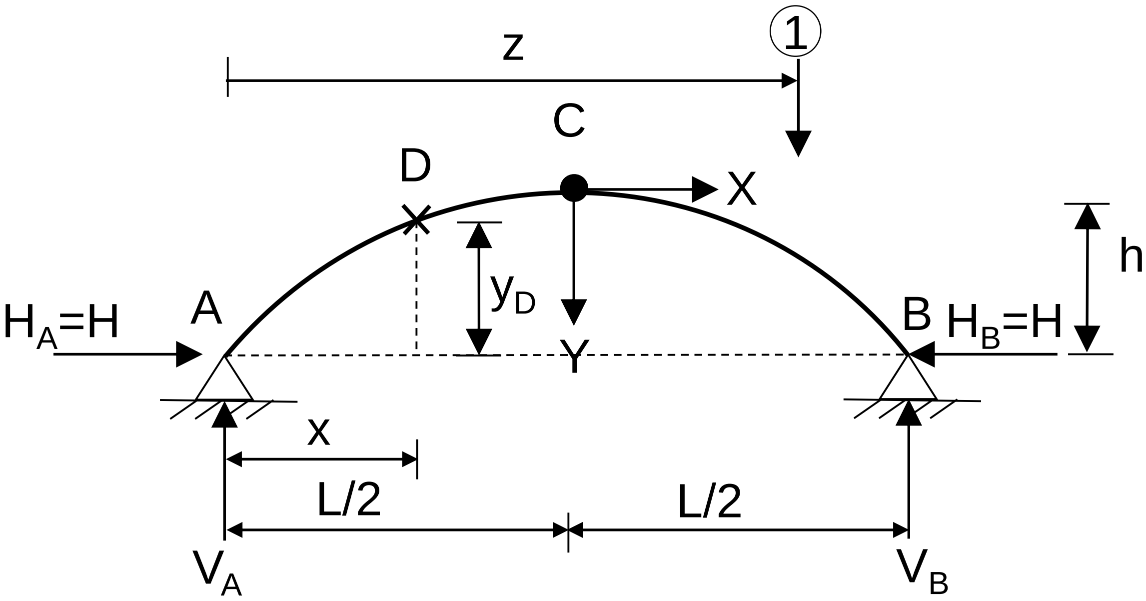
$$\Sigma M_{C\,(left)} = 0$$

$$Or, -V_A * \frac{L}{2} + H * h = 0$$

$$Or, \frac{L-z}{L} * \frac{L}{2} = H * h$$

$$Or, Hh = \frac{(L-z) * L}{2L}$$

$$Or, H = \frac{(L-z)}{2h}$$




Source: Bhavikatti, S. S. (2011). *Structural Analysis –I* (4th ed.).New Delhi: Vikas Publishing House.

# 6.4 ILDs for reactions, bending moments, shearing forces, and normal forces in three-hinged arches

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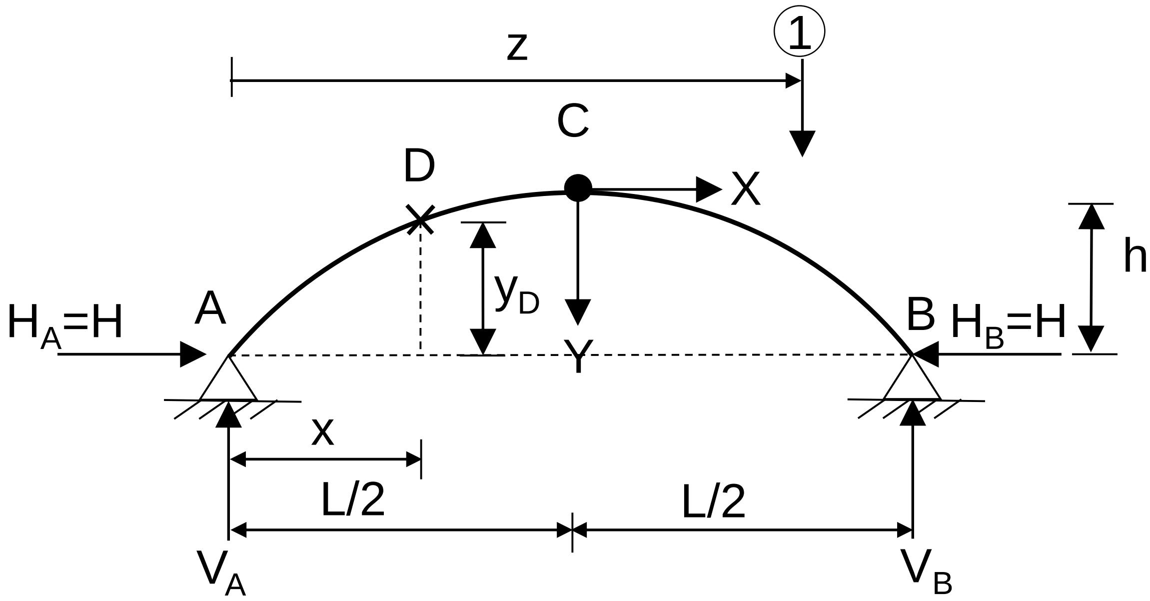
*(ii) When unit load is in portion CB (L/2 ≤ z ≤ L)*

$$Or, H = \frac{(L-z)}{2h}$$

Linear variation



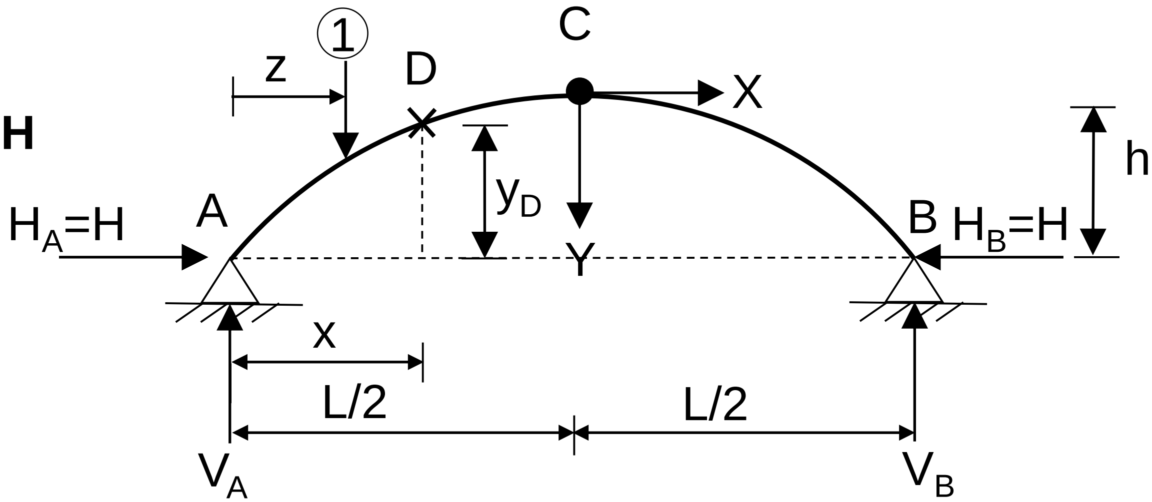

Source: Bhavikatti, S. S. (2011). *Structural Analysis –I* (4th ed.).New Delhi: Vikas Publishing House.

Using boundary conditions,

| z=L/2 | H=L/4h |
|---|---|
| z=L | H=0 |

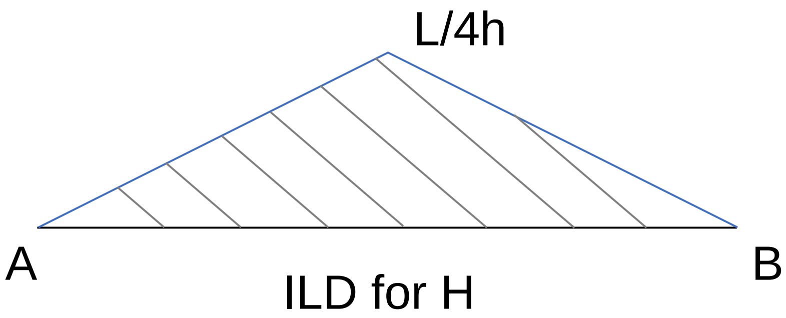
# 6.4 ILDs for reactions, bending moments, shearing forces, and normal forces in three-hinged arches

**[A] ILD for H**




Source: Bhavikatti, S. S. (2011). *Structural Analysis –I* (4th ed.).New Delhi: Vikas Publishing House.

