Engineering Hydrology

Week-7

CHAPTER -3 FLOOD ROUTING

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Lecture contents of the last weeks (Week-3-5)

CHAPTER -2 Rainfall-runoff Relationships

2.1 Hydrological Models

2.2 Rational Method

2.3 Rainfall intensity and Time of Concentration

2.4 SCS Curve Number and Time-Area Methods

2.5 Stream Flow Hydrograph

2.6 The Unit Hydrograph (UH)

2.7 Applications of Unit Hydrograph

2.8 Synthetic Unit Hydrographs

2.9 Hydrology of Ungauged Catchment

Home assignment:

I hope you are sure you area able to exercise on:

- **Hydrological modeling**: definition, importance and classification
- **Rainfall**: Intensity, duration, frequency, effective rainfall and time of concentration
- **Rational method**: symbols, units and problem analysis
- **SCS-CN methods**: symbols, units and problem analysis
    - Compare and contrast rainfall, effective rainfall and direct runoff.
        - Relate parameters under these methods
        - Try to solve mathematical expression and problem solving on these methods for effective rainfall
- **Baseflow separation**: definitions and methods
    - What does baseflow mean?
    - Why it is important?
    - Compare and contrast baseflow separation methods graphically.
- **Hydrograph**: Factors affecting, UH., SUH
- **Data transfer for ungauged watersheds**

Lecture contents of the week

HAPTER -3 FLOOD ROUTING

3.4 Channel Routing

3.4.1 Muskingum Method of Routing

3.4.2 Application of The Muskingum Method

3.5 Hydraulic Routing

Workout examples

Lecture Learning Outcomes

Course Learning Outcomes: After completion of this Lecture, you will be able to:

CLO-1: Apply measurement techniques of the components of the hydrologic cycle, water balance and filling of missed data;

CLO-2: Examine rainfall-runoff relationship and hydrograph;

- **Apply flood routing**

CLO-3: Examine the probability of occurrence;
CLO-4: Analyze the water movement in to, over, and through the soil surface;
CLO-5: Design capacity of reservoir;
CLO-6: Design runoff volume and time of distribution of the runoff hydrograph from urbanization effect.

Channel routing

On part 6 of the presentation, the following topics have covered:

- Application of flood routing
- Simple non-storage routing
- Reservoir or Level Pool Routing
- Channel Routing
- Today we will cover some parts routing and remembering some equations to solve practical problems

Channel routing

Basic Equations: remember the equation of **continuity is used**:

$$I - Q = \frac{dS}{dt}$$

$$\bar{I}\,\Delta t - \bar{Q}\,\Delta t = \Delta S$$

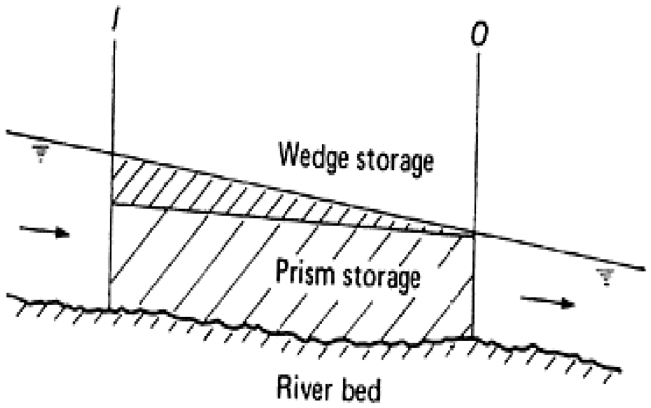
where, $\bar{I} = (I_1 + I_2)/2$, $\bar{Q} = (Q_1 + Q_2)/2$, and $\Delta S = S_2 - S_1$

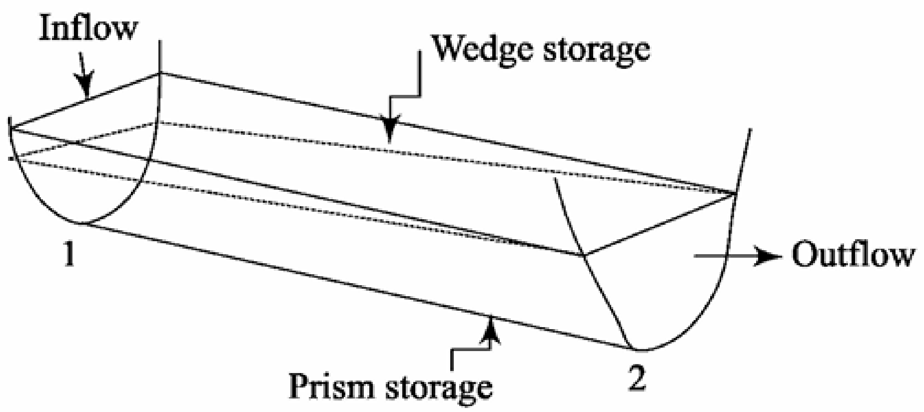
$$\left(\frac{I_1 + I_2}{2}\right)\Delta t - \left(\frac{Q_1 + Q_2}{2}\right)\Delta t = S_2 - S_1$$

