Theory of Structures - I

Chapter 7. Suspension Cable Systems [Part II of II]

Lecturer: Dr. Sanjeema Bajracharya

Contents

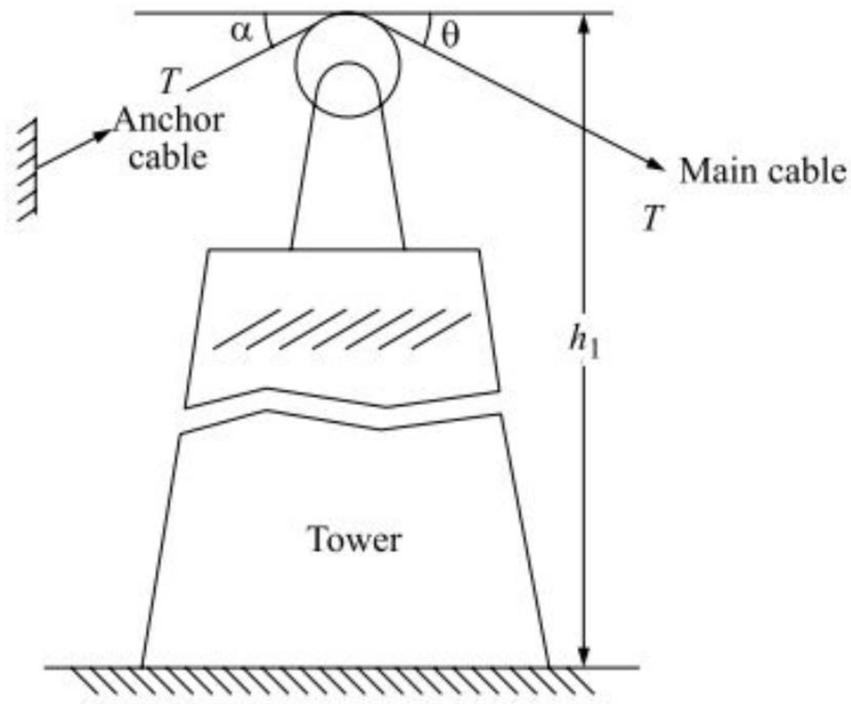
7.2 Elements of a simple suspension bridge

[B] Support

The forces on the anchor cables and trusses depend upon the type of support given to the cables.

[B.1] Guided pulley support

Let the inclination of the main cable to the horizontal be θ, and that of anchor cable be α.
Assuming the pulley as frictionless,
Tension in the anchor cable
= Tension in the main cable
= T



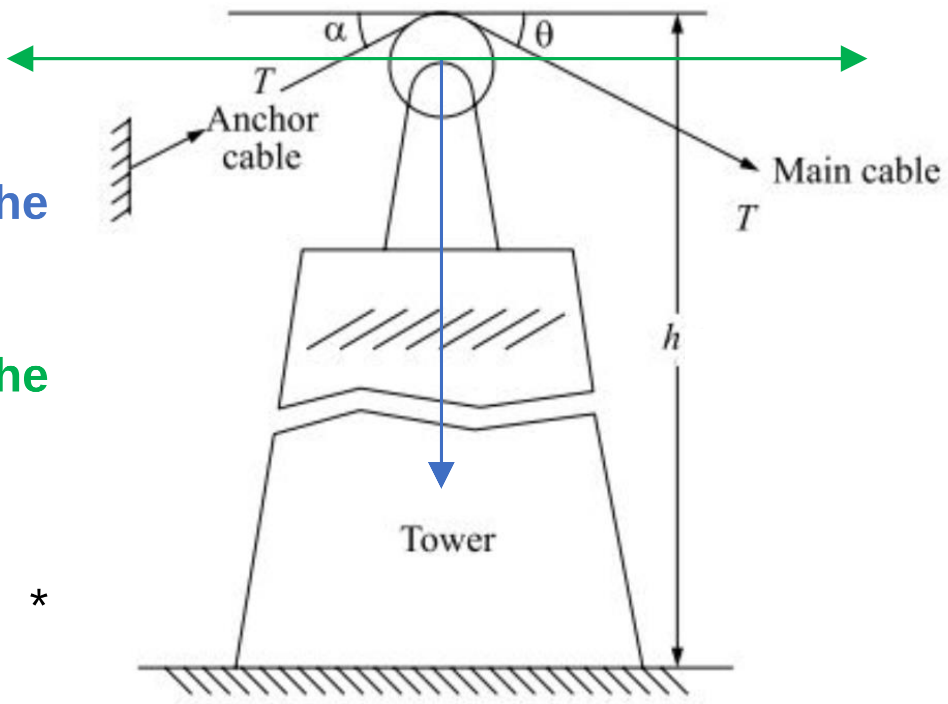
Source: Bhavikatti, S. S. (2011). *Structural Analysis –I* (4th ed.).New Delhi: Vikas Publishing House.

7.2 Elements of a simple suspension bridge

[B] Support

The forces on the anchor cables and trusses depend upon the type of support given to the cables.

[B.1] Guided pulley support



Source: Bhavikatti, S. S. (2011). *Structural Analysis –I* (4th ed.).New Delhi: Vikas Publishing House.

Then,

Vertical load transmitted to the tower = $Tsin\theta + Tsin\alpha$

Horizontal load transmitted to the tower = $Tcos\theta - Tcos\alpha$

Bending moment on the tower

= Horizontal force on the tower * height of the tower

= $(Tcos\theta - Tcos\alpha) * \mathrm{h}$

7.2 Elements of a simple suspension bridge

[B] Support

[B.2] Roller support

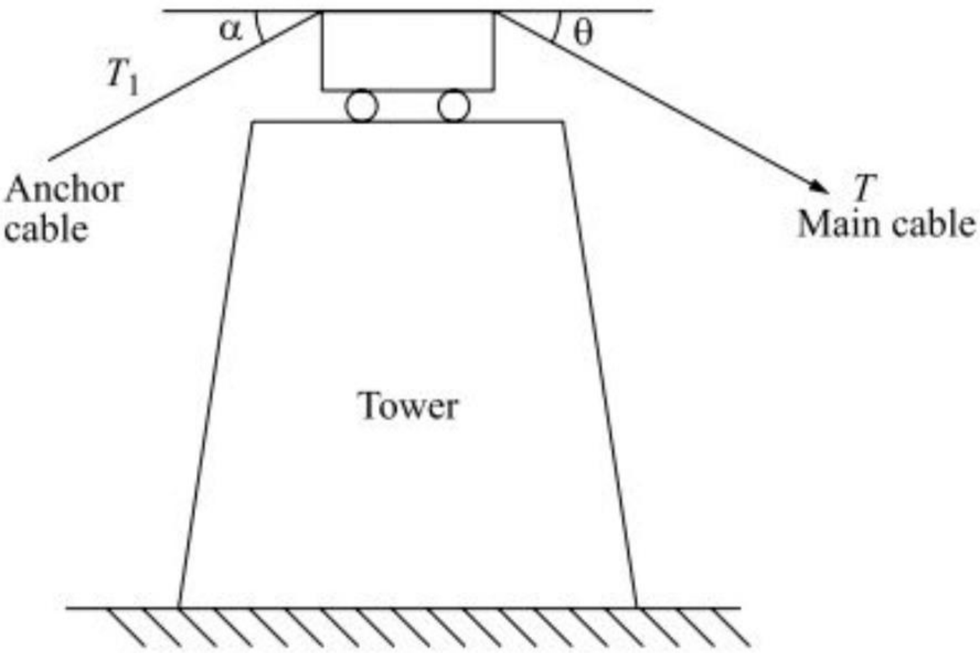
In this case, the anchor cable and main cable don't have the same tension.

Let T be the tension in the main cable and T_1 be the tension in the anchor cable.

Assuming the saddle to have frictionless rollers,

$$T_1 cos\alpha = T cos\theta$$

$$T_1 = T(\frac{cos\theta}{cos\alpha})$$



Source: Bhavikatti, S. S. (2011). *Structural Analysis –I* (4th ed.).New Delhi: Vikas Publishing House.

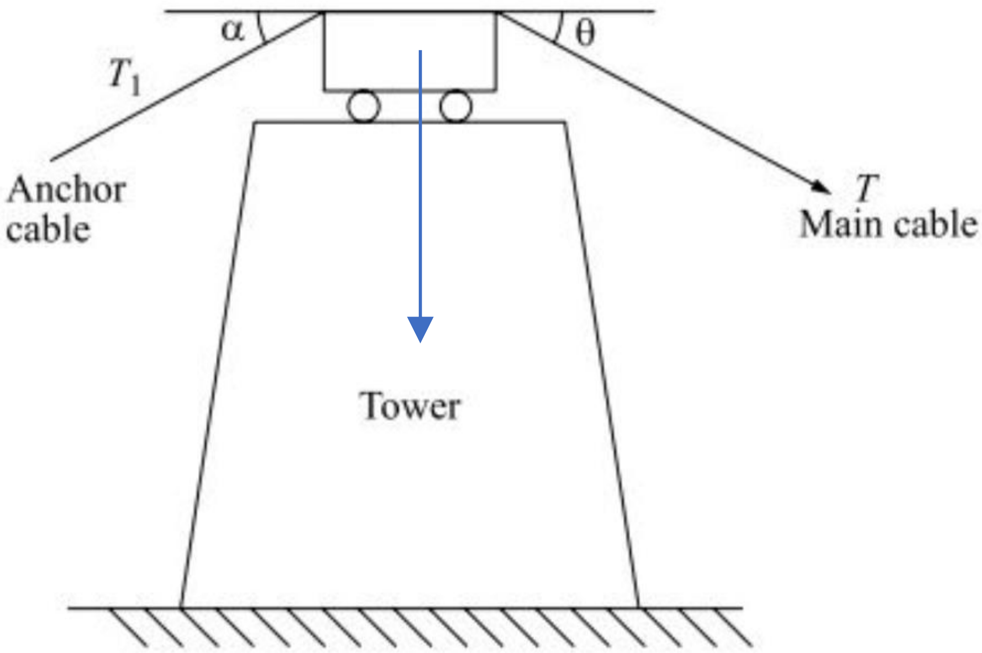
7.2 Elements of a simple suspension bridge

[B] Support

[B.2] Roller support

Then,
Since the saddle has frictionless rollers, there is **no horizontal force**, and hence, **no bending moment** on the tower.

Vertical load transmitted to the tower = $T sin\theta + T_1 sin\alpha$



Source: Bhavikatti, S. S. (2011). *Structural Analysis –I* (4th ed.).New Delhi: Vikas Publishing House.

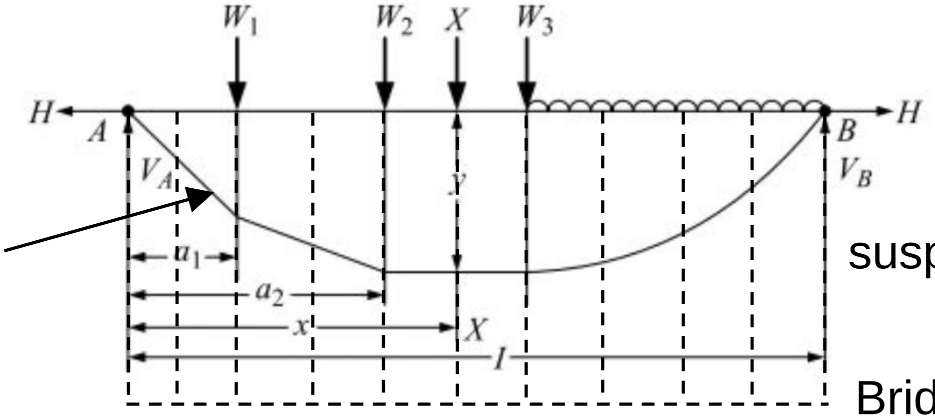
7.3 Suspension bridge with three-hinged stiffening girder

Stiffening girder:

As we discussed in the week 11 lecture, under a system of loads, the cable takes the shape of a bending moment diagram in a simply supported beam. Now, when rolling loads pass across the bridge, the BMD changes with the position of loads, and hence, the *profile of the cable changes*.