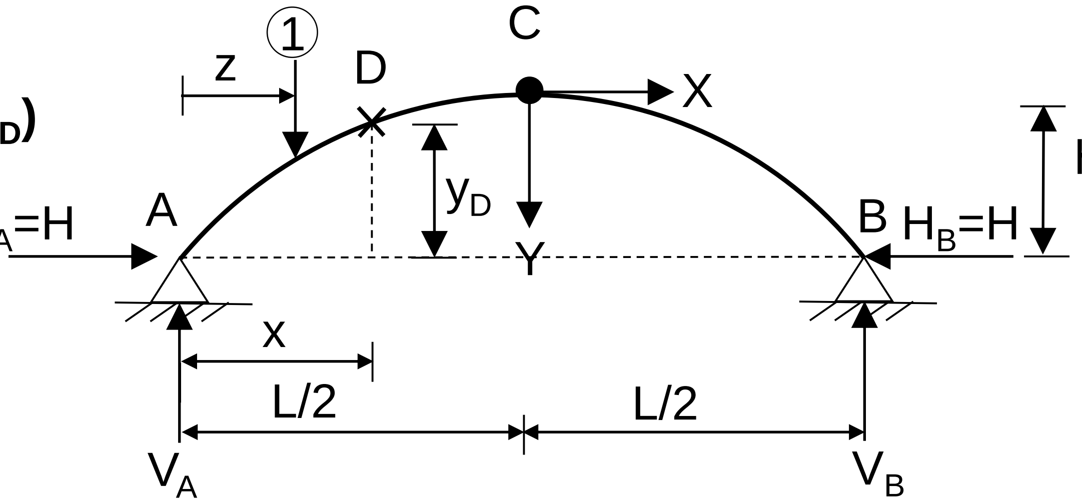
| AC ($0 \le z \le L/2$) | CB ($L/2 \le z \le L$) |
|---|---|
| $H = \dfrac{z}{2h}$ | $H = \dfrac{(L-z)}{2h}$ |
| z=0 | H=0 |
| z=L/2 | H=L/4h |
| z=L | H=0 |



# 6.4 ILDs for reactions, bending moments, shearing forces, and normal forces in three-hinged arches

**[B] ILD for moment at D ($M_D$)**

BM at any given section in the arch
= Beam moment –H*y




Source: Bhavikatti, S. S. (2011). *Structural Analysis –I* (4th ed.).New Delhi: Vikas Publishing House.

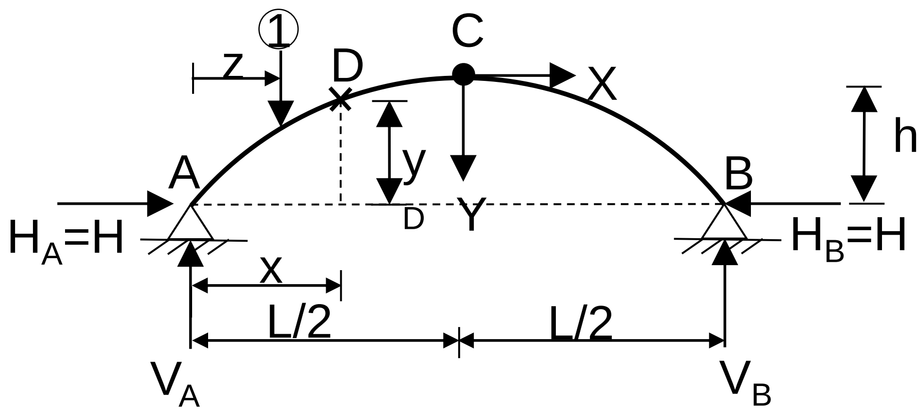
Therefore,
$M_D$=Beam moment at D – H*$y_D$

Hence,
ILD for $M_D$ in arch
= ILD for Beam moment at D – (ILD for H)*$y_D$

# 6.4 ILDs for reactions, bending moments, shearing forces, and normal forces in three-hinged arches

ILD for $M_D$ in arch
= ILD for Beam moment at D
– (ILD for H)*$y_D$

**[B] ILD for moment at D ($M_D$)**

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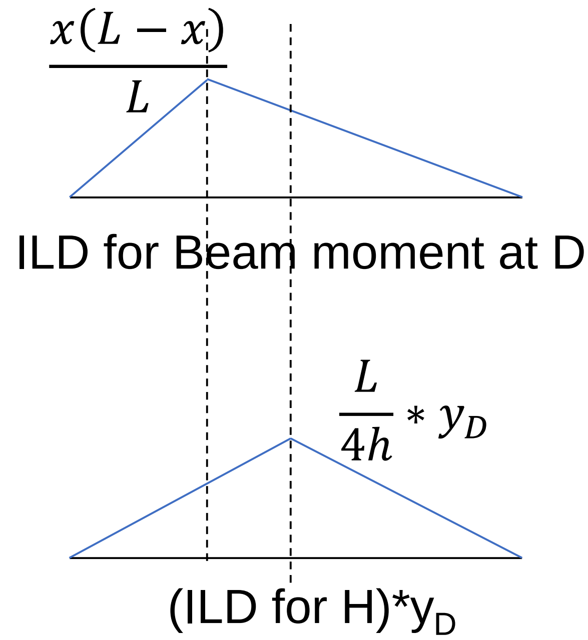


# 6.4 ILDs for reactions, bending moments, shearing forces, and normal forces in three-hinged arches

ILD for $M_D$ in arch
= ILD for Beam moment at D
– (ILD for H)*$y_D$

**[B] ILD for moment at D ($M_D$)**

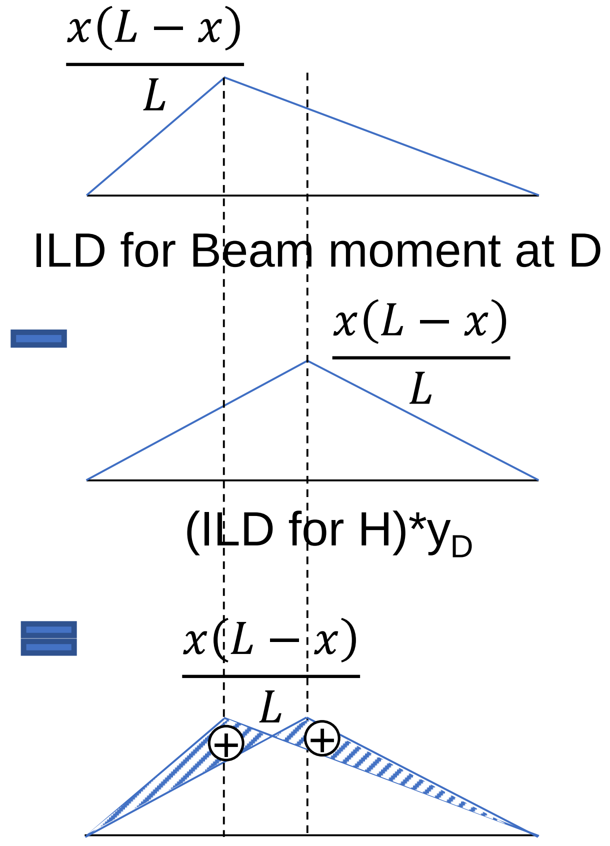
In case of a parabolic arch,

$$y_D = \frac{4hx(L-x)}{L^2}$$

$$\Rightarrow \frac{L}{4h} * y_D = \frac{L}{4h} * \frac{4hx(L-x)}{L^2} = \frac{x(L-x)}{L}$$

Which is the same as the maximum ordinate for ILD for beam moment at D.