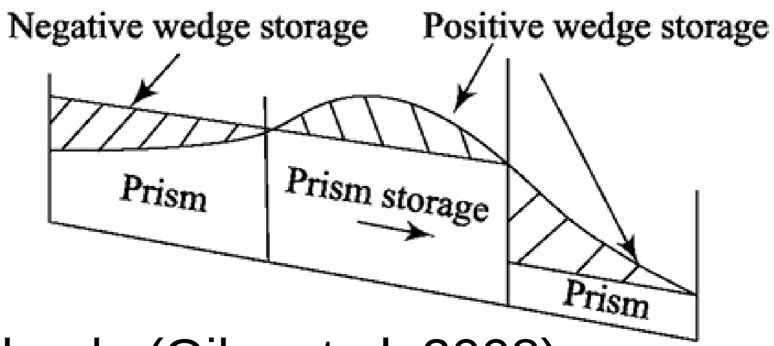
- Δt must be shorter than the time of transit of the flood wave through the reach.

Channel routing

Storages in the Channel,

- the storage in the reach may be split up in two parts:
  - ***prism storage***
  - ***wedge storage***
- the **prism** storage is constant
- the **wedge** storage changes from a **positive** value at an advancing flood to a **negative** value during a receding flood.
- 



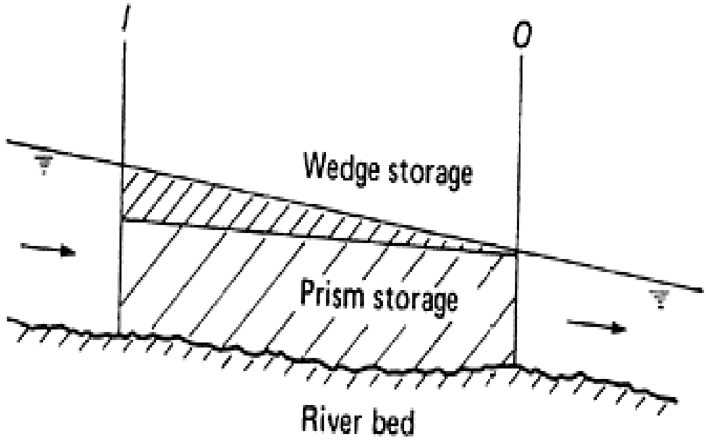


Source: Textbook, (Ojha et al, 2008)

Channel routing

Storages in the Channel,

- The prism storage (S_p) is similar to a reservoir and can be expressed as a function of the **outflow** discharge, $S_p = f(Q)$.
- The **wedge storage** can be accounted for by expressing it as $S_w = f(I)$.
- The **total storage** in the channel reach can then be expressed as: $S = K\,[x\,I^m + (1 - x)Q^m]$
- where, ***K*** and ***x*** are coefficients, and rn is a constant exponent.
  - the value of m varies from **0.6** for rectangular channels to about **1.0** for natural channels.



Source: Textbook, (Ojha et al, 2008)

Channel routing

- The total storage in the channel reach can be generally represented by:

Storage in wedge = $KX(I - Q)$

Storage in prism = KQ

So, Storage $S = KX(I - Q) + KQ$

And this can then be expressed as:

$$S = K\,(x\,I^m + (1 - x)Q^m)$$

where

K and **x** are coefficients and **m** is a constant exponent. It has been found that the value of m varies from **0.6** for rectangular channels to value of about **1.0** for natural channels.

Channel routing

- **Muskingum Equation**: is the equation that assumes Using m = 1.0 in:

$$Then, \qquad S = K\,(xI + (1 - x)Q)$$

Where,

- the parameter $\boldsymbol{x}$ is known as weighting factor and it takes a value between 0 and 0.5.
- It accounts for the storage portion of the routing.
- When x = 0, the storage is only the function of discharge and

$$S = KQ$$

Channel routing

Estimation of *K* and *x*

- Using the continuity equation in **finite difference** form, we can write the increment in storage at any time, ***t***, and time element, Δ***t***, can be calculated:

$$\Delta S = S_2 - S_1 = \{(I_1 + I_2)\Delta t\}/2 - \{(Q_1 + Q_2)\,\Delta t\}/2$$

- For a given channel reach by selecting a routing interval ***Δt*** and using the **Muskingum** equation, the change in storage can be determined.

$$S_1 = K(xI_1 + (1 - x)\,Q_1)$$

$$S_2 = K(xI_2 + (1 - x)\,Q_2)$$

- By choosing a trial value of x, values of S at any time t are plotted against

corresponding ***[XI + (1 - x)Q]*** values**.**

- **for natural channels, the value of x lies between 0 and 0.3.**

Muskingum Method of Routing (1960)