As a result, the *bridge deck suspended from such cables will swing up and down* and thus, *will not be useful*.

changing profile of the cable as the loads move across it.



suspenders

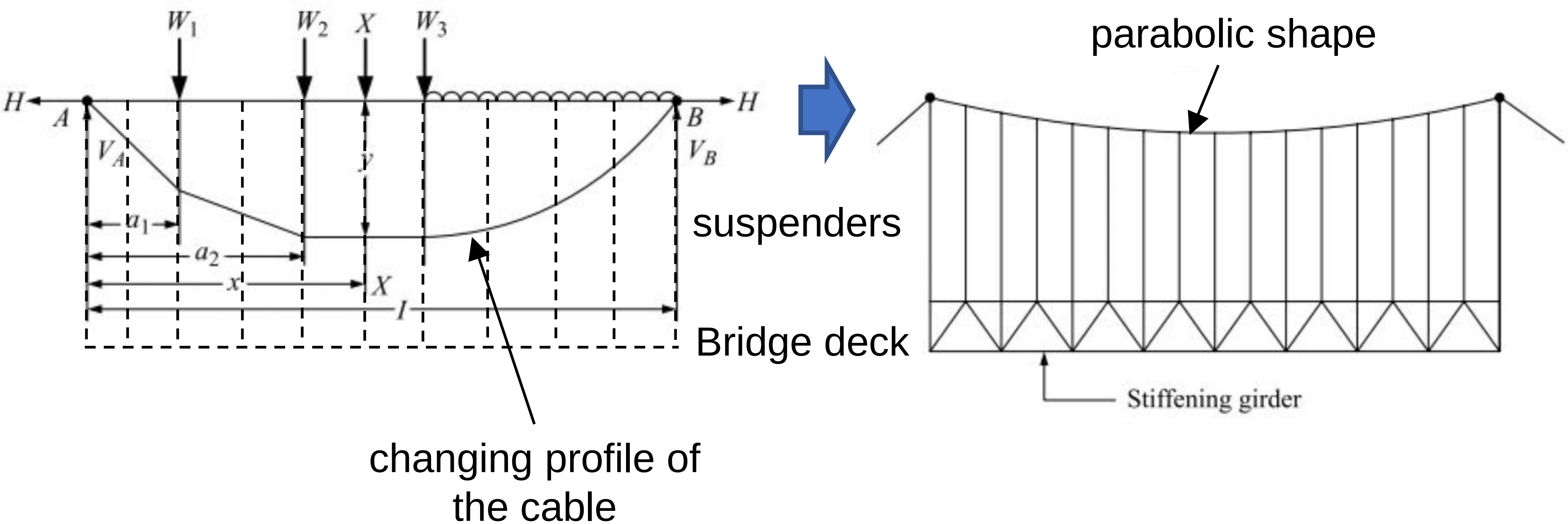
Bridge deck

Source: Reddy, C.S. (2011). *Basic Structural Analysis (3rd ed.).* New Delhi: Tata McGraw Hill.

7.3 Suspension bridge with three-hinged stiffening girder

Stiffening girder:

In order to *make the cable retain its parabolic shape* throughout the passage of loads, it is necessary to transmit the moving loads to the suspension cable as a UDL. This is *achieved by providing stiffening girders* on either side of the roadway, which may be 3-hinged or 2-hinged.



Source: Reddy, C.S. (2011). *Basic Structural Analysis (3rd ed.).* New Delhi: Tata McGraw Hill.

7.3 Suspension bridge with three-hinged stiffening girder

Stiffening girder:

Dead load:

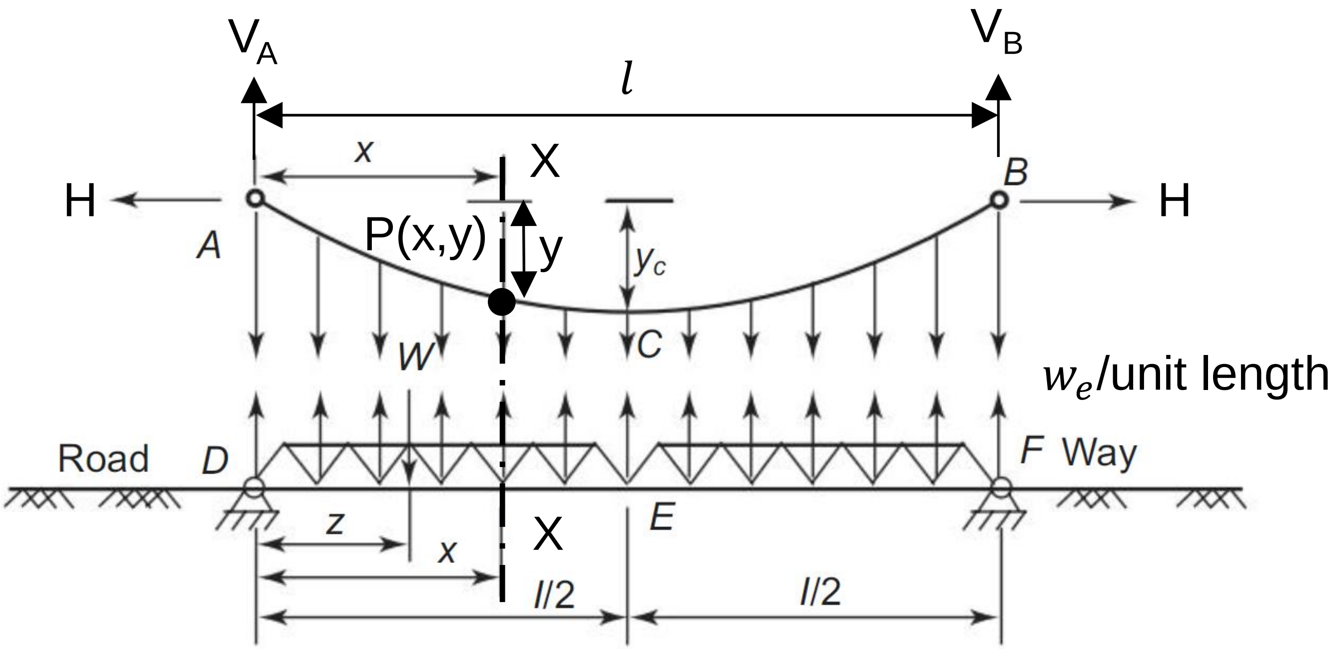
The uniformly distributed dead load of the roadway and stiffening girders is transmitted to the cables through the hanger cables and is taken up entirely by *tension in the cables*. The *stiffening girder*, which is being held all along by the suspenders *will not be subjected to any SF or BM at any section due to the UDL dead loa*d.

Live load:

Any rolling load that crosses the bridge will be transmitted to the cables through the girders and hanger cables as UDL so that the *cables will retain their parabolic shape*, while the *girder will have to resist a certain amount of SF and BM due to rolling loads*.

Source: Reddy, C.S. (2011). *Basic Structural Analysis (3rd ed.).* New Delhi: Tata McGraw Hill.

7.3 Suspension bridge with three-hinged stiffening girder



Source: Reddy, C.S. (2011). *Basic Structural Analysis (3rd ed.).* New Delhi: Tata McGraw Hill.

Let us consider a concentrated load W moving from left to right across the suspension bridge with the stiffening girder. Here, **ACB** is the suspension cable and **DEF** is the 3-hinged stiffening girder.

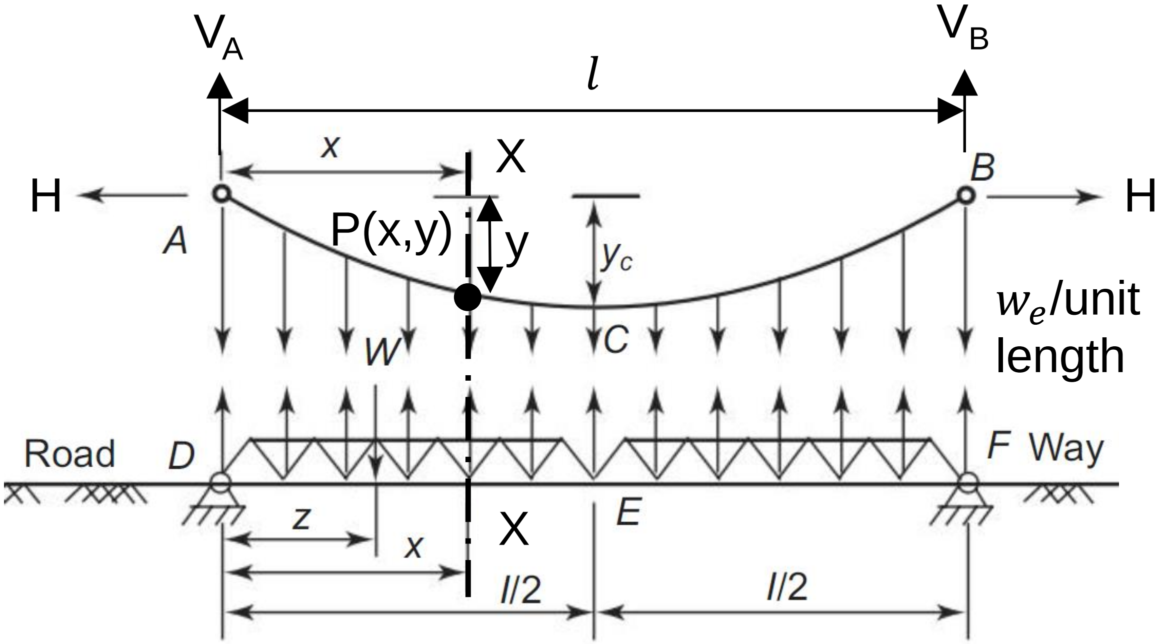
Let the load W be at a distance z from support D at any instant of time, and considering a section X-X at a distance x from A or D.

7.3 Suspension bridge with three-hinged stiffening girder

For the cable,
Taking A as the origin, the equation of the cable profile may be written as:

$$y = \frac{4hx(L-x)}{L^2}$$

Let w_e be the equivalent UDL on the cable.



Source: Reddy, C.S. (2011). *Basic Structural Analysis (3rd ed.).* New Delhi: Tata McGraw Hill.

The vertical reaction at A or B will be:

$$V_A = V_B = \frac{w_e l}{2}$$

The horizontal reaction will be:

$$H = \frac{w_e l^2}{8h}$$

7.3 Suspension bridge with three-hinged stiffening girder

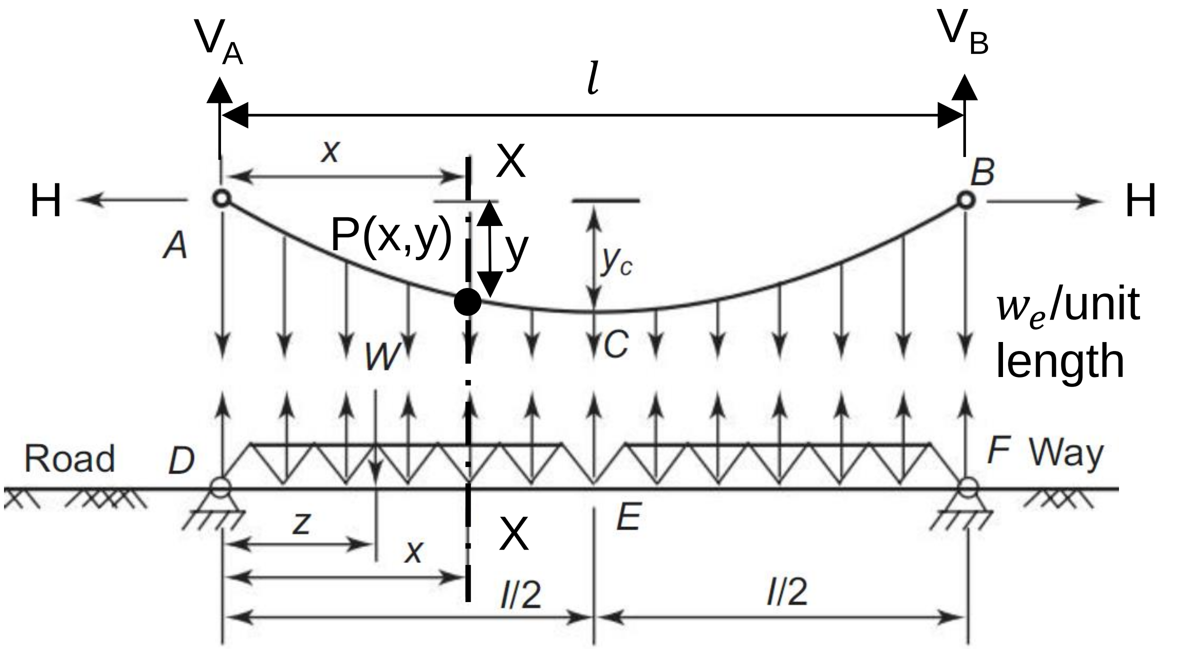
For the cable,

$$V_A = V_B = \frac{w_e l}{2}$$

$$H = \frac{w_e l^2}{8h}$$

The maximum tension in the cable occurs at A or B:

$$T_{max} = \sqrt{V^2 + H^2}$$



Source: Reddy, C.S. (2011). *Basic Structural Analysis (3rd ed.).* New Delhi: Tata McGraw Hill.