Hence, we get ILD for $M_D$ in arch is:




ILD for $M_D$

# 6.4 ILDs for reactions, bending moments, shearing forces, and normal forces in three-hinged arches

**[C] ILD for normal thrust at D ($N_D$)**

We have,

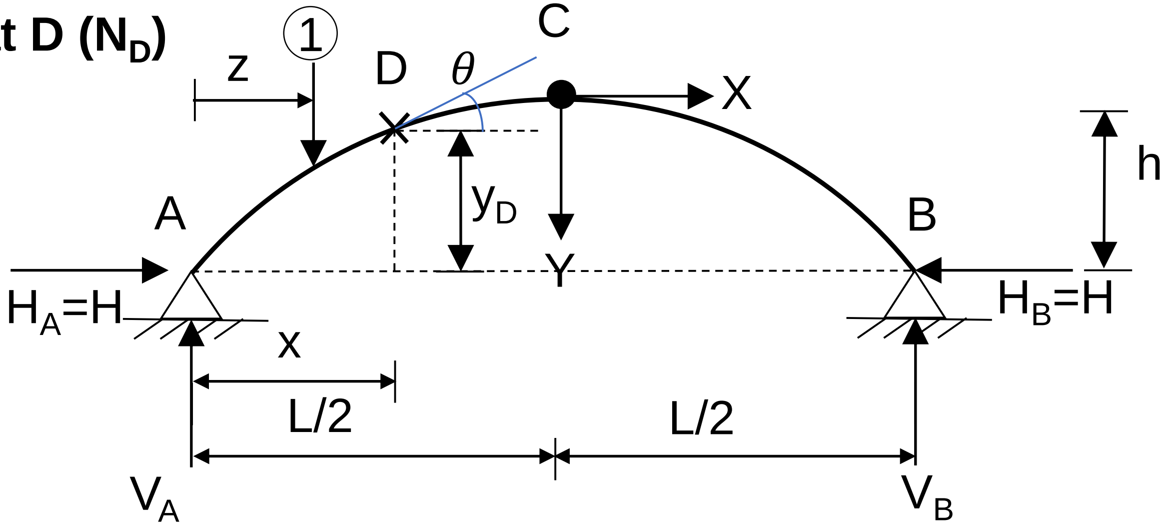
$$N_D = V sin\theta + H cos\theta$$

Where,

$\theta$ = slope of the arch with the horizontal at section D

$V$ = vertical shear at D

$H$ = horizontal thrust




Source: Bhavikatti, S. S. (2011). *Structural Analysis –I* (4th ed.).New Delhi: Vikas Publishing House.

# 6.4 ILDs for reactions, bending moments, shearing forces, and normal forces in three-hinged arches

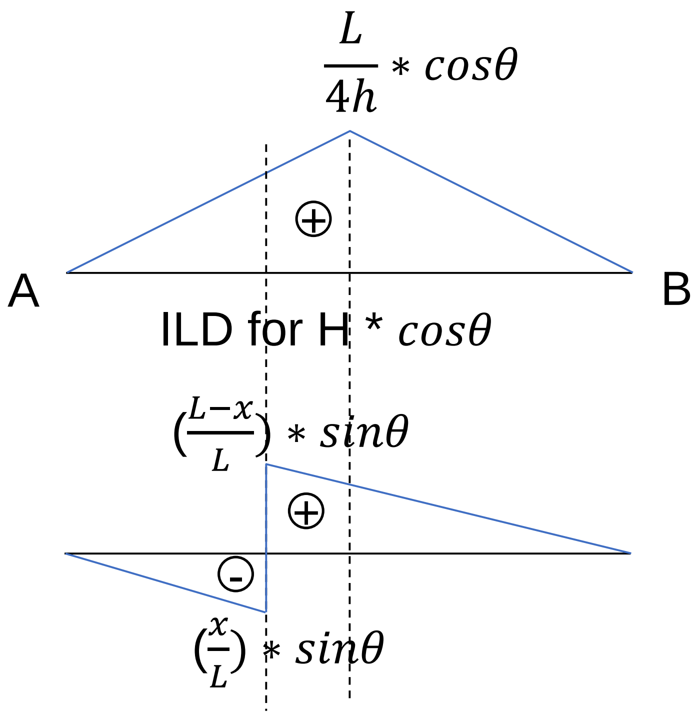
**[C] ILD for normal thrust at D ($N_D$)**

$$N_D = Vsin\theta + Hcos\theta$$

Here,
ILD for $Hcos\theta$ = ILD for H*$cos\theta$

And,

ILD for $Vsin\theta$ = ILD for V for equivalent beam * $sin\theta$




ILD for V * $sin\theta$

Source: Bhavikatti, S. S. (2011). *Structural Analysis –I* (4th ed.).New Delhi: Vikas Publishing House.

# 6.4 ILDs for reactions, bending moments, shearing forces, and normal forces in three-hinged arches

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$$N_D = Vsin\theta + Hcos\theta$$

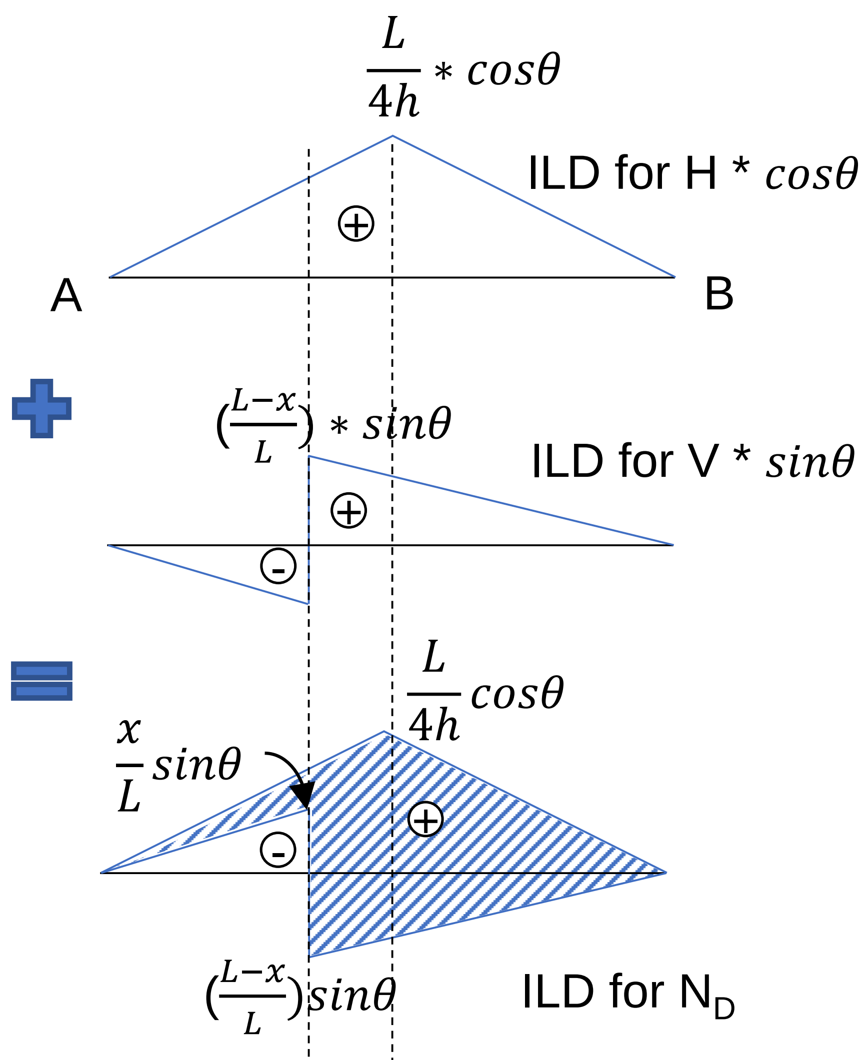
Here,
ILD for $Hcos\theta$ = ILD for H*$cos\theta$

And,

ILD for $Vsin\theta$ = ILD for V for equivalent beam * $sin\theta$

Hence, we get

ILD for $N_D$:




Source: Bhavikatti, S. S. (2011). *Structural Analysis –I* (4th ed.).New Delhi: Vikas Publishing House.