- Substituting the above Muskingum equations in to finite difference equation and after rearrangements gives:

$$S_2 - S_1 = K[x(I_2 - I_1) + (1 - x)(Q_2 - Q_1)]$$

- The continuity equation for the reach is:

$$S_2 - S_1 = \left(\frac{I_1 + I_2}{2}\right)\Delta t - \left(\frac{Q_1 + Q_2}{2}\right)\Delta t$$

$$C_1 = \frac{\Delta t + 2Kx}{\Delta t + 2K - 2Kx}$$

$$C_2 = \frac{\Delta t - 2Kx}{\Delta t + 2K - 2Kx}$$

$$C_3 = \frac{-\Delta t + 2k - 2KX}{\Delta t + 2k - 2kx}$$

$$Q_{i+1} = c_1 I_i + c_2 I_{i+1} + c_3 Q_i$$

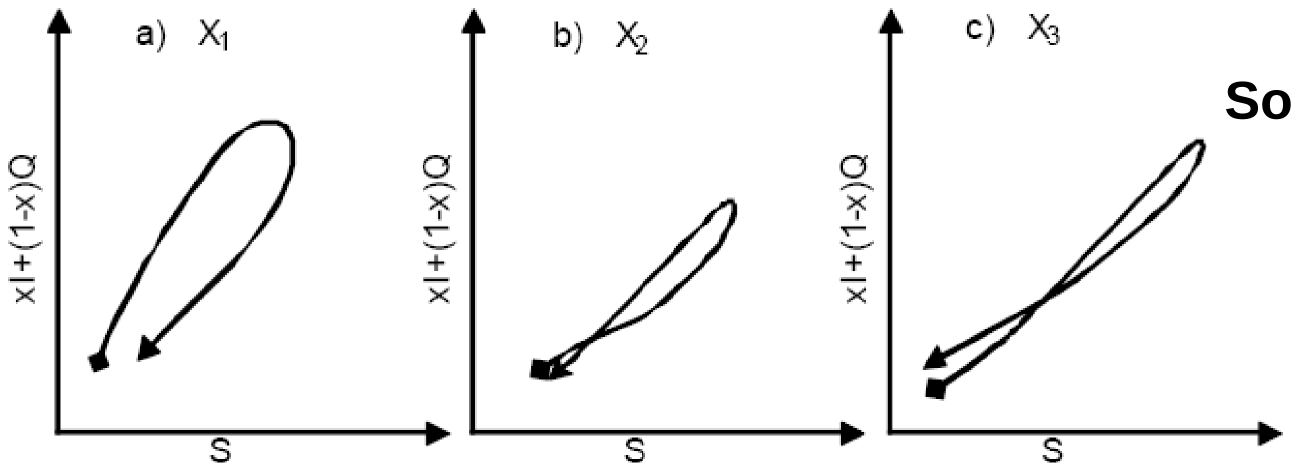
where,

Note that ΣC=1, i.e C1+C2+C3=1

Channel routing

- It has been found that best results will be obtained when routing interval should be so chosen that; **$K > \Delta t > 2kx$**
- In order to use equation Muskingum equ. for Q_{i+1}, it is necessary to know **K** and **x** for calculating the coefficients **C** . Using **recorded hydrographs** of a flood at the beginning and end of the river reach.

Channel routing



Source: Textbook, (Ojha et al, 2008)

x and **k** can be determined if upstream and downstream hydrographs are available:

1.Compute the **change in storage** for each time interval

2. Compute **cumulative storage** over time

3. Plot **storage vs (xI + (1-x)Q)** for a range of values of x

4. Select value of **x** which produces the narrowest "loop" and calculate **K** as the best fit slope.

i.e K = ΔS/ Δ(xI + (1-x)Q)

Hydraulic Routing

- The **equation of continuity** for unsteady flow in a reach with no lateral flow is given by the differential forms:

$$\frac{\partial h}{\partial t} + u\frac{\partial h}{\partial x} + h\frac{\partial u}{\partial x} = 0$$

where, u = velocity of flow, and h = depth of flow

- The equation of motion for a flood wave is derived from the application of the **momentum equation** as:

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + g\frac{\partial h}{\partial x} - (S_0 - S_f)g = 0$$

where, S_0 = channel bed slope, and S_f = slope of the energy line.

Spreadsheet application

Excel spreadsheet application is helpful for engineering problem solution since:

- To produce own the software that can automate solution
- Skills and techniques transferable
- Easily adapted connection and encoding formula
- It is easy and manageable to solve time series data
- To apply interconnected processes

Spreadsheet application

The next three typical problems have solved using excel spreadsheet

- Follow the procedures how to setup excel spreadsheet on your desktop.
- We’ll have done together step by step during the practical session.
- Or it was displayed on the video lecture

Workout examples

- **Example-1** ***The inflow and outflow hydrographs for a reach of a river are given below.***
- ***Determine*** *the value of the Muskingum coefficients* ***K*** *and* ***x*** *for the reach.*

Time (hr)	Inflow (m^3/s)	Outflow (m^3/s)
0	35	39
24	125	52
48	575	287
72	740	624
96	456	638
120	245	394
144	144	235
168	95	142
192	67	93
216	50	60

Workout examples

- **Example-1** ***Encoding on excel:***

E	F	G	H	I	J	K	L	M	N
							(xI+(1-x)Q) (m^3/sec)		
Time (hr)	Inflow (m^3/sec)	Outflow (m^3/sec)	I-Q	AVERAGE I-Q	ΔS=(AVERAGEI-Q)*Δt (m^3/sec) . Day	S=cum.of ΔS (m^3/sec) . Day	for x=0.2	for x=0.3	for x=0.25
1	2	3	4	5	6	7	8	9	10
0	35	39	-4			0	38.2	37.8	38
				34.5	34.5				
24	125	52	73			34.5	66.6	73.9	70.25
				180.5	180.5				
48	575	287	288			215	344.6	373.4	359
				202	202				
72	740	624	116			417	647.2	658.8	653
				-33	-33				
96	456	638	-182			384	601.6	583.4	592.5
				-165.5	-165.5				
120	245	394	-149			218.5	364.2	349.3	356.75
				-120	-120				
144	144	235	-91			98.5	216.8	207.7	212.25
				-69	-69				
168	95	142	-47			29.5	132.6	127.9	130.25
				-36.5	-36.5				
192	67	93	-26			-7	87.8	85.2	86.5
				-18	-18				
216	50	60	-10			-25	58	57	57.5

fx =F4-G4

fx =AVERAGE(H4,H6)

fx =0.2*F4+(1-0.2)*G4

Example

Solution;