7.3 Suspension bridge with three-hinged stiffening girder

For the cable,

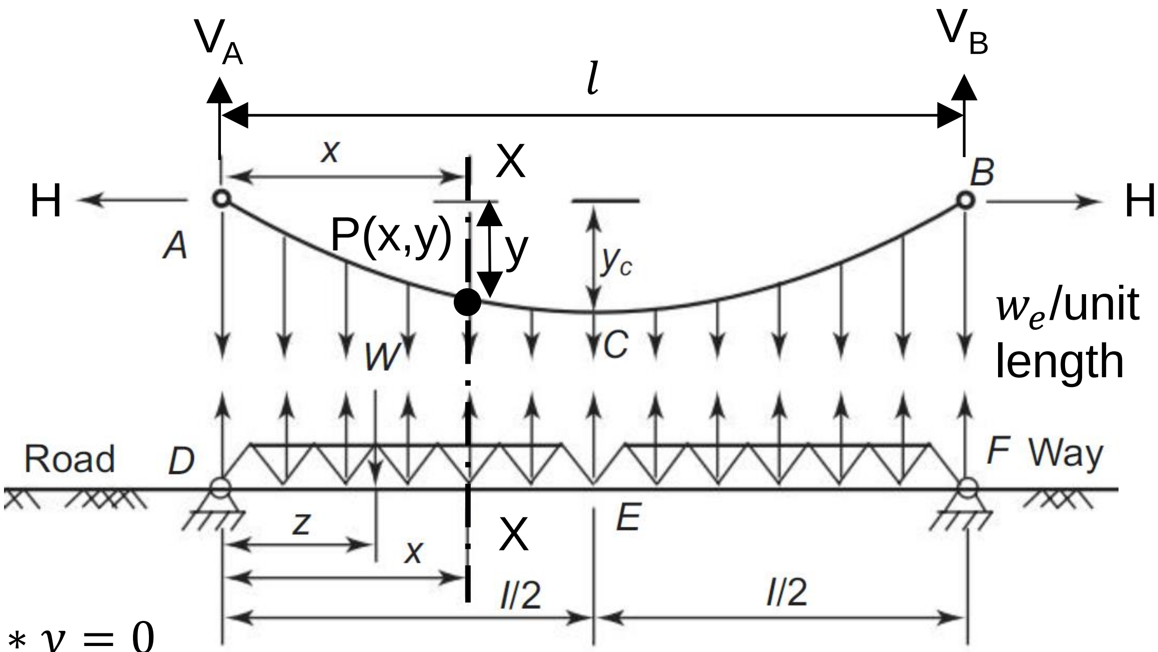
Now, consider a point P(x, y) on the cable and taking moments about P,

$$\Sigma M_P = 0$$

$$Or, \frac{w_e l}{2} * x - w_e * x * \frac{x}{2} - H * y = 0$$

$$Or, \frac{w_e l}{2} * x - \frac{w_e x^2}{2} = H * y$$

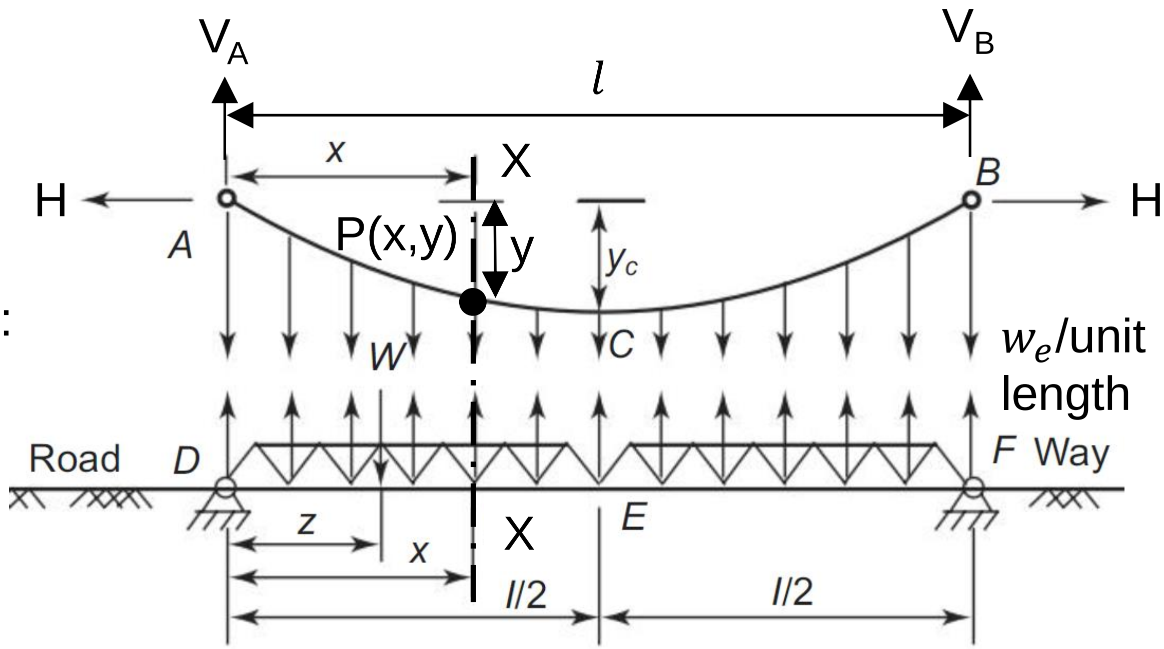
$$\therefore Hy = \frac{w_e l x}{2} - \frac{w_e x^2}{2} - - - (1)$$



Source: Reddy, C.S. (2011). *Basic Structural Analysis (3rd ed.).* New Delhi: Tata McGraw Hill.

7.3 Suspension bridge with three-hinged stiffening girder

For the stiffening girder,



Source: Reddy, C.S. (2011). *Basic Structural Analysis (3rd ed.).* New Delhi: Tata McGraw Hill.

Vertical reaction at D due to:

- **Load, W**

$$V_D' = \frac{W(l-z)}{l}$$

- **Upward w_e**

$$V_D'' = \frac{w_e l}{2}$$

Vertical reaction at F due to:

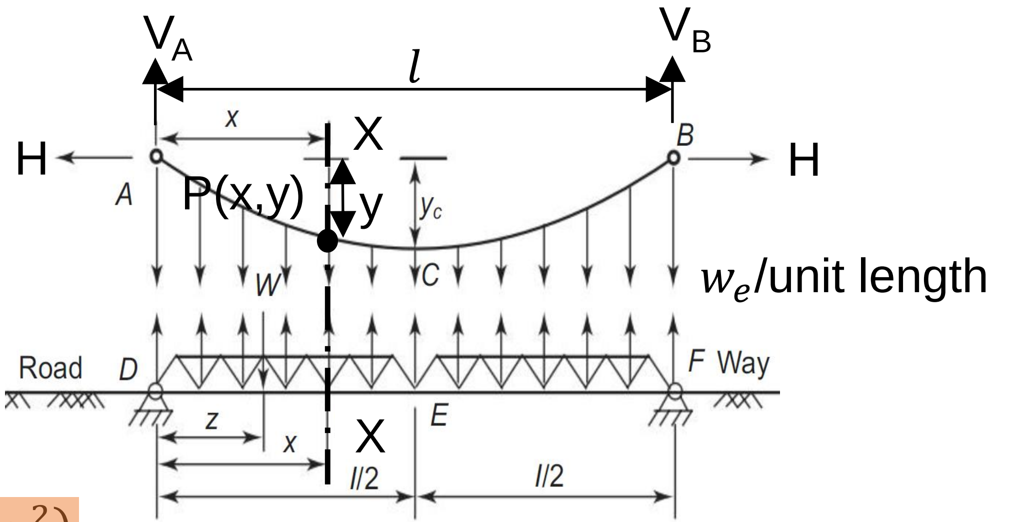
- **Load, W**

$$V_F' = \frac{Wz}{l}$$

- **Upward w_e**

$$V_F'' = \frac{w_e l}{2}$$

7.3 Suspension bridge with three-hinged stiffening girder



Source: Reddy, C.S. (2011). *Basic Structural Analysis (3rd ed.).* New Delhi: Tata McGraw Hill.

For the stiffening girder,

Then, moment at section X-X is:

For z≤x,

$$M_x = V_D' x - W(x-z) - \left\{\frac{w_e l}{2} x - \frac{w_e x^2}{2}\right\} --(2)$$

For z≥x,

$$M_x = V_D' x - \left\{\frac{w_e l}{2} x - \frac{w_e x^2}{2}\right\} --(3)$$

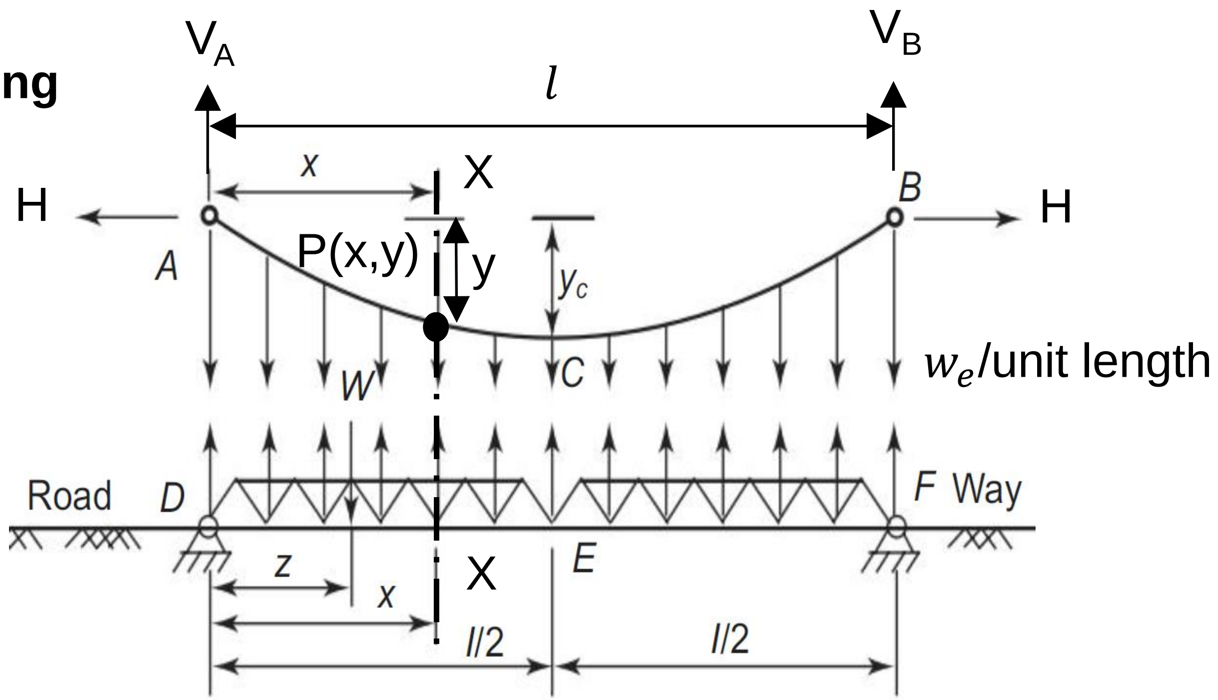
Equation for BM at a section in a SSB due to load, W

BM due to w_e only

$= H * y$ [From equation (1)]

7.3 Suspension bridge with three-hinged stiffening girder

For the stiffening girder,



Source: Reddy, C.S. (2011). *Basic Structural Analysis (3rd ed.).* New Delhi: Tata McGraw Hill.