# 6.4 ILDs for reactions, bending moments, shearing forces, and normal forces in three-hinged arches

**[D] ILD for radial shear at D ($Q_D$)**

We have,

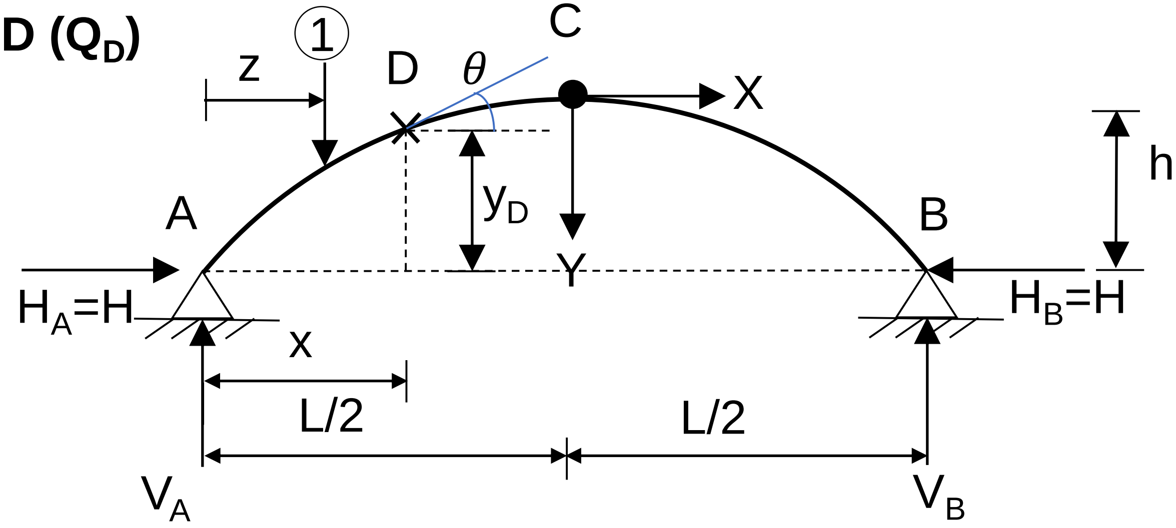
$$Q_D = V cos\theta - H sin\theta$$

Where,

$\theta$ = slope of the arch with the horizontal at section D

$V$ = vertical shear at D

$H$ = horizontal thrust

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# 6.4 ILDs for reactions, bending moments, shearing forces, and normal forces in three-hinged arches

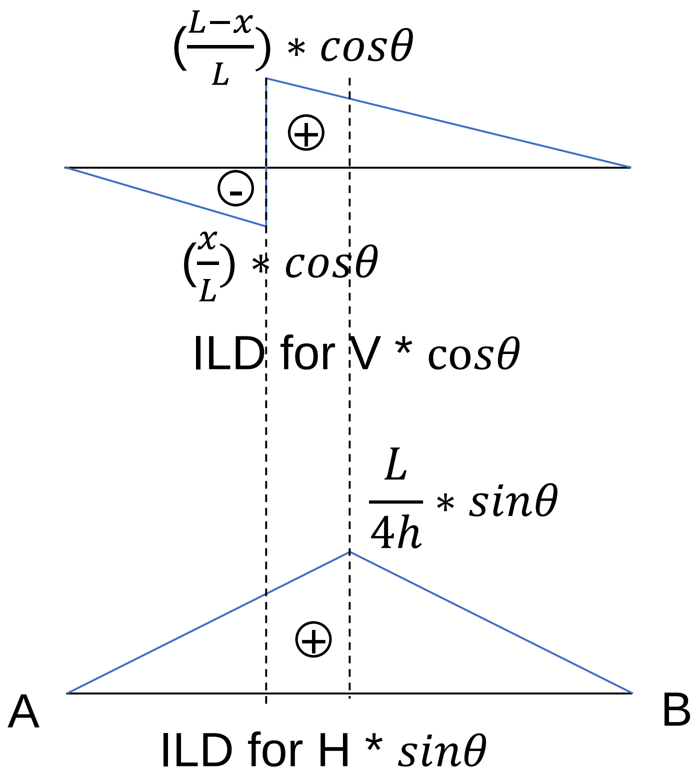
**[D] ILD for radial shear at D ($Q_D$)**

$$Q_D = V cos\theta - H sin\theta$$

Here,
ILD for $V cos\theta$ = ILD for V for equivalent beam *$cos\theta$

And,

ILD for $H sin\theta$ =ILD for H*$sin\theta$

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# 6.4 ILDs for reactions, bending moments, shearing forces, and normal forces in three-hinged arches

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Here,
ILD for $Vcos\theta$ = ILD for V for equivalent beam *$cos\theta$

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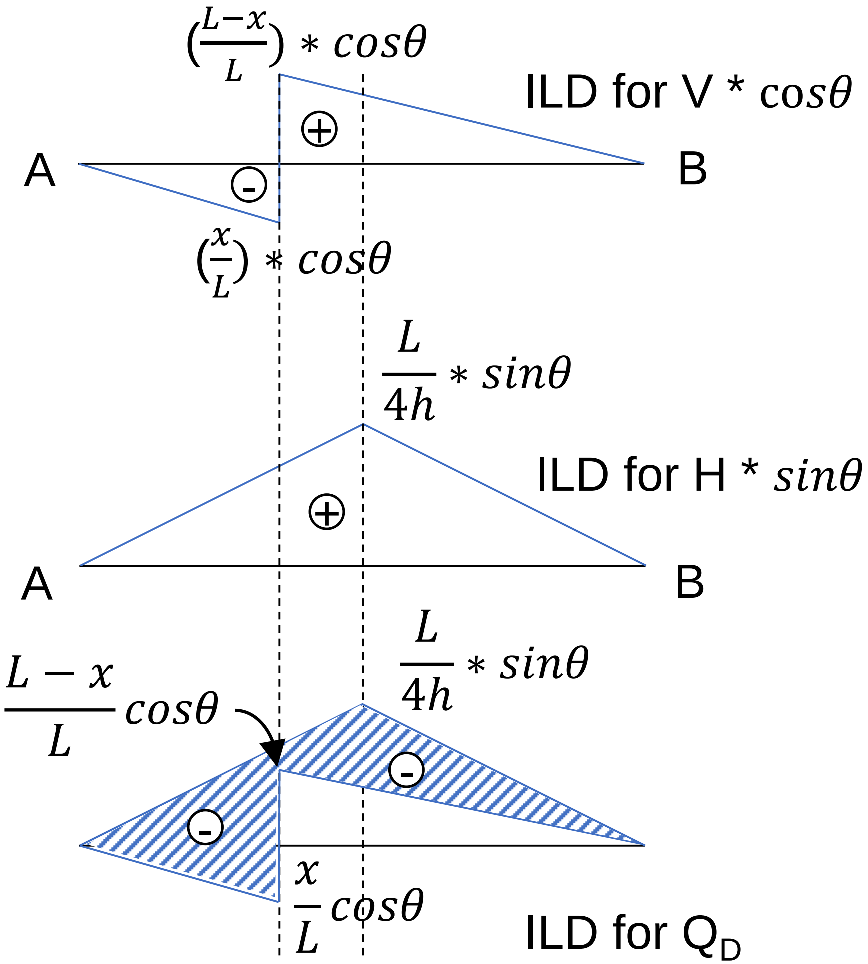
Hence, we get

ILD for $Q_D$:




Source: Bhavikatti, S. S. (2011). *Structural Analysis –I* (4th ed.).New Delhi: Vikas Publishing House.