- From the daily readings of the inflow and outflow hydrographs, a routing period
  t =24 hr = 1 day is taken.
- The **mean storage** is determined and then the **cumulative storage *S*** *is tabulated.*
- For trial values of *x = 0.2, 0.25 and 0.3, the values of* $[xI + (1 - x)\,O]$
  are computed in the table below table.
  Storage loops for the reach, *i.e., curves of* *S* *vs* $[xI + (1 - x)\,O]$ *for* each trial value of *x are plotted .*
- By inspection, the middle value of *x = 0.25* approximates a straight line and hence this value of *x is chosen.*
- *K is determined by measuring* **the slope of the median** straight line which is found to be **0.7 day**.
- Hence, for the given reach of the river, the values of the Muskingum coefficients are

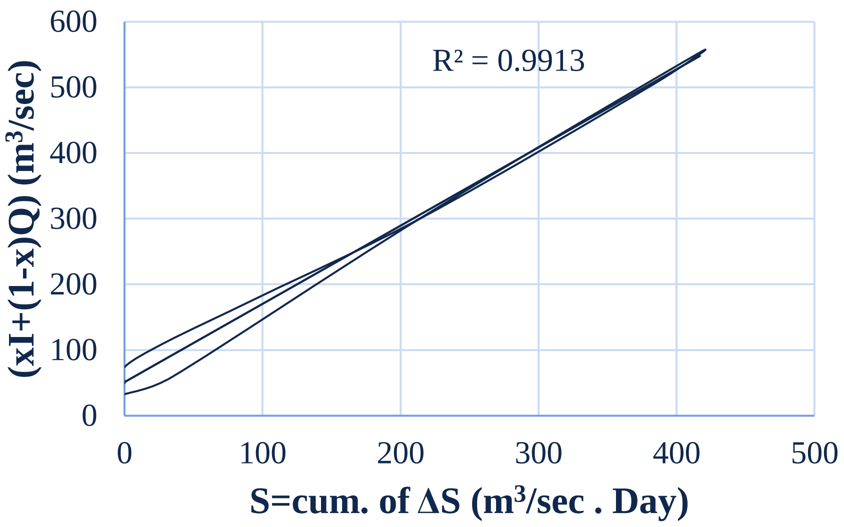
$$x = 0.25,\ K = 0.7 \text{ day}$$

Example

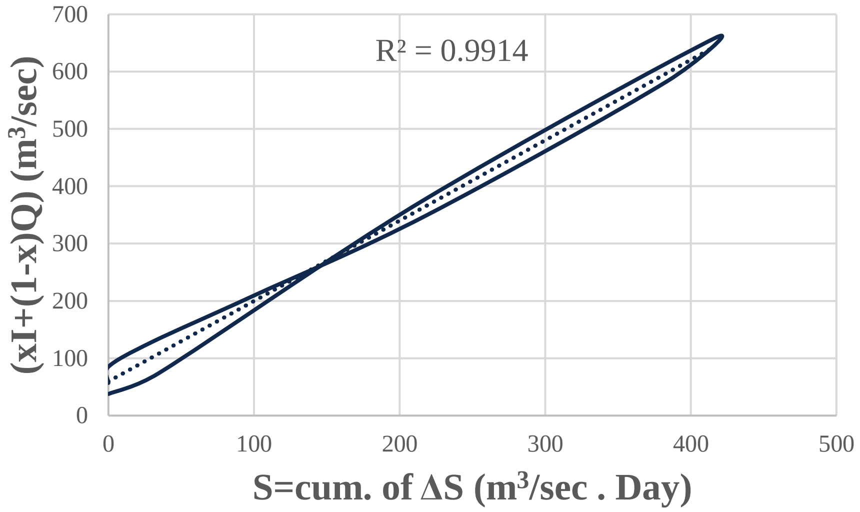
Time (hr)	S=cum. of ΔS (m^3/sec . Day)	(xI+(1-x)Q) (m^3/sec)		
		for x=0.2	for x=0.3	for x=0.25
0	0	32.35	37.8	38
24	34.5	58.8	73.9	70.25
48	215	301.55	373.4	359
72	417	553.6	658.8	653
96	384	505.9	583.4	592.5
120	218.5	305.1	349.3	356.75
144	98.5	181.55	207.7	212.25
168	29.5	111.3	127.9	130.25
192	0	73.85	85.2	86.5
216	0	49	57	57.5