Hence,

$$\therefore M_x = \mu_x - Hy$$

Where,

μ_x = Moment due to W at section X in a SSB

H = Horizontal tension in the cable

y = Ordinate of the cable profile at section X

7.4 Influence Line Diagrams

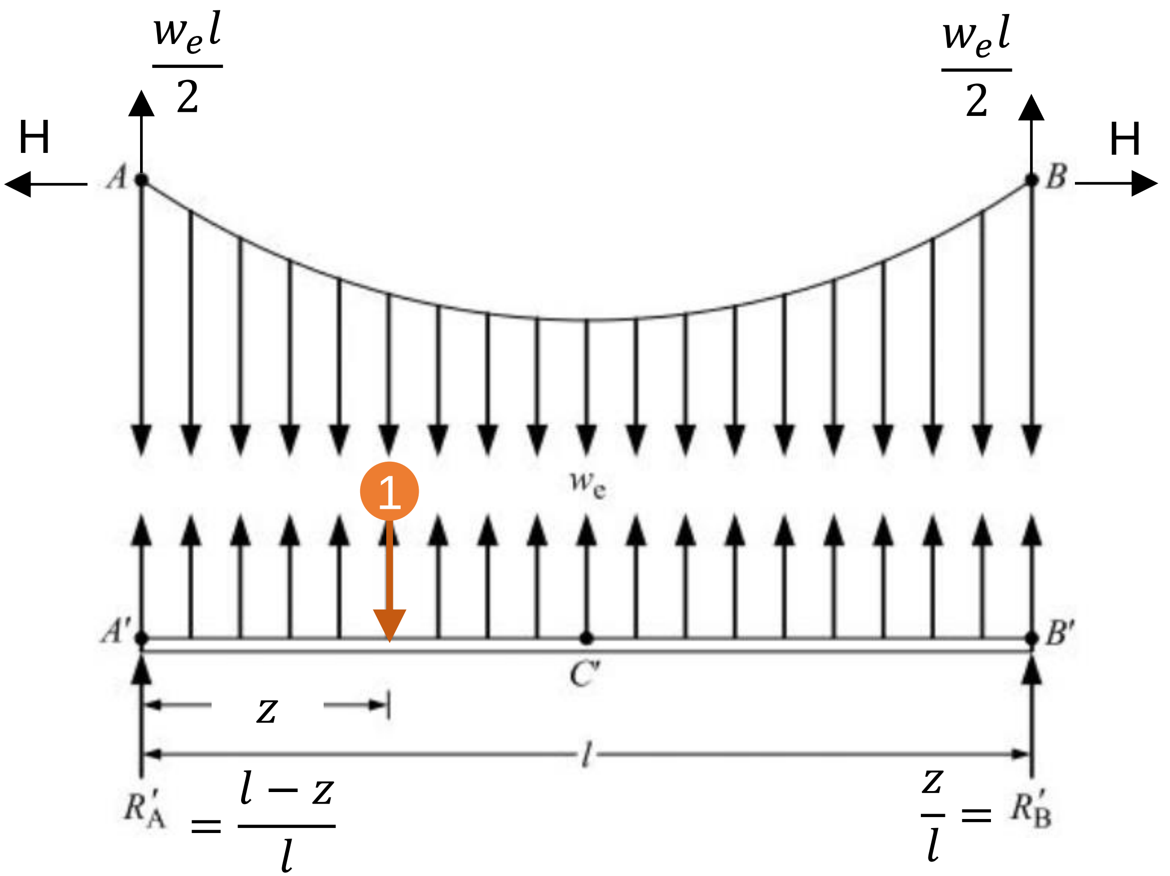
[A] ILD for w_e:

Let unit load be at a distance z from end A'. Reaction at B' due to this load is given by:

$$R'_A = \frac{l - z}{l}$$

$$R'_B = \frac{z}{l}$$

Let w_e be the equivalent load exerted by suspenders.



Source: Bhavikatti, S. S. (2011). *Structural Analysis –I* (4th ed.).New Delhi: Vikas Publishing House.

7.4 Influence Line Diagrams

[A] ILD for w_e:

Load in portion A'C'

$$\Sigma M'_C = 0$$

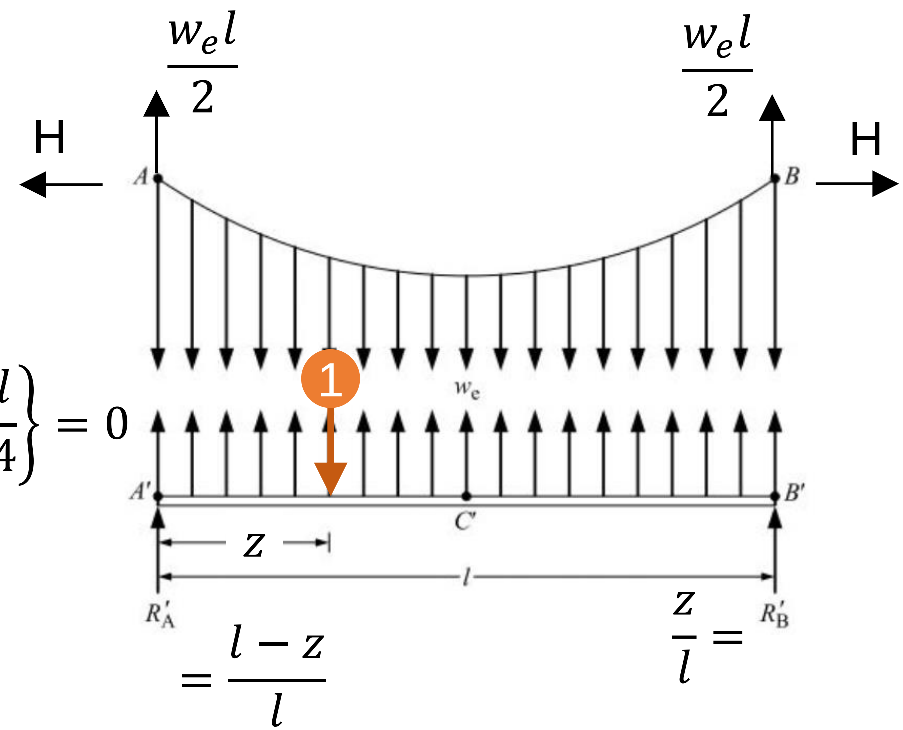
$$Or, \frac{l-z}{l} * \frac{l}{2} - 1 * \left(\frac{l}{2} - z\right) - \left\{\frac{w_e l}{2} * \frac{l}{2} - w_e * \frac{l}{2} * \frac{l}{4}\right\} = 0$$

R'_A ; $Unit\ load$; V_A ; Due to $UDL\ w_e$

$$Or, \frac{l-z}{2} - \frac{l-2z}{2} = \frac{w_e l^2}{8}$$

$$Or, \frac{l-z-l+2z}{2} = \frac{w_e l^2}{8}$$

$$Or, \frac{z}{2} = \frac{w_e l^2}{8}$$



Source: Bhavikatti, S. S. (2011). *Structural Analysis –I* (4th ed.).New Delhi: Vikas Publishing House.

7.4 Influence Line Diagrams

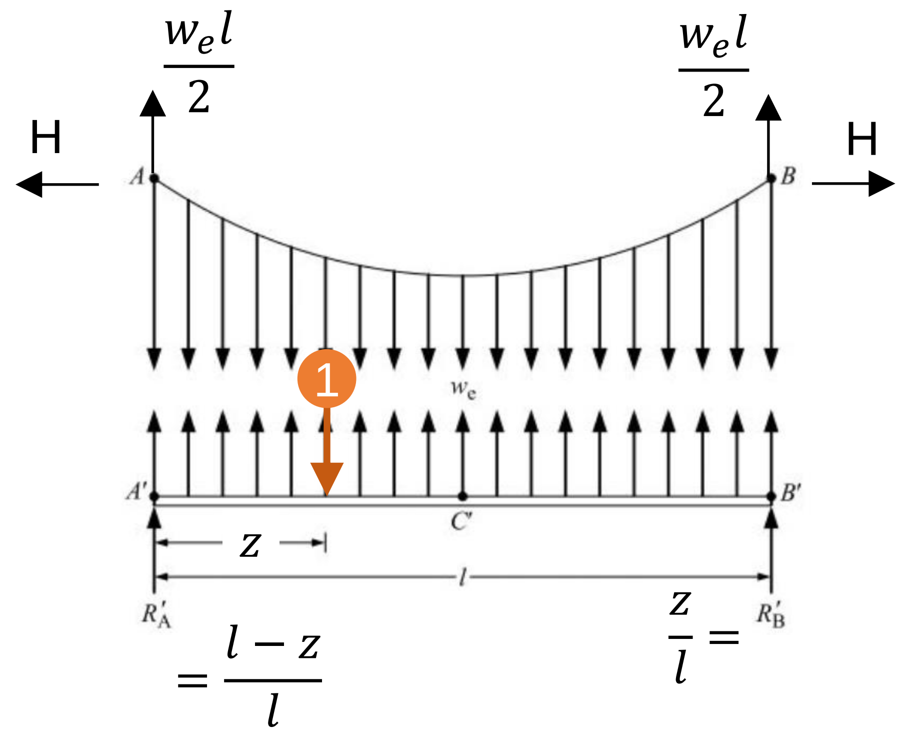
[A] ILD for w_e:

Load in portion A'C'

$$\Rightarrow w_e = \frac{4z}{l^2}, linear\ variation$$

At z=0, $w_e = 0$

At z=$\frac{l}{2}$, $w_e = \frac{2}{l}$



Source: Bhavikatti, S. S. (2011). *Structural Analysis –I* (4th ed.).New Delhi: Vikas Publishing House.