------------End of Part II of II for Chapter 6-------------

# Chapter 7. Suspension Cable Systems [Part I of II]

## Contents

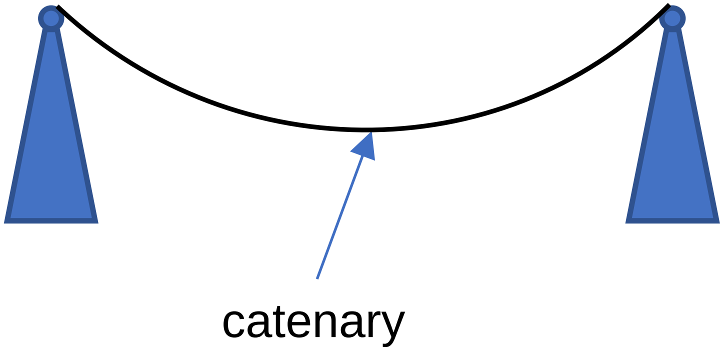


} Part-I

# 7.1 Catenary and Parabolic Cables

**Catenary**

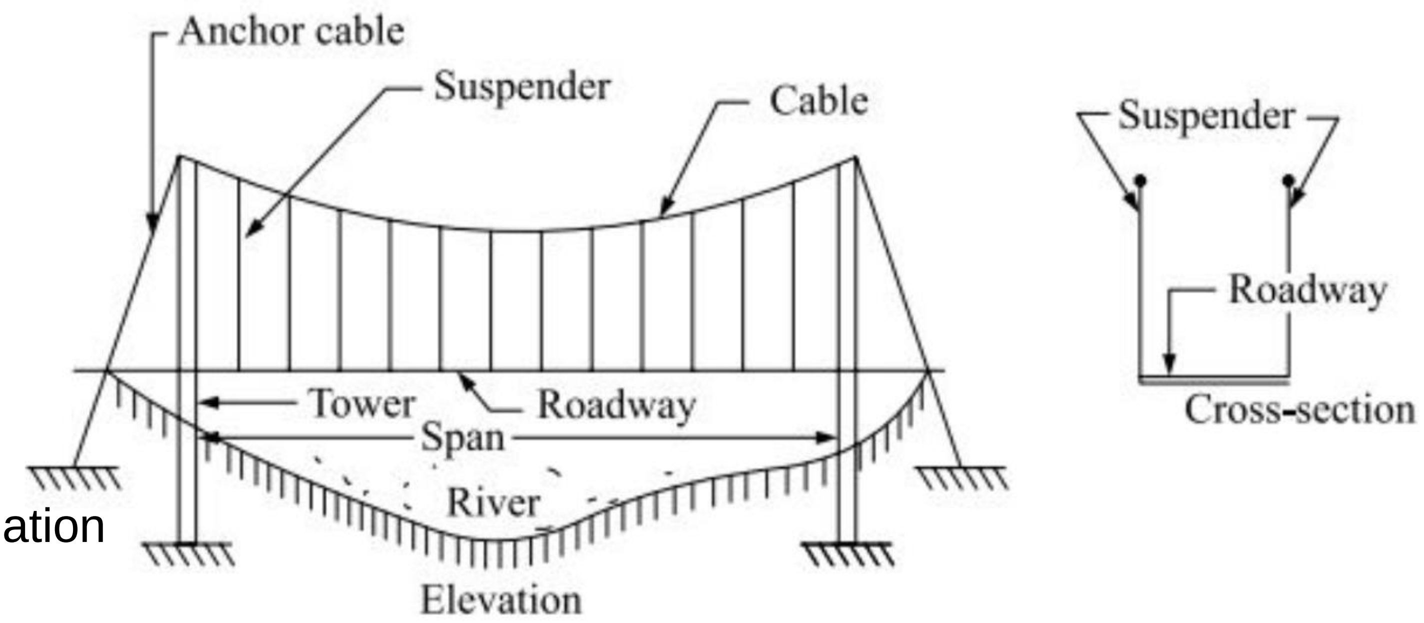
It is the curve that an idealized hanging chain or cable assumes under its own weight when supported only at its ends in a uniform gravitational field.




When a suspension bridge is being built and the main cables are attached to the towers, the curve is a catenary. But, when the cables are attached to the deck with hangers, it is no longer a catenary. The curves of the cables then become the curve of a parabola.

Source: Bhavikatti, S. S. (2011). *Structural Analysis –I* (4th ed.).New Delhi: Vikas Publishing House.

# 7.2 Elements of a simple suspension bridge



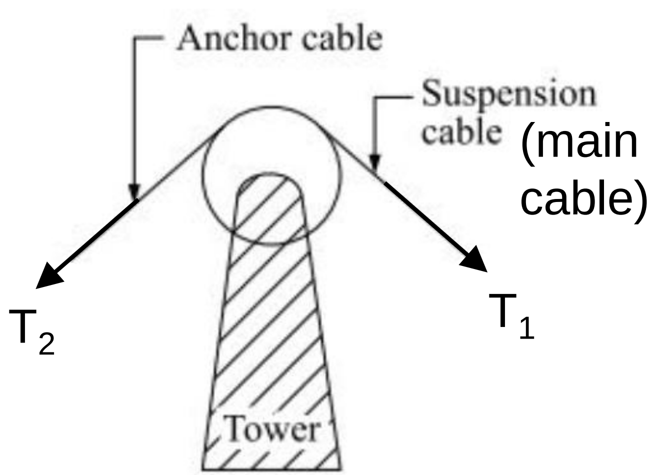

Foundation

Source: Bhavikatti, S. S. (2011). *Structural Analysis –I* (4th ed.).New Delhi: Vikas Publishing House.

- A suspension bridge consists of two cables with a number of suspenders (hangers), which support the roadway.
- The main cables pass over the supporting towers are anchored by back stays (anchor cables) to a firm foundation.

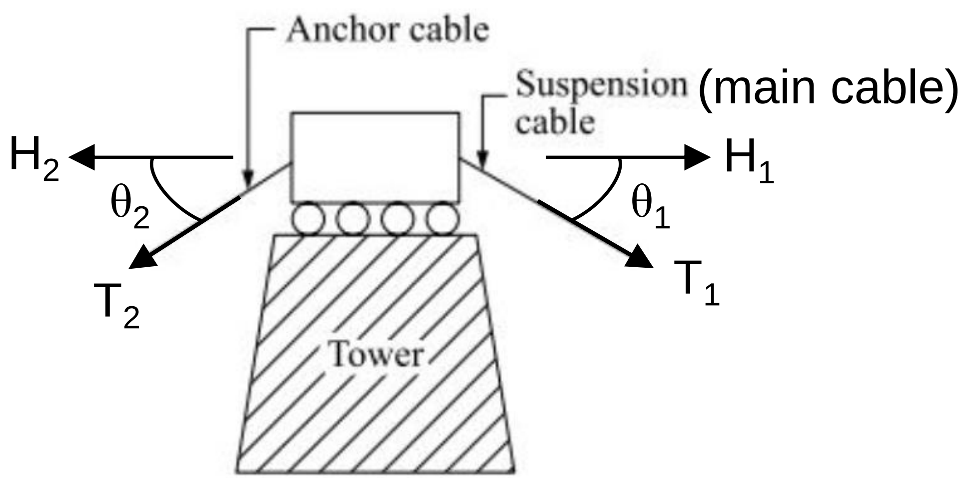
# 7.2 Elements of a simple suspension bridge

- The cables will pass over the supporting towers either over a frictionless pulley or a saddle supported on frictionless rollers, giving freedom for horizontal movement.