Example

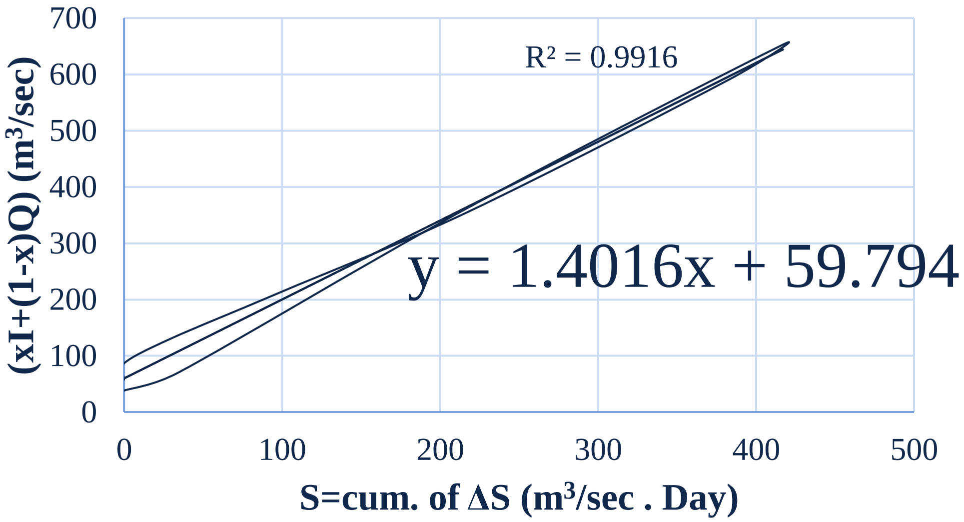
x="0.2"



x=0.3



x="0.25"



Example

Time(hr)	S=cum.of ΔSin m3/sec . Day	(xI+(1-x)Q) in m3/sec for x=0.2	for x=0.3	for x=0.25
0	0	32.35	37.8	38
24	34.5	58.8	73.9	70.25
48	215	301.55	373.4	359
72	417	553.6	658.8	653
96	384	505.9	583.4	592.5
120	218.5	305.1	349.3	356.75
144	98.5	181.55	207.7	212.25
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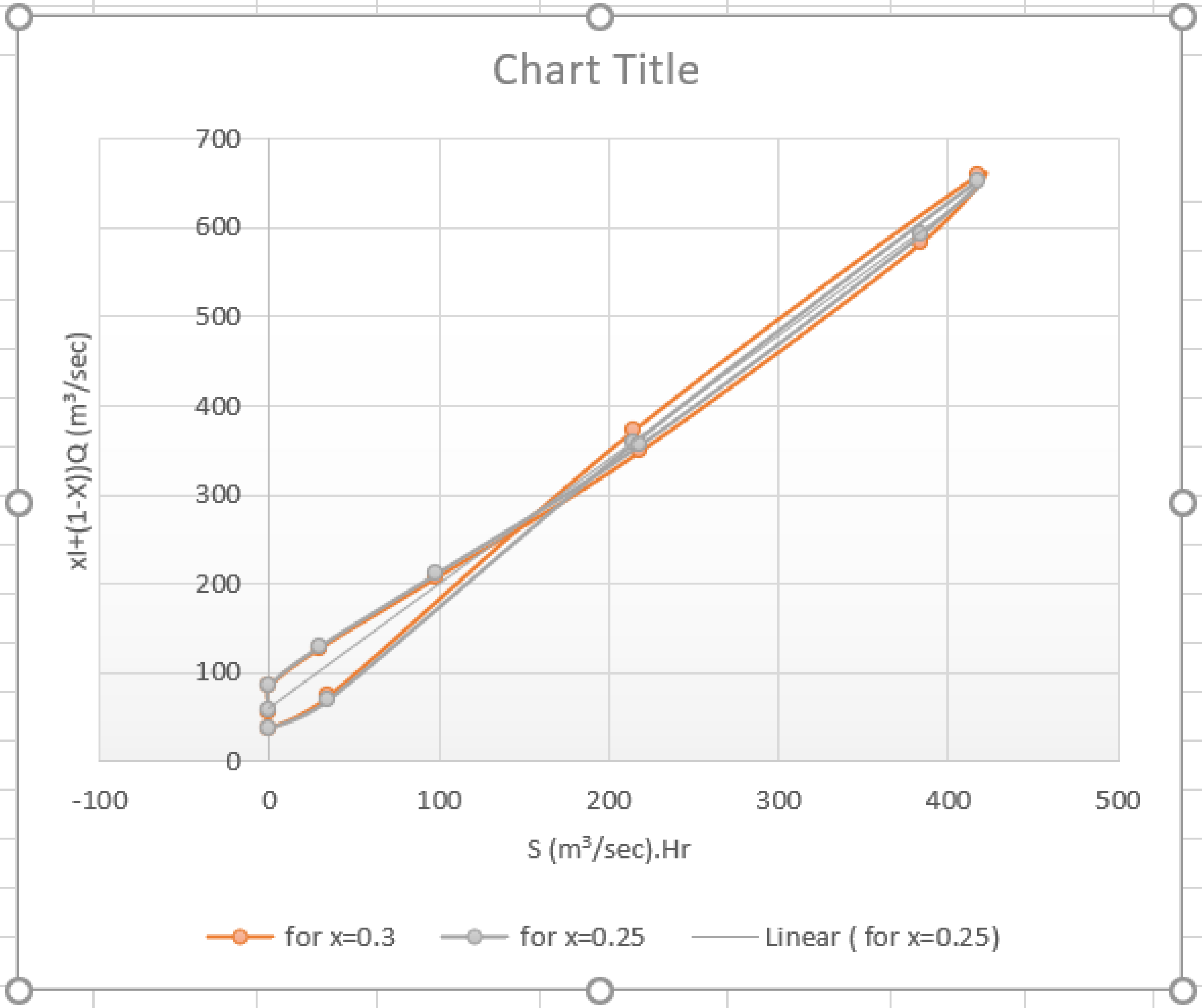


When the 3 trials were drawn on the same graph, it is easy to have visual comparison.