7.4 Influence Line Diagrams

[A] ILD for w_e:

Load in portion C'B'

$$\Sigma M_C' = 0$$

$$Or, \frac{l-z}{l} * \frac{l}{2} - \left\{\frac{w_e l}{2} * \frac{l}{2} - w_e * \frac{l}{2} * \frac{l}{4}\right\} = 0$$

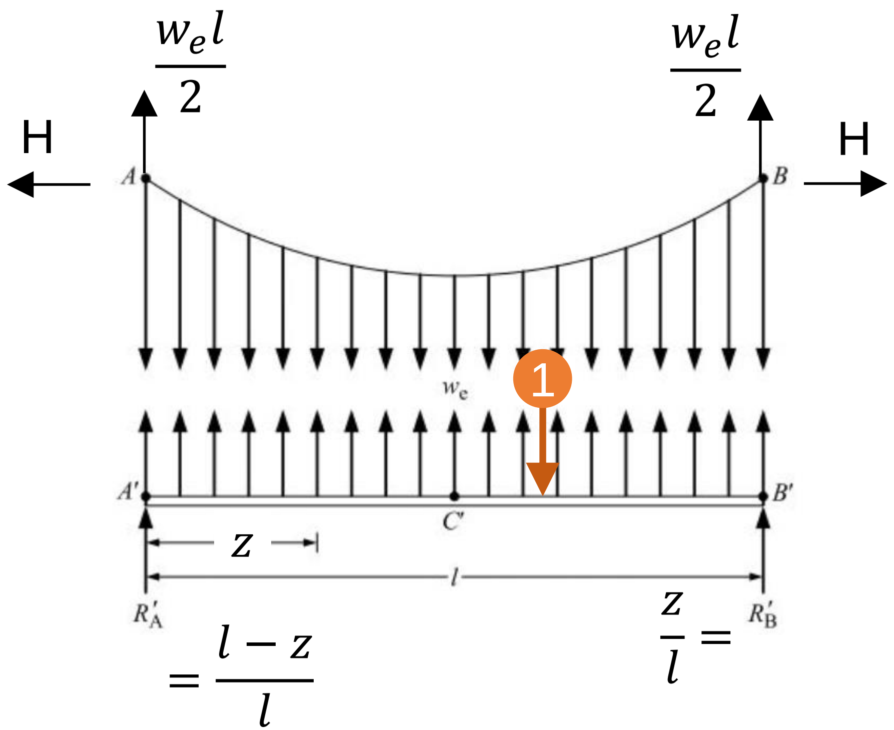
$\frac{l-z}{l}$ ⇩ R_A' ; $\frac{w_e l}{2}$ ⇩ V_A ; $w_e * \frac{l}{2} * \frac{l}{4}$ ⇩ Due to $UDL\ w_e$

$$Or, \frac{l-z}{2} = \frac{w_e l^2}{8}$$

$$Or, w_e = \frac{4(l-z)}{l^2}, linear\ variation$$

At z=$\frac{l}{2}$, $w_e = \frac{2}{l}$

At z= l, $w_e = 0$



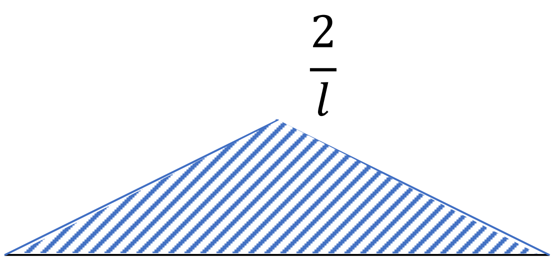
Source: Bhavikatti, S. S. (2011). *Structural Analysis –I* (4th ed.).New Delhi: Vikas Publishing House.

7.4 Influence Line Diagrams

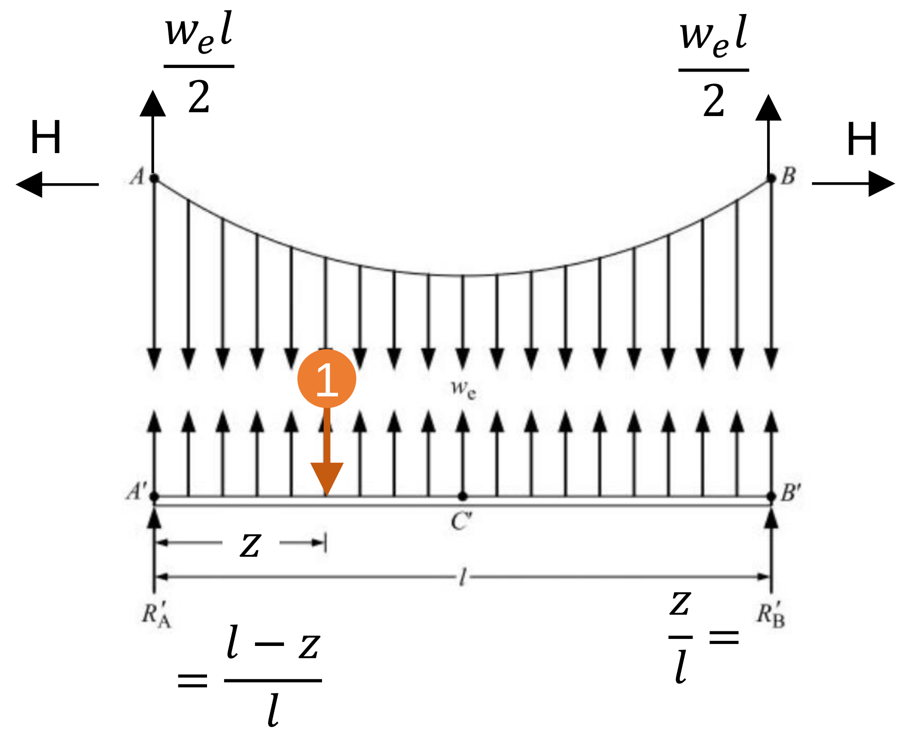
[A] ILD for w_e:

Hence, we get

$z = 0$	$w_e = 0$
$z = \frac{l}{2}$	$w_e = \frac{2}{l}$
$z = l$	$w_e = 0$



ILD for w_e



Source: Bhavikatti, S. S. (2011). *Structural Analysis –I* (4th ed.).New Delhi: Vikas Publishing House.

7.4 Influence Line Diagrams

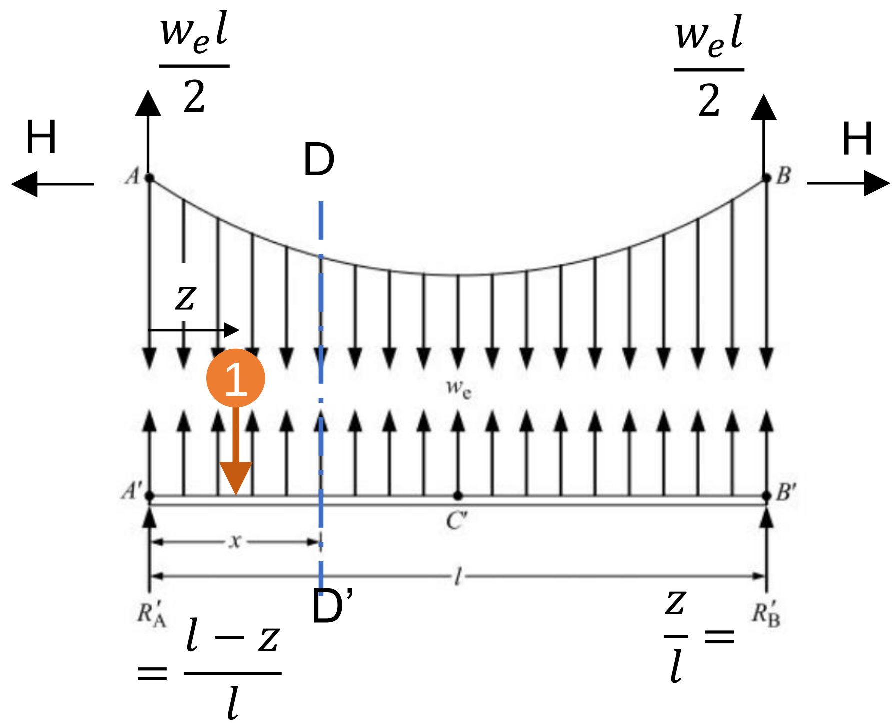
[B] ILD for BM in the girder:

Let BM at D’ at a distance x from support A’ is required.

<u>Unit load between A’ and D’</u>

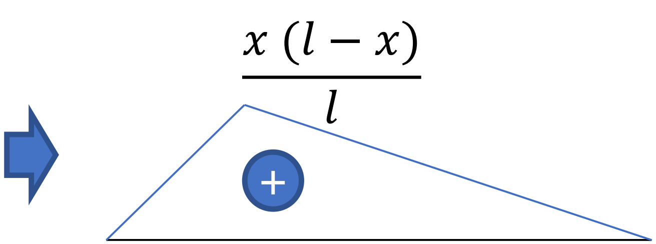
M = Moment due to given unit load + Moment due to w_e alone

∴ ILD for M = ILD for M due to unit load + ILD for M due to w_e alone



Source: Bhavikatti, S. S. (2011). *Structural Analysis –I* (4th ed.).New Delhi: Vikas Publishing House.

We know,
ILD for M due to unit load is a triangle with maximum ordinate $\frac{x\,(l-x)}{l}$ at section D’, at a distance x from A’.



7.4 Influence Line Diagrams

[B] ILD for BM in the girder:

Unit load between A' and D'

We have,

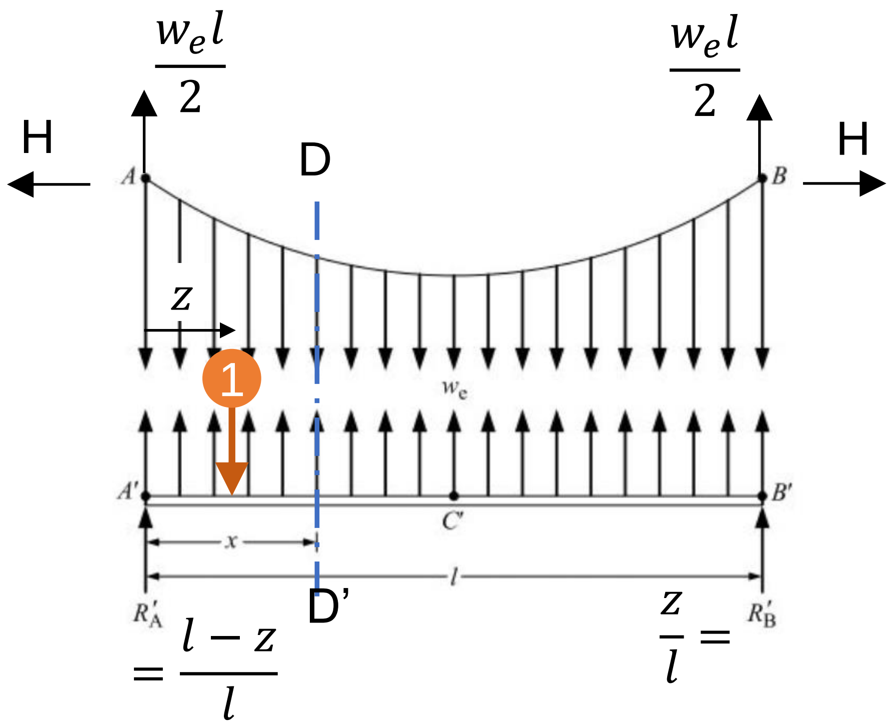
M due to w_e alone = $-w_e * \frac{x}{2} * (l-x)$

And,

$w_e = \frac{4z}{l^2}$

∴ M due to w_e alone = $-\frac{4z}{l^2} * \frac{x}{2} * (l-x)$

$= -\frac{2xz}{l^2} * (l-x)$; varies linearly with x



Source: Bhavikatti, S. S. (2011). *Structural Analysis –I* (4th ed.).New Delhi: Vikas Publishing House.