Guided pulley support/
Frictionless pulley

$T_1 = T_2$



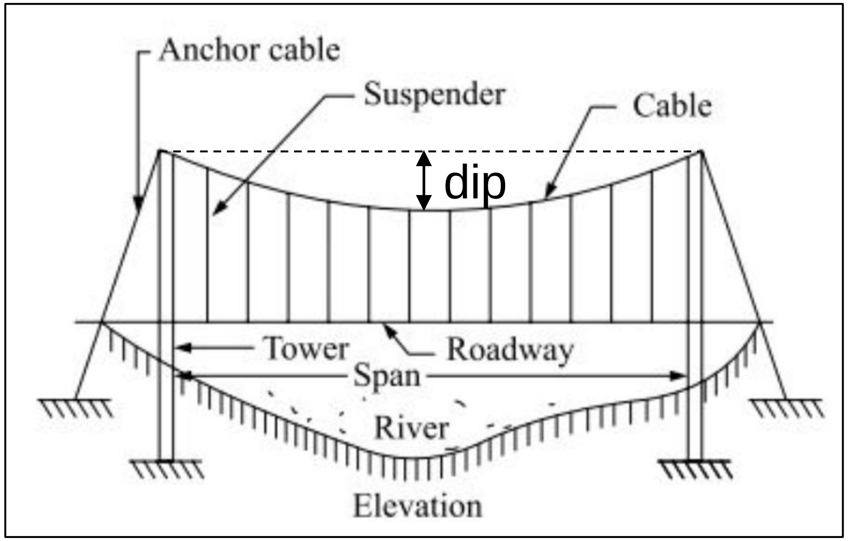

Roller pulley support/
Saddle on rollers

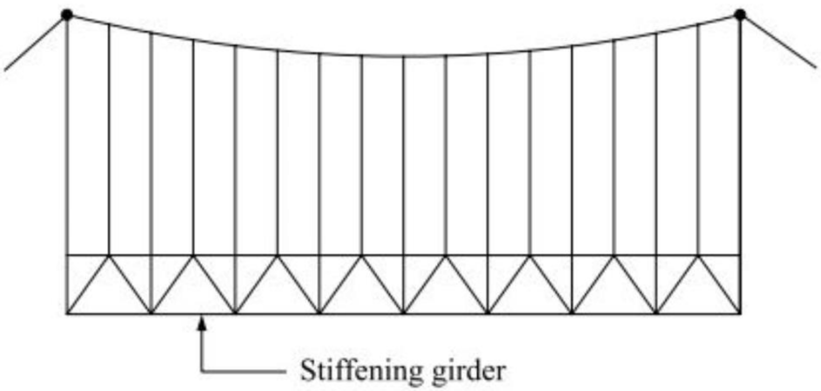
$T_1 \neq T_2$ ; $H_1 = H_2$
➔ $T_1 \cos\theta_1 = T_2 \cos\theta_2$

Source: Bhavikatti, S. S. (2011). *Structural Analysis –I* (4th ed.).New Delhi: Vikas Publishing House.
Reddy, C.S. (2011). *Basic Structural Analysis (3rd ed.).* New Delhi: Tata McGraw Hill.

# 7.2 Elements of a simple suspension bridge

- The central sag or dip of the cable varies from (1/10)th to (1/15)th of the span.
- For heavy traffic and large spans, stiffening girders are provided to support the roadway.

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# 7.2 Elements of a simple suspension bridge

- Since the number of suspenders are very large, the load on the cable can be taken as a Uniformly Distributed Load (UDL), for which the cable assumes the shape of a parabola, similar to a Bending Moment Diagram (BMD) of a Simply Supported Beam (SSB) under UDL.

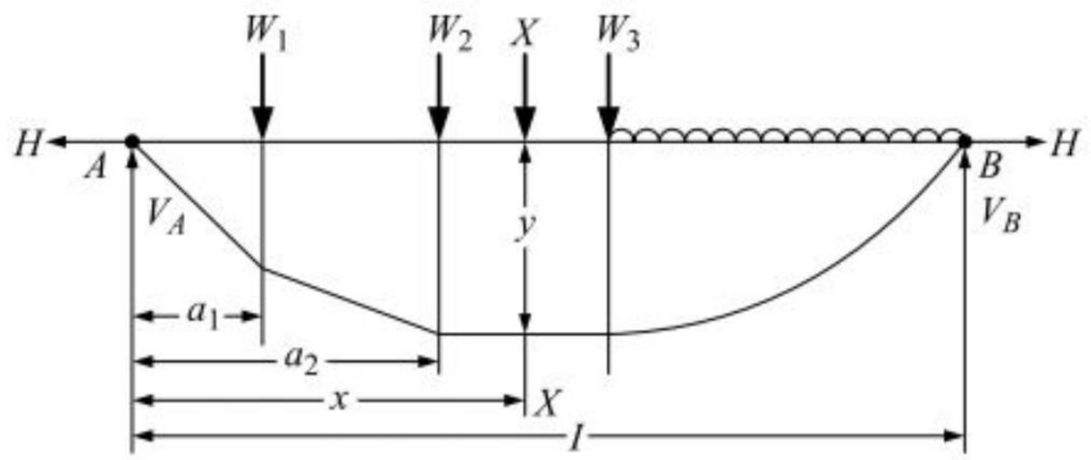
**[A] Cable:**

- flexible structural element
- cannot resist any bending moment
- resist load only by tension
- adjust their shape to the loads such that the bending moment is zero at any point (which is achieved by developing horizontal thrust at the support and thus developing appropriate deflection)

Source: Bhavikatti, S. S. (2011). *Structural Analysis –I* (4$^{th}$ ed.).New Delhi: Vikas Publishing House.

# 7.2 Elements of a simple suspension bridge

**[A] Cable**
**[A.1] Equation of the cable/General cable theorem**



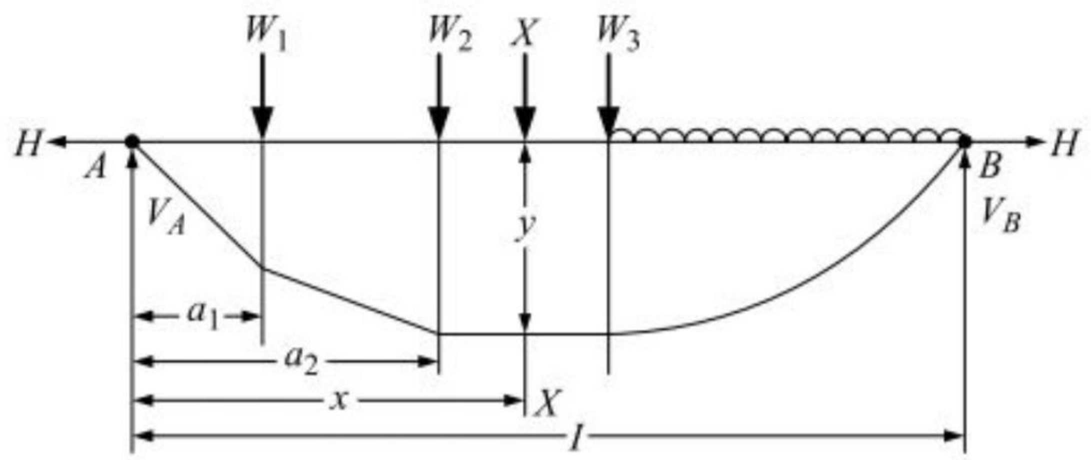

Source: Bhavikatti, S. S. (2011). *Structural Analysis –I* (4th ed.).New Delhi: Vikas Publishing House.

Let us consider a cable as shown in the figure, subjected to a number of loads $W_1$, $W_2$, $W_3$,… at a distance $a_1$, $a_2$, $a_3$, …from support A. Let the horizontal force developed be H and $V_A$ and $V_B$ be the vertical reactions at supports A and B.

At a section X-X, at a distance x from support A, let the deflection be y.

# 7.2 Elements of a simple suspension bridge

**[A] Cable**
**[A.1] Equation of the cable/General cable theorem**




Source: Bhavikatti, S. S. (2011). *Structural Analysis –I* (4th ed.).New Delhi: Vikas Publishing House.

Then, the bending moment at section X-X is:

$$M_x = V_A * x - W_1 * (x - a_1) - W_2 * (x - a_2) - H * y$$

Since the cable is flexible, $M_x = 0$

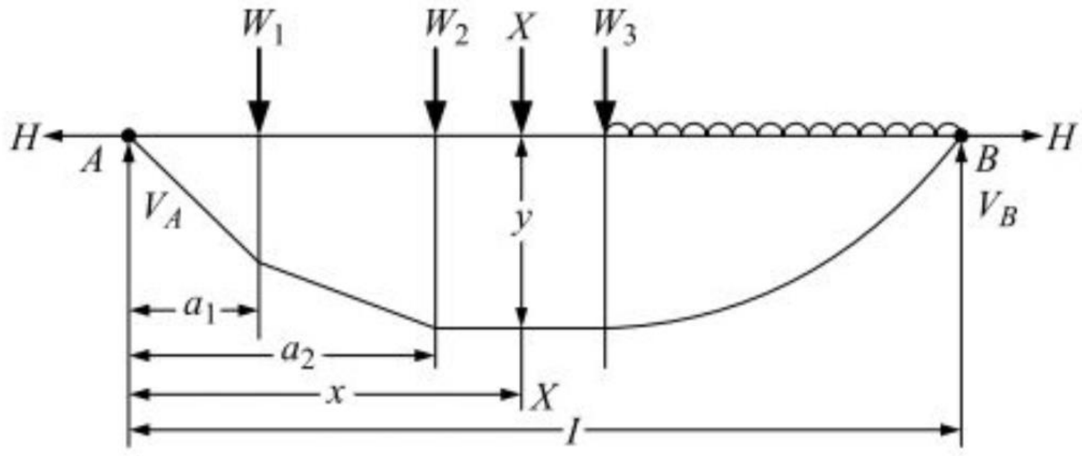
$\therefore Hy = V_A * x - W_1 * (x - a_1) - W_2 * (x - a_2)$ ⇨ Beam moment

# 7.2 Elements of a simple suspension bridge

**[A] Cable**
**[A.1] Equation of the cable/General cable theorem**

$$\therefore Hy = Beam\ moment$$

Hence, "at any point on a cable acted upon by vertical loads, the product of the horizontal force developed, H and its vertical distance to the cable chord equals the bending moment that would occur at that section if the loads carried by the cable were acting on an end support beam of the same span as that of the cable."