Workout examples

- **Example-2:**

The following inflow and outflow hydrographs were observed in a river reach. Estimate the values of K and x applicable to this reach for use in the Muskingum equation.

Time (h)	0	6	12	18	24	30	36	42	48	54	60	66
Inflow (m^3/s)	5	20	50	50	32	22	15	10	7	5	5	5
Outflow (m^3/s)	5	6	12	29	38	35	29	23	17	13	9	7

Workout examples

- **Example-2**:

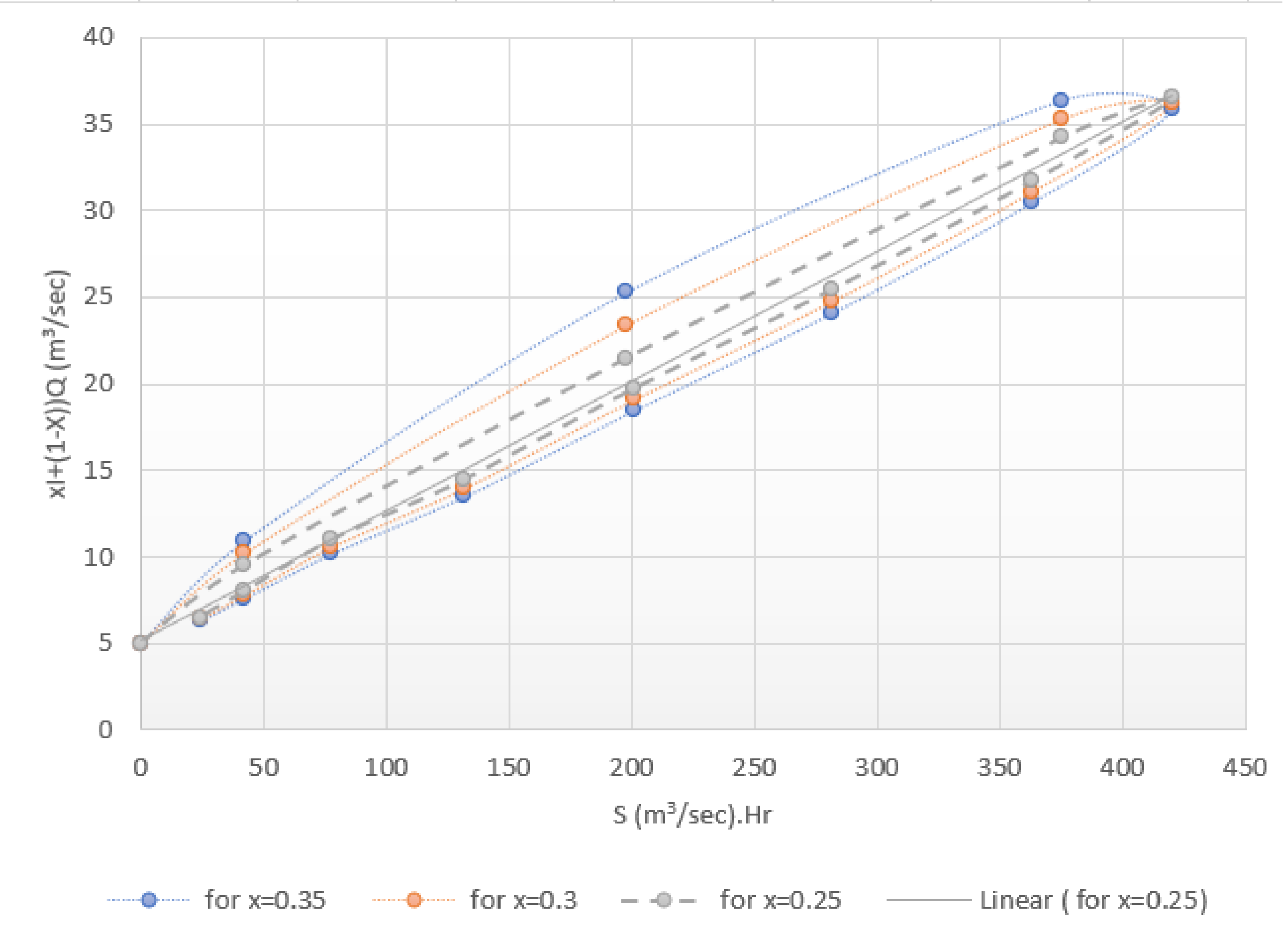
  Solved on excel

A	B	C	D	E	F	G	H	I	J
Time (hr)	Inflow (m^3/sec)	Outflow (m^3/sec)	I-Q	AVERAGE I-Q	ΔS=(AVERAGEI-Q)*Δt (m^3/sec . Hr)	S=cum.of ΔS (m^3/sec . Hr)	for x=0.35	for x=0.3	for x=0.25
1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
0	5	5	0			0	5	5	5
				7	42				
6	20	6	14			42	10.9	10	9.5
				26	156				
12	50	12	38			198	25.3	23	21.5
				29.5	177				
18	50	29	21			375	36.35	35	34.25
				7.5	45				
24	32	38	-6			420	35.9	36	36.5
				-9.5	-57				
30	22	35	-13			363	30.45	31	31.75
				-13.5	-81				
36	15	29	-14			282	24.1	25	25.5
				-13.5	-81				
42	10	23	-13			201	18.45	19	19.75
				-11.5	-69				
48	7	17	-10			132	13.5	14	14.5
				-9	-54				
54	5	13	-8			78	10.2	11	11