7.4 Influence Line Diagrams

[B] ILD for BM in the girder:

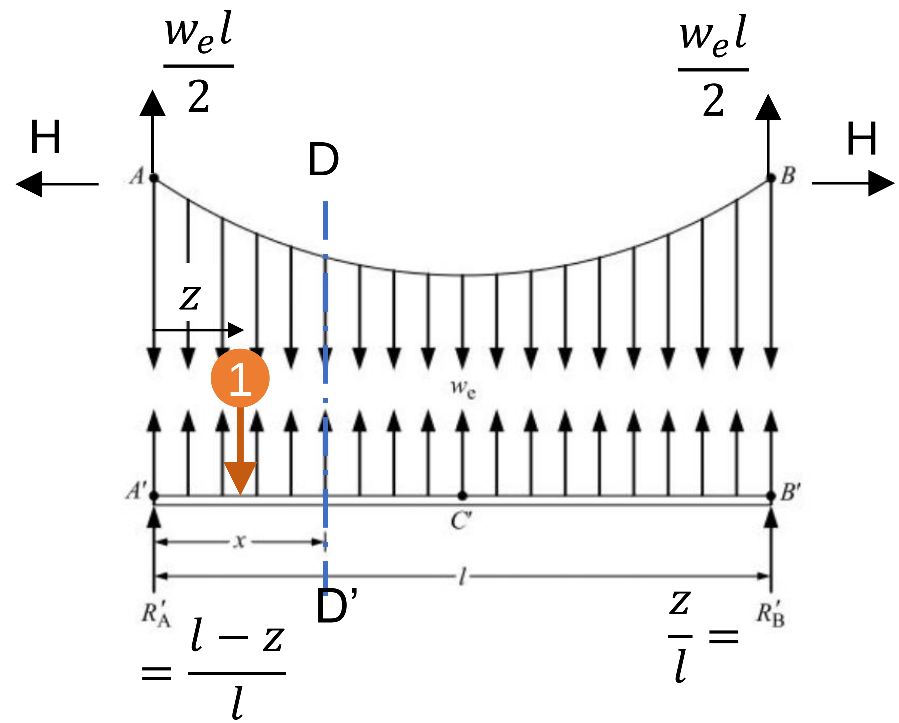
Unit load between A’ and D’

∴ M due to w_e alone = $-\frac{2xz}{l^2} * (l-x)$; varies linearly with x

At z=0, M due to $w_e = 0$

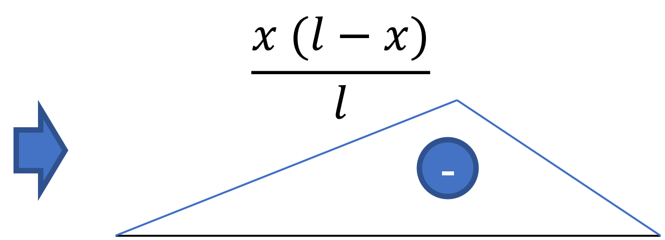
At z=$\frac{l}{2}$, M due to $w_e = -\frac{2x*\frac{l}{2}}{l^2} * (l-x)$

$= -\frac{x\,(l-x)}{l}$



Source: Bhavikatti, S. S. (2011). *Structural Analysis –I* (4th ed.).New Delhi: Vikas Publishing House.

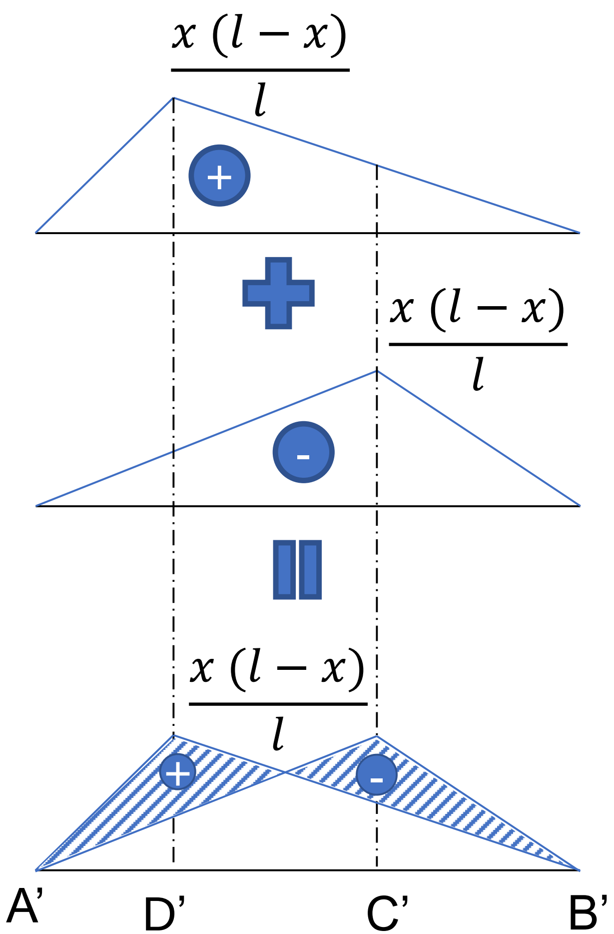
∴ ILD for moment due to we alone is also a triangle with maximum ordinate $-\frac{x\,(l-x)}{l}$ at mid span of the girder C’

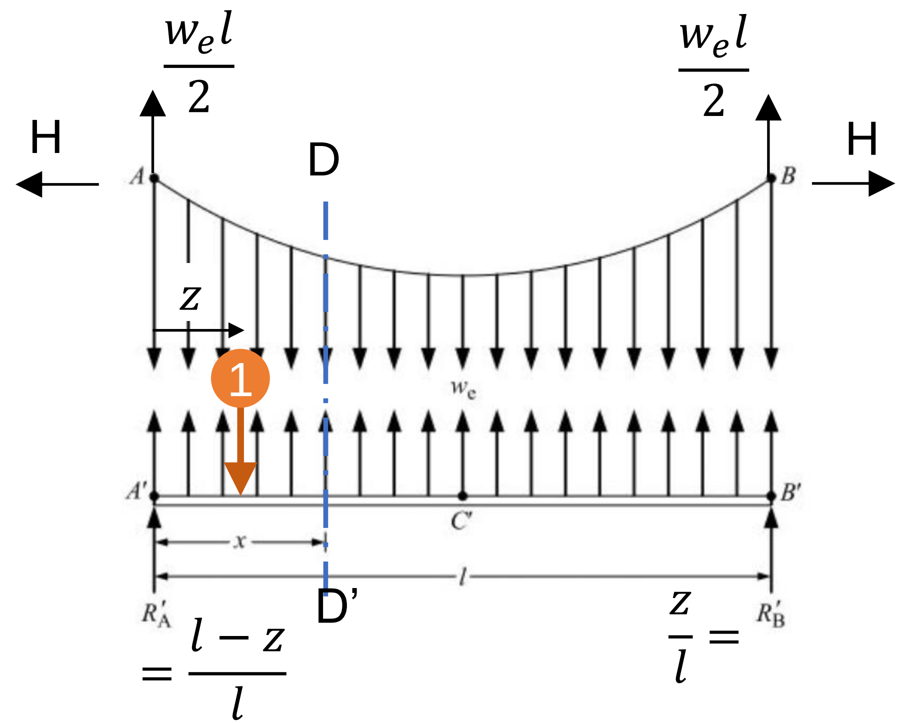


7.4 Influence Line Diagrams

[B] ILD for BM in the girder:

Hence, ILD for M in the girder is:





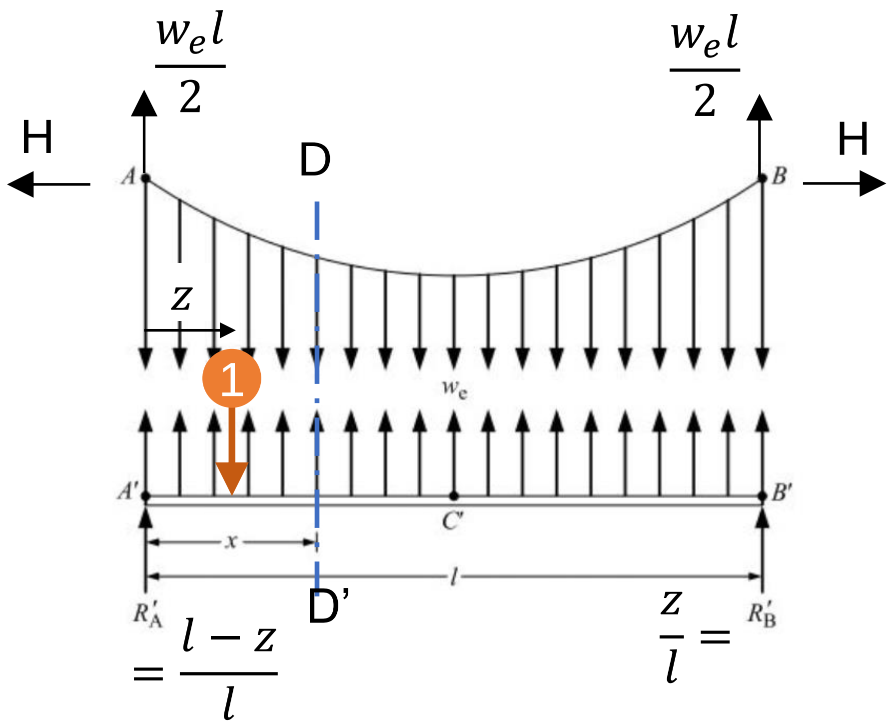
Source: Bhavikatti, S. S. (2011). *Structural Analysis –I* (4th ed.).New Delhi: Vikas Publishing House.

7.4 Influence Line Diagrams

[C] ILD for SF in the girder:

SF in the girder at D’ at a distance x from A’
=
SF in the girder due to given unit load
+
SF due to w_e alone

∴ ILD for SF in the girder at D’
= ILD for SF in the girder due to unit load
+
ILD for SF in the girder due to w_e alone

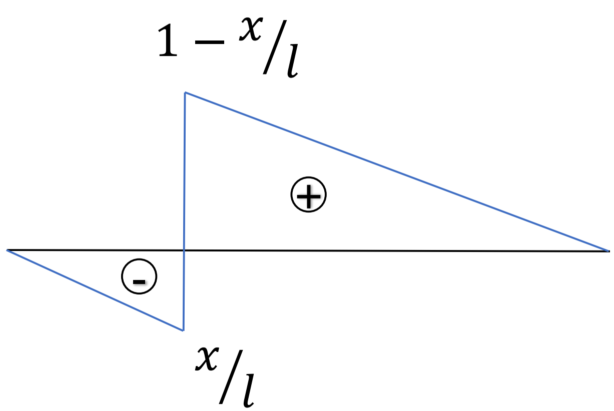


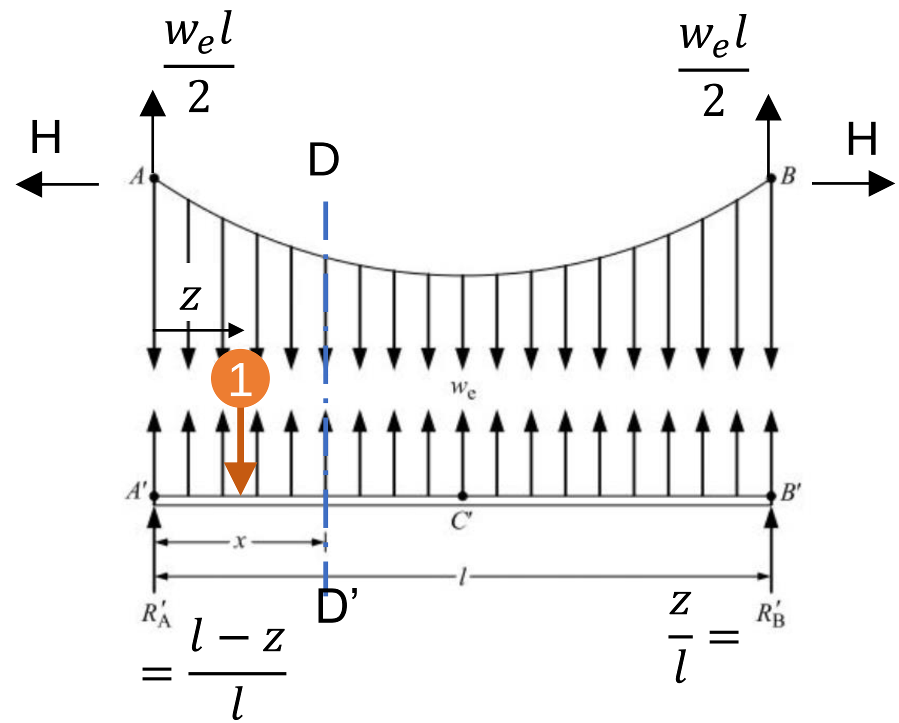
Source: Bhavikatti, S. S. (2011). *Structural Analysis –I* (4th ed.).New Delhi: Vikas Publishing House.

7.4 Influence Line Diagrams

[C] ILD for SF in the girder:

We have,
ILD for SF in the girder due to unit load:





Source: Bhavikatti, S. S. (2011). *Structural Analysis –I* (4th ed.).New Delhi: Vikas Publishing House.