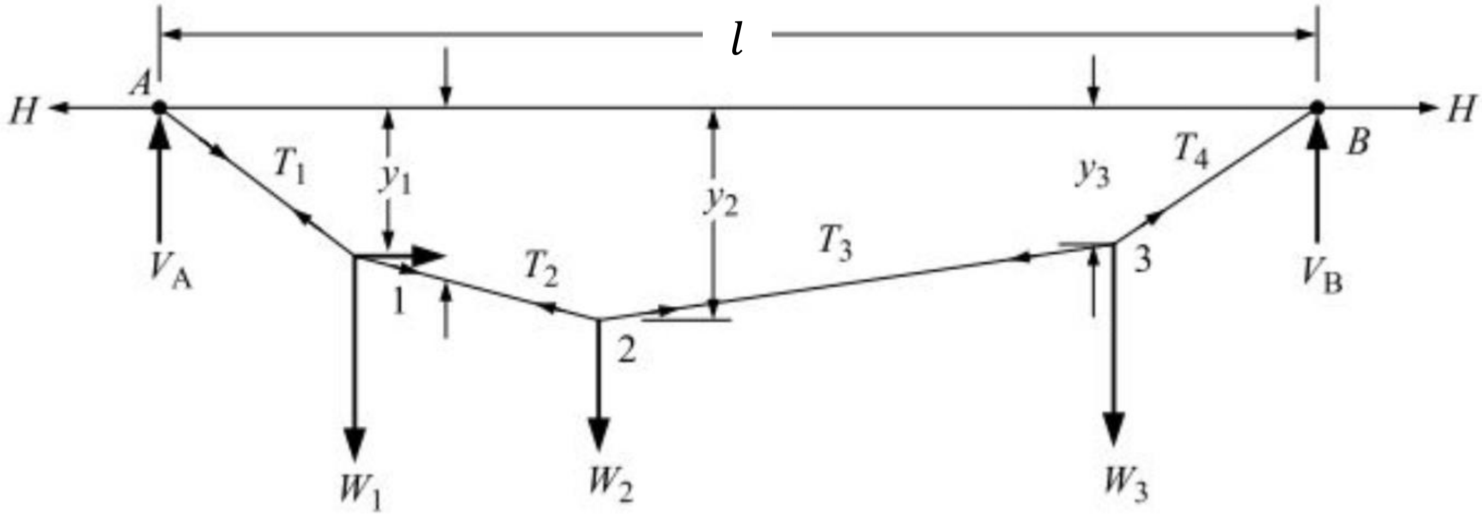



Source: Bhavikatti, S. S. (2011). *Structural Analysis –I* (4th ed.).New Delhi: Vikas Publishing House.

Therefore, considering any segment of the cable and using the above equation, a loaded cable can be analyzed.

# 7.2 Elements of a simple suspension bridge

**[A] Cable**
**[A.2] Cable subjected to concentrated loads**




Source: Bhavikatti, S. S. (2011). *Structural Analysis –I* (4th ed.).New Delhi: Vikas Publishing House.

Let us consider a cable of length L, spanning over a horizontal gap $l$, subjected to the concentrated loads $W_1$, $W_2$, and $W_3$ at points 1, 2, and 3, respectively.

Let $V_A$ and $V_B$ be the vertical reactions and H be the horizontal reaction at the supports. Then,

# 7.2 Elements of a simple suspension bridge

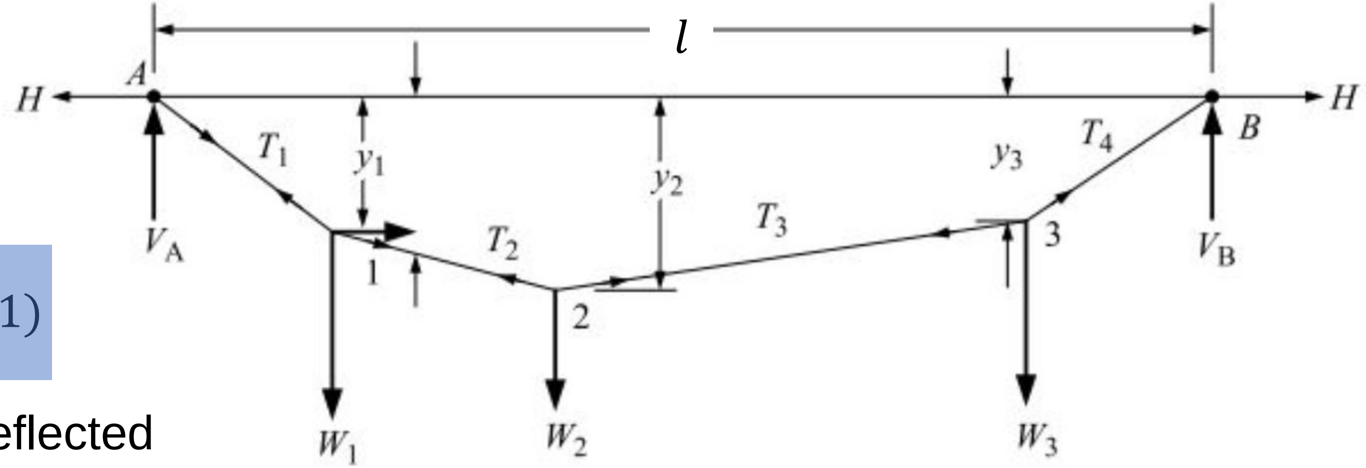
**[A] Cable**
**[A.2] Cable subjected to concentrated loads**

We have,

$$Hy = M_{beam}$$

$$y = \frac{M_{beam}}{H} - - (1)$$

Hence, the deflected shape (given by the values of y) is similar to the moment diagram of the beam.




Source: Bhavikatti, S. S. (2011). *Structural Analysis –I* (4th ed.).New Delhi: Vikas Publishing House.

Now, if $M_1$, $M_2$, and $M_3$ are the beam moments at load points 1, 2, and 3, the deflections $y_1$, $y_2$, and $y_3$ are given by:

$$y_1 = \frac{M_1}{H};\ y_2 = \frac{M_2}{H};\ y_3 = \frac{M_3}{H} - - - (2)$$

# 7.2 Elements of a simple suspension bridge

**[A] Cable**
**[A.2] Cable subjected to concentrated loads**

Hence, if the horizontal thrust is known or position of the cable at any point is known, then the deflections at all the points can be calculated and the deflected shape of the cable can be found.

$$\therefore Actual\ length\ of\ cable = \Sigma\ Length\ of\ each\ segment$$

After finding deflections, slope of the various segments can be found, and using the equations of equilibrium of load points 1, 2, and 3, the forces in various segments of the cable can be found.