Workout examples

- **Example-2: Solved on excel**

S=cum.of ΔS (m^3/sec . Hr)	($xI+(1-x)Q$) (m^3/sec) for x=0.35	for x=0.3	for x=0.25
7	8	9	10
0	5	5	5
42	10.9	10.2	9.5
198	25.3	23.4	21.5
375	36.35	35.3	34.25
420	35.9	36.2	36.5
363	30.45	31.1	31.75
282	24.1	24.8	25.5
201	18.45	19.1	19.75
132	13.5	14	14.5
78	10.2	10.6	11
42	7.6	7.8	8
24	6.3	6.4	6.5



Workout examples

- **Example-2: Solved on text**

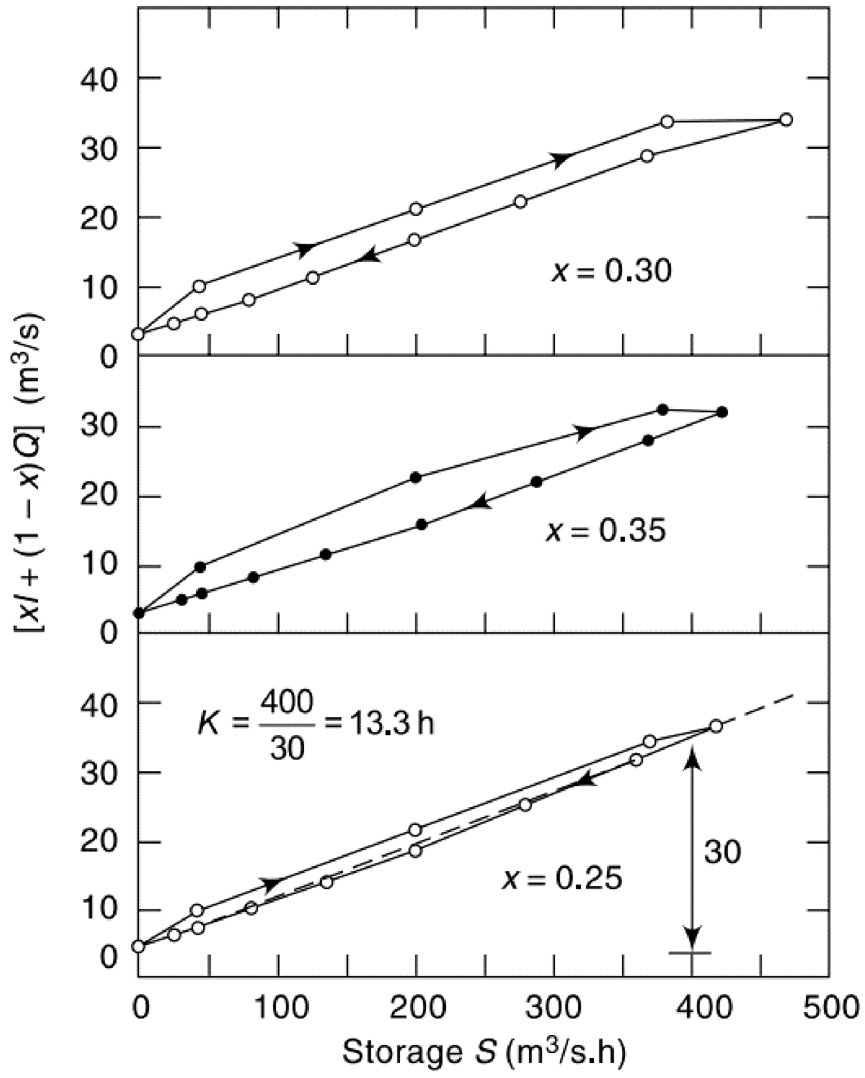
SOLUTION: Using a time increment $\Delta t = 6$ h, the calculations are performed in a tabular manner as in Table 8.3. The incremental storage ΔS and S are calculated in columns 6 and 7 respectively. It is advantageous to use the units [(m^3/s).h] for storage terms.

As a first trial $x = 0.30$ is selected and the value of $[x I + (1 - x) Q]$ evaluated (column 8) and plotted against S in Fig. 8.9. Since a looped curve is obtained, further trials are performed with $x = 0.35$ and 0.25. It is seen from Fig. 8.9 that for $x = 0.25$ the data very nearly describe a straight line and as such $x = 0.25$ is taken as the appropriate value for the reach.

From Fig. 8.9, $K = 13.3$ h

Workout examples

- **Example-2: Solved on text**



Determination of K and x for a channel reach

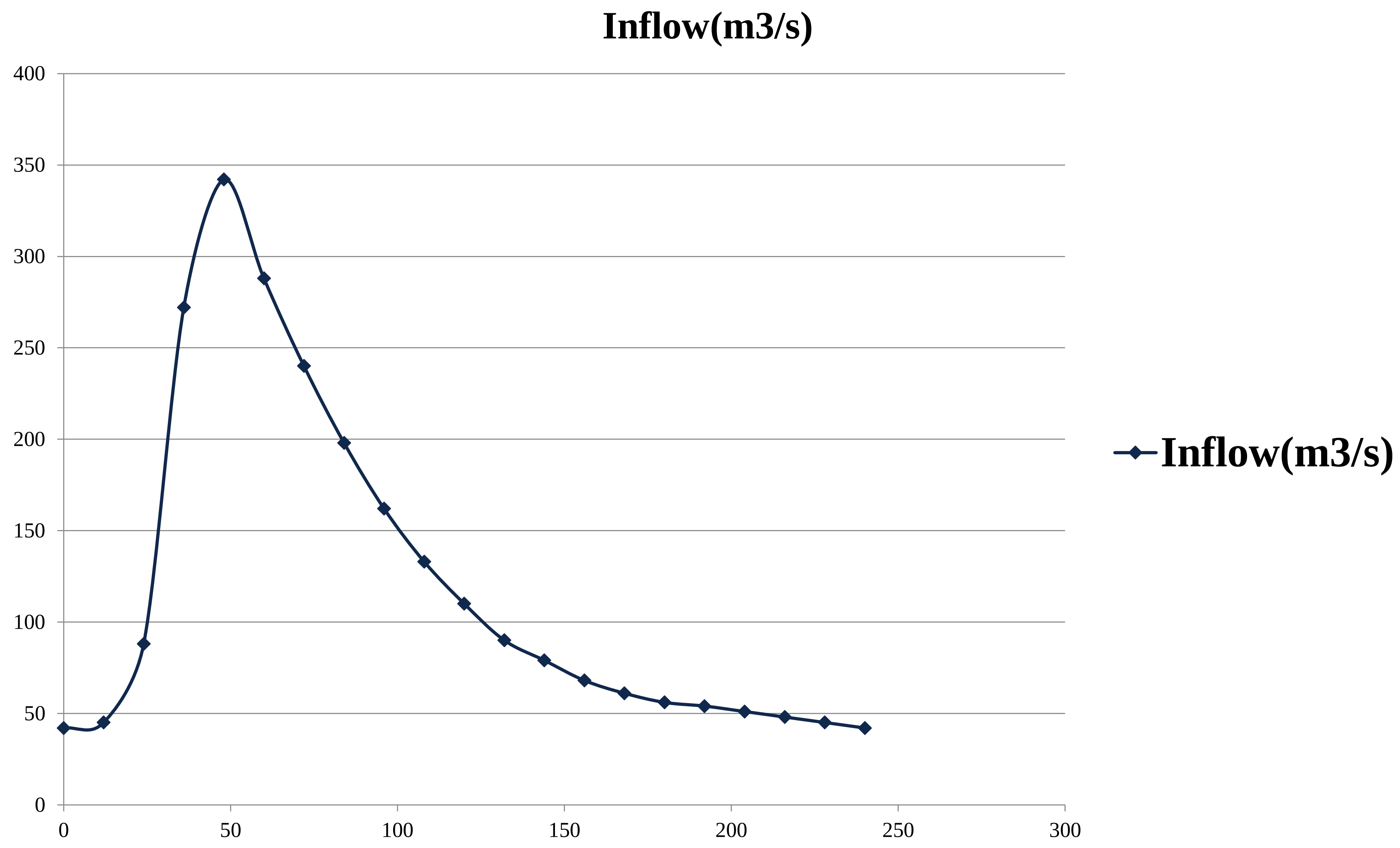
Workout examples

- **Example-3:** The inflow hydrograph readings for a stream reach are given below for which the Muskingum coefficients of K = 36 hr and x = 0.15 apply.
- Route the flood through the reach and determine the outflow hydrograph.
- Also determine the reduction in peak and the time of peak of outflow.

Workout examples

Time (hr)	Inflow (m^3/s)	Time (hr)	Inflow (m^3/s)
0	42	132	90
12	45	144	79
24	88	156	68
36	272	168	61
48	342	180	56
60	288	192	54
72	240	204	51
84	198	216	48
96	162	228	45
108	133	240	42
120	110		

Workout examples



Workout examples

- ***Solution***;
- $\boldsymbol{O_2 = C_0 I_2 + C_1 I_1 + C_2 O_1}$
- $\boldsymbol{O_2 = c_1 I_1 + c_2 I_2 + c_3 O_1}$
- *x = 0.15, K = 36 hr = 1.5 day; take the routing period (from the inflow hydrograph readings) as* 12 hr = 1/2 day=0.5day.
- Compute *C_1, C_2 and C_3 as follows:*