7.4 Influence Line Diagrams

[C] ILD for SF in the girder:

We have,

SF due to w_e alone = $-w_e * \left(\frac{l}{2} - x\right)$

And,

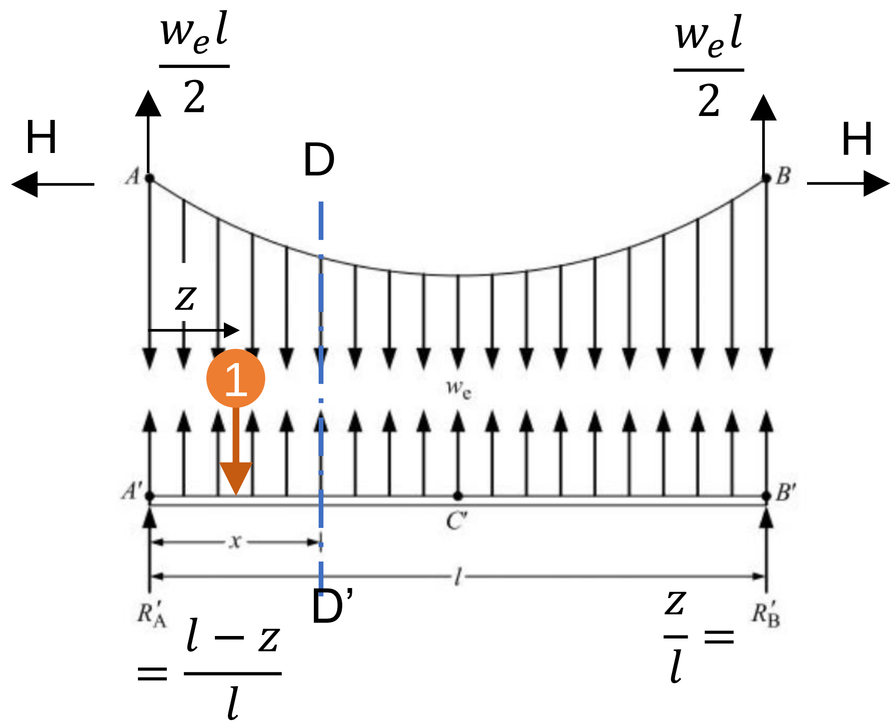
$w_e = \frac{4z}{l^2}$

$\therefore$ SF due to w_e alone = $-\frac{4z}{l^2} * \left(\frac{l}{2} - x\right)$ varies linearly with x

At z=0, SF due to $w_e = 0$

At z=$\frac{l}{2}$, SF due to $w_e = -\frac{4*\frac{l}{2}}{l^2} * \left(\frac{l}{2} - x\right)$

$$= -\frac{(l-2x)}{l}$$



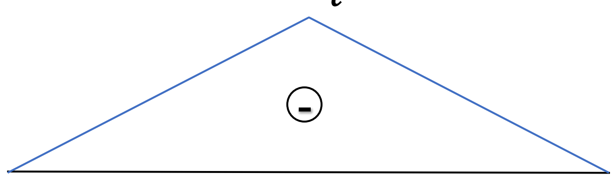
Source: Bhavikatti, S. S. (2011). *Structural Analysis –I* (4th ed.).New Delhi: Vikas Publishing House.

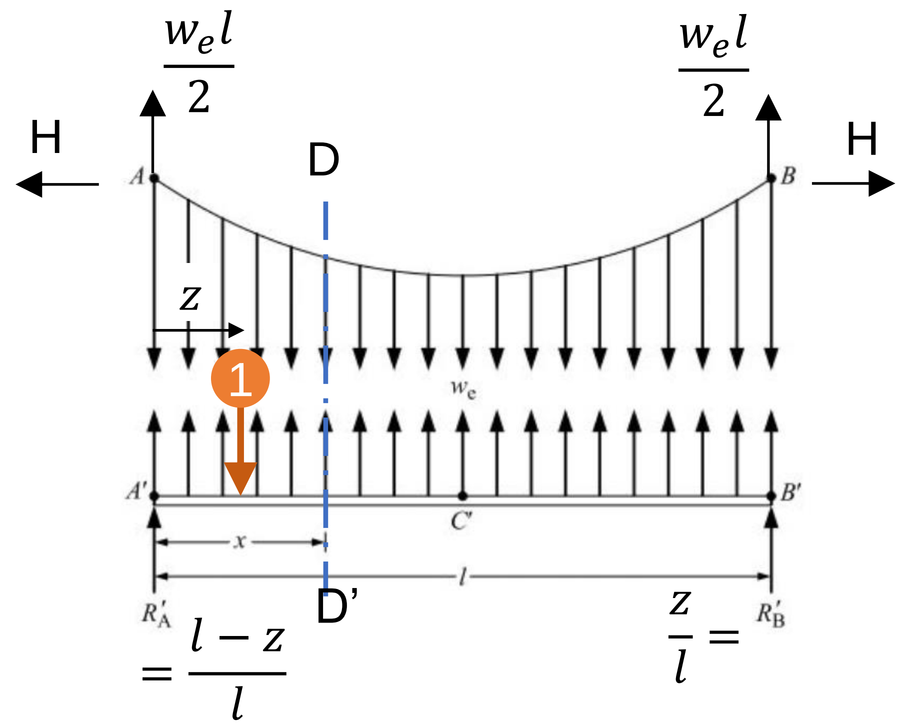
7.4 Influence Line Diagrams

[C] ILD for SF in the girder:

Hence,
ILD for SF due to w_e alone is a triangle with maximum ordinate $\frac{(l-2x)}{l}$ at the center.

$$\frac{(l-2x)}{l}$$

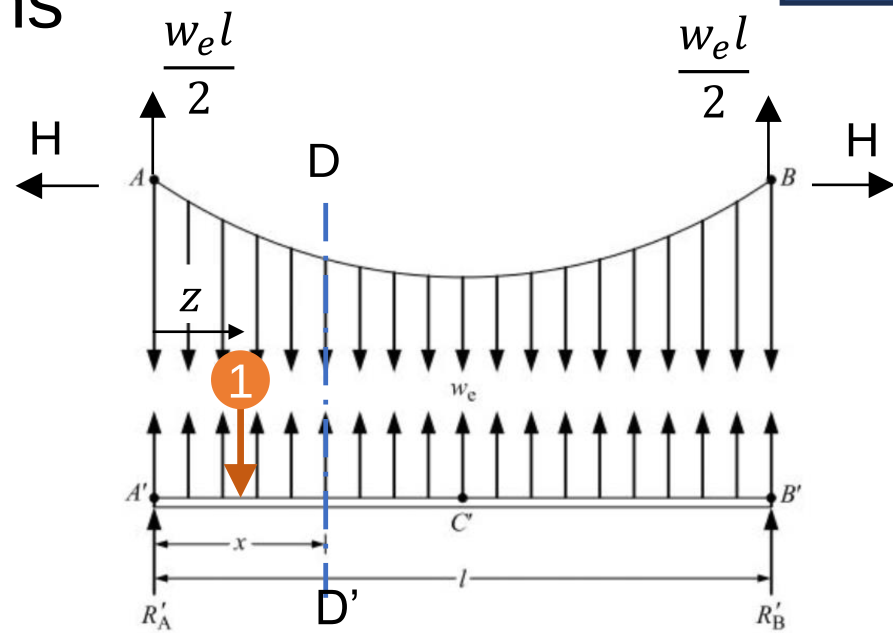




Source: Bhavikatti, S. S. (2011). *Structural Analysis –I* (4th ed.).New Delhi: Vikas Publishing House.

7.4 Influence Line Diagrams

[C] ILD for SF in the girder:

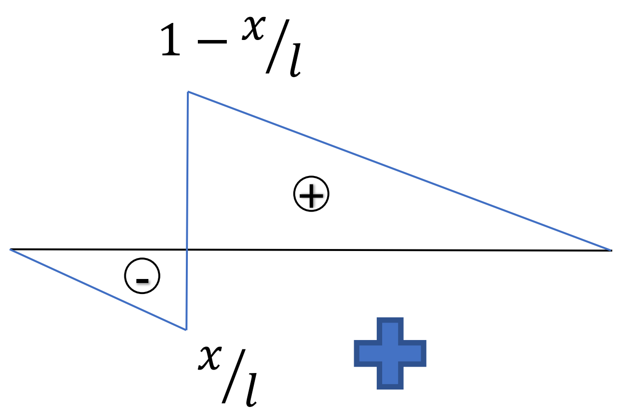


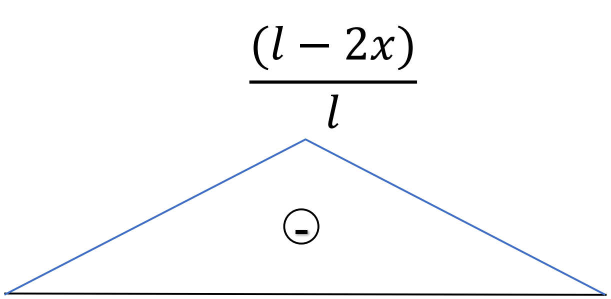
Source: Bhavikatti, S. S. (2011). *Structural Analysis –I* (4th ed.).New Delhi: Vikas Publishing House.

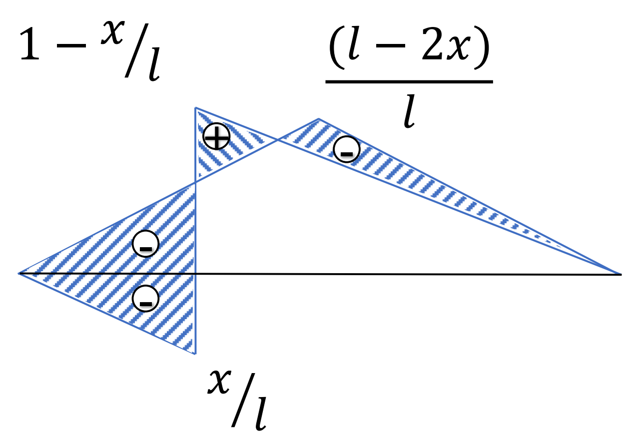
Then,

For $x < \frac{l}{4}$

ILD for SF at D' in the girder is:







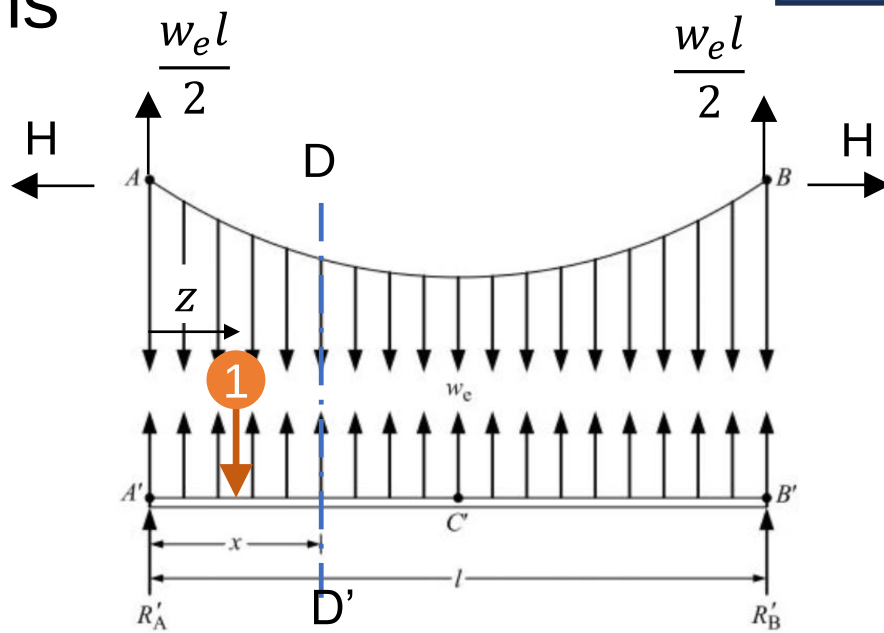
7.4 Influence Line Diagrams

[C] ILD for SF in the girder:

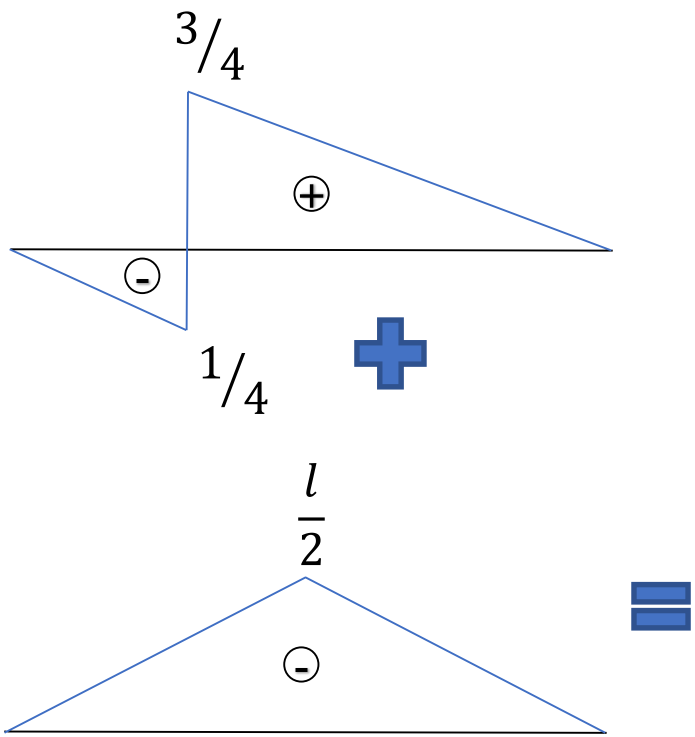
Then,

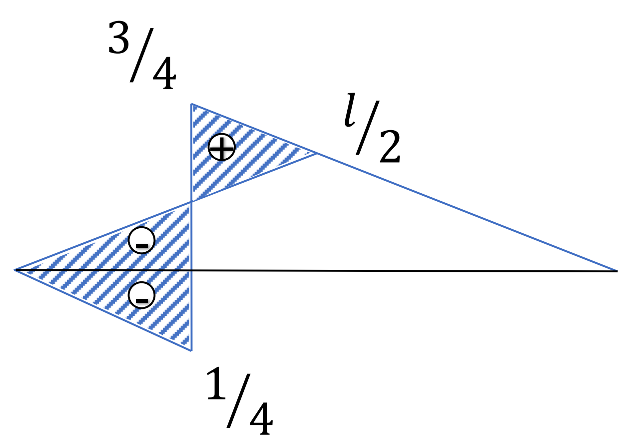
For $x = \frac{l}{4}$

ILD for SF at D' in the girder is:

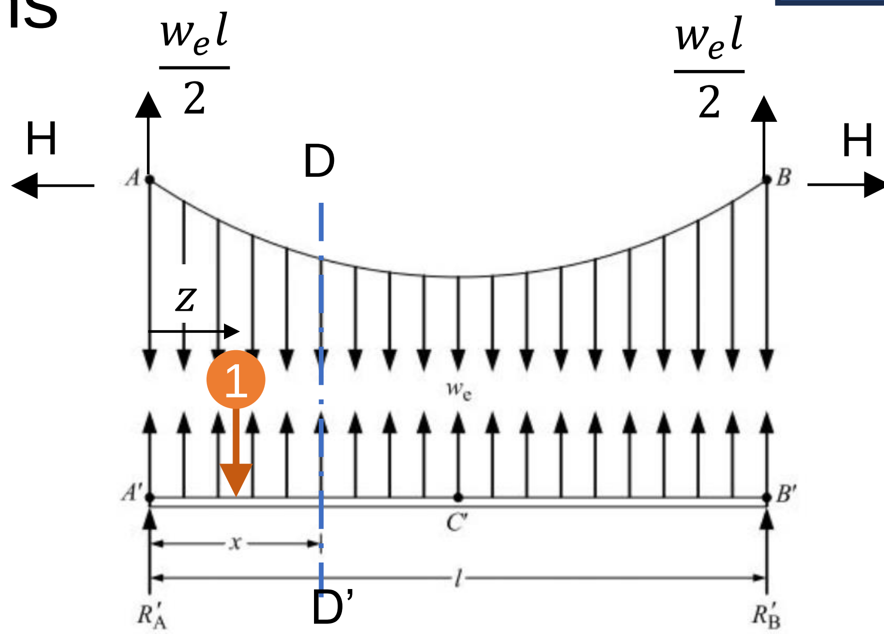


Source: Bhavikatti, S. S. (2011). *Structural Analysis –I* (4th ed.).New Delhi: Vikas Publishing House.





7.4 Influence Line Diagrams



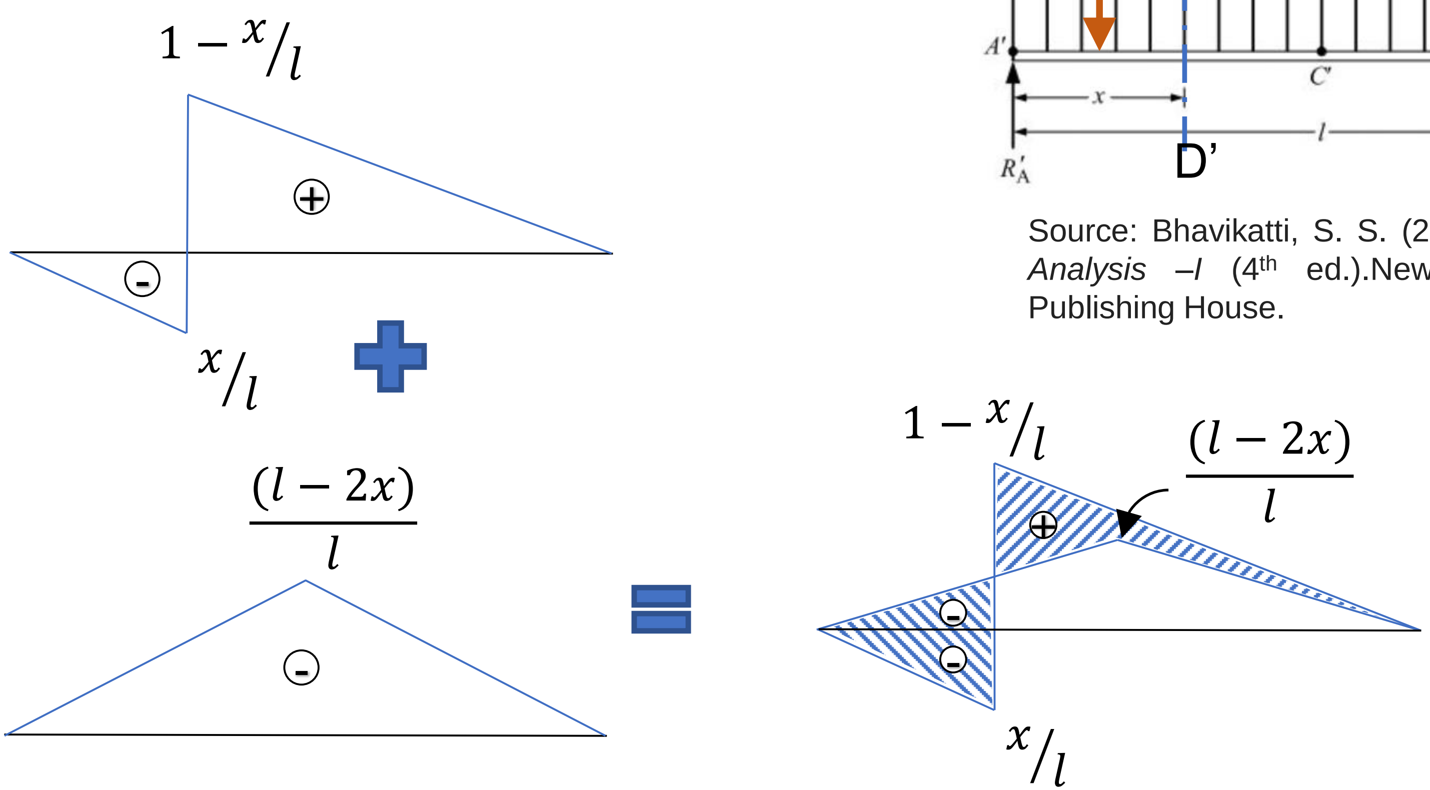
Source: Bhavikatti, S. S. (2011). *Structural Analysis –I* (4th ed.).New Delhi: Vikas Publishing House.

[C] ILD for SF in the girder:

Then,

For $x > \frac{l}{4}$

ILD for SF at D' in the girder is:



7.4 Influence Line Diagrams

[D] ILD for H

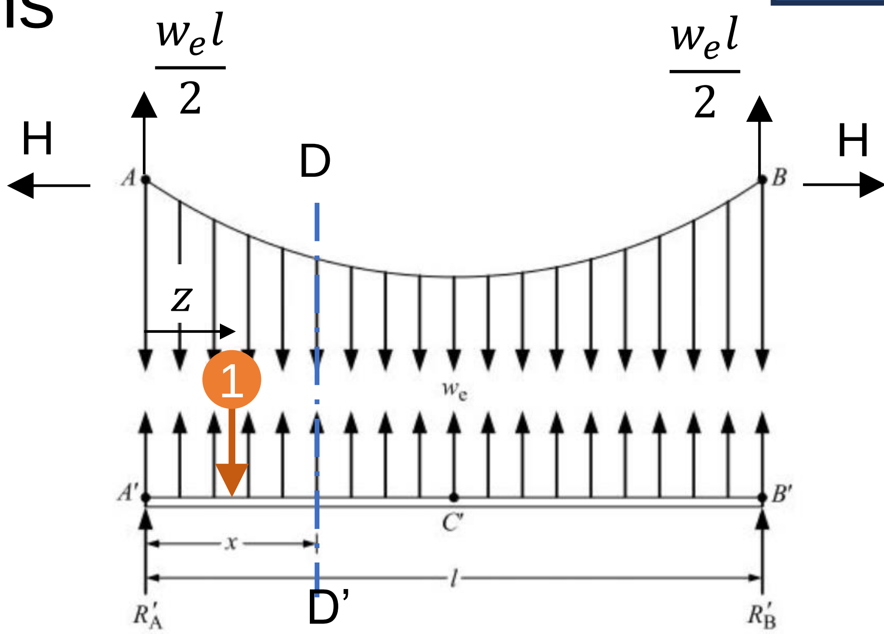
We have,

$$H = \frac{w_e l^2}{8h}$$

And,

$$w_e = \frac{4z}{l^2}$$

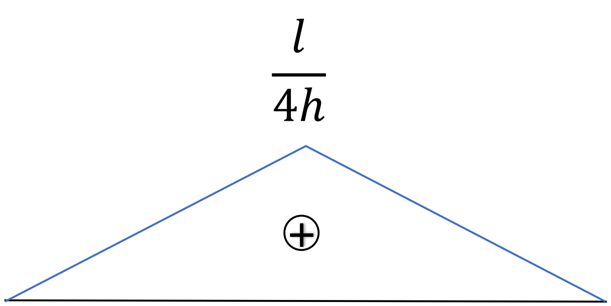
$$\Rightarrow H = \frac{\frac{4z}{l^2} * l^2}{8h} = \frac{z}{2h}$$



Source: Bhavikatti, S. S. (2011). *Structural Analysis –I* (4th ed.).New Delhi: Vikas Publishing House.

At z=0, H= 0

At z=$\frac{l}{2}$, H= $\frac{l}{4h}$



ILD for H

------------End of Lecture #12-----------------

------------End of Part II of II for Chapter 7-------------

References

[1] Bhavikatti, S. S. (2011). *Structural Analysis –I* (4th ed.).New Delhi: Vikas Publishing House.
[2] Reddy, C.S. (2011). *Basic Structural Analysis (3rd ed.).* New Delhi: Tata McGraw Hill.