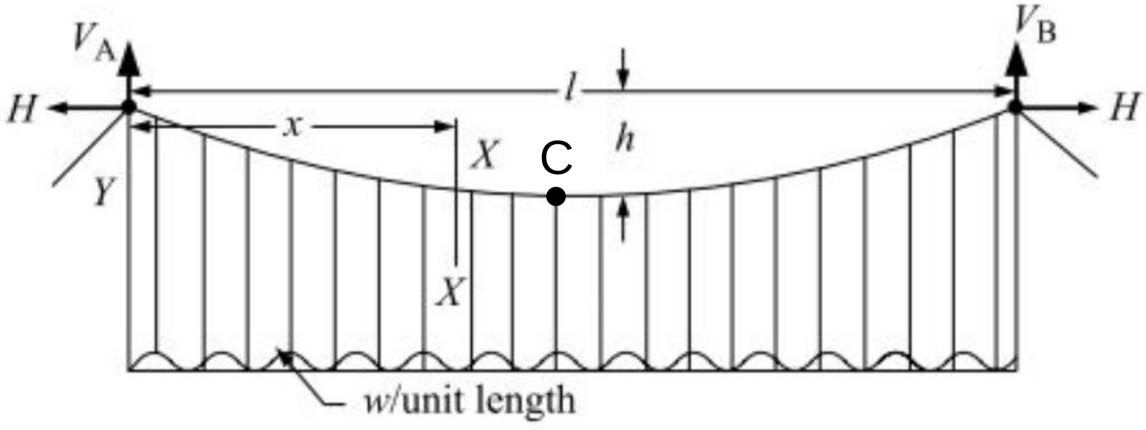
Source: Bhavikatti, S. S. (2011). *Structural Analysis –I* (4th ed.).New Delhi: Vikas Publishing House.

# 7.2 Elements of a simple suspension bridge

**[A] Cable**

**[A.3] Cable subjected to a UDL**

Let a cable of length L be supported at points A and B, which are at a horizontal distance $l$ and are the same level. The cable is subjected to a udl w/unit length.




Source: Bhavikatti, S. S. (2011). *Structural Analysis –I* (4th ed.).New Delhi: Vikas Publishing House.

Then,

$$V_A = V_B = \frac{wl}{2}$$

Taking moment about central point,

$$\Sigma M_{C\,(left)} = 0$$

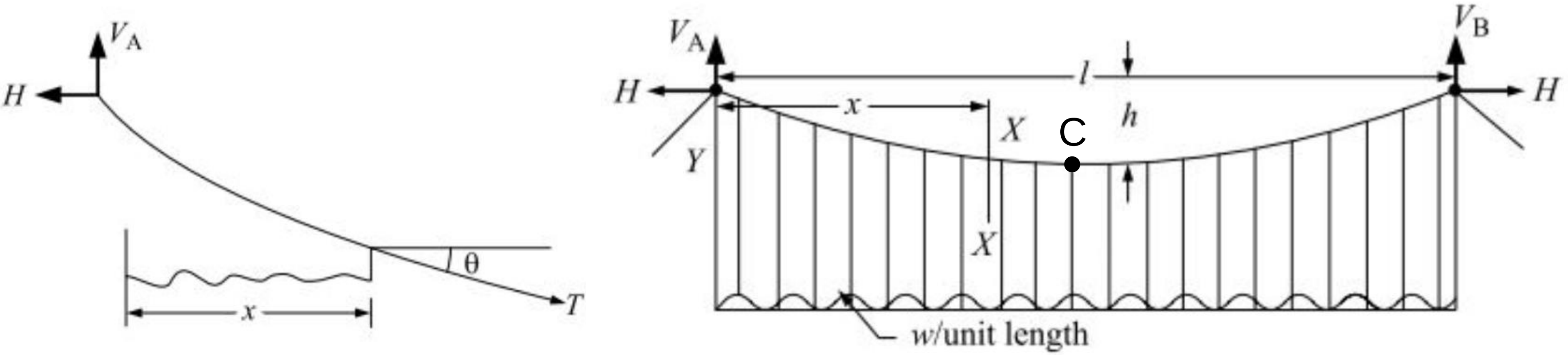
$$Or, -V_A * \frac{l}{2} + H * h + \frac{wl}{2} * \frac{l}{4} = 0$$

$$Or, Hh = \frac{wl}{2} * \frac{l}{2} - \frac{wl^2}{8}$$

$$\therefore H = \frac{wl^2}{8h}$$

# 7.2 Elements of a simple suspension bridge

**[A] Cable**
**[A.3] Cable subjected to a UDL**




Source: Bhavikatti, S. S. (2011). *Structural Analysis –I* (4th ed.).New Delhi: Vikas Publishing House.

If V is the shear at any section X-X at a distance x from A, then

$$T = \sqrt{V^2 + H^2}$$

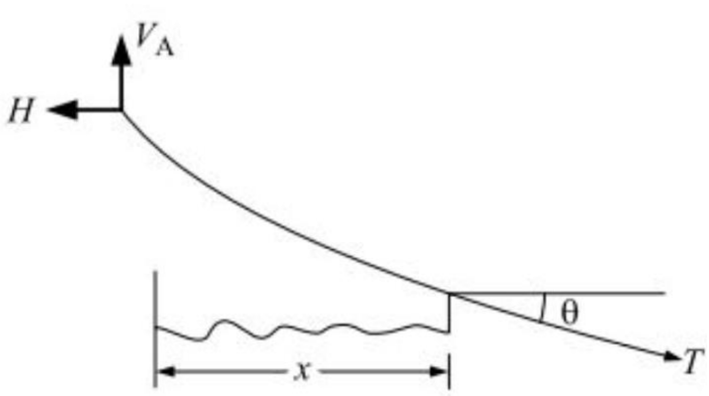
We know,

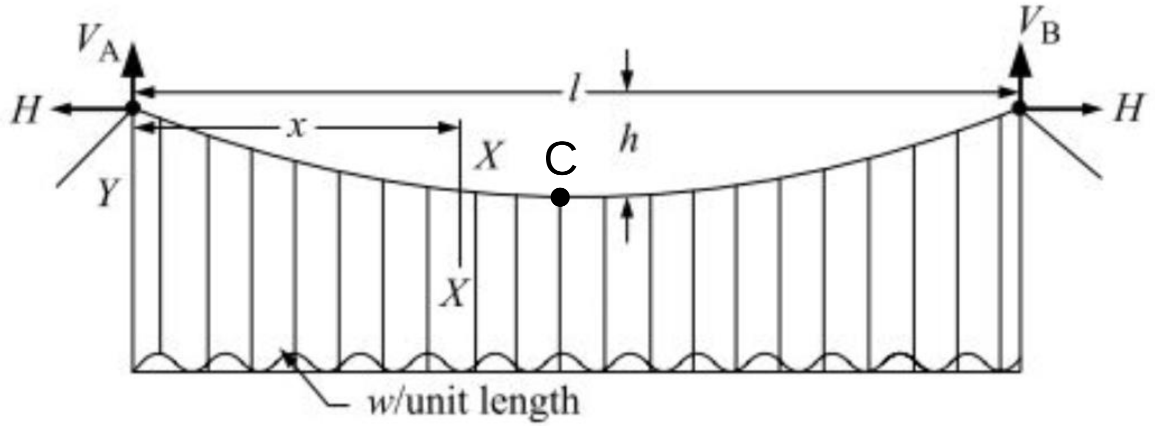
$$V_{max} = \frac{wl}{2} (at\ the\ supports)$$

$$\therefore T_{max} = \sqrt{(\frac{wl}{2})^2 + (\frac{wl^2}{8h})^2} = \frac{wl}{2}\sqrt{1 + \frac{l^2}{16h^2}}$$

# 7.2 Elements of a simple suspension bridge

**[A] Cable**
**[A.3] Cable subjected to a UDL**




Source: Bhavikatti, S. S. (2011). *Structural Analysis –I* (4th ed.).New Delhi: Vikas Publishing House.

Next,

$$V_{min} = 0 \; (at\ the\ center)$$

$$\therefore T_{min} = \sqrt{V^2 + H^2} = \sqrt{(0)^2 + H^2} = H$$

At any point,

$$V = T sin\theta$$

------------End of Lecture #11-----------------

------------End of Part I of II for Chapter 7-------------

# References

[1] Bhavikatti, S. S. (2011). *Structural Analysis –I* (4th ed.).New Delhi: Vikas Publishing House.
[2] Reddy, C.S. (2011). *Basic Structural Analysis (3rd ed.).* New Delhi: Tata McGraw Hill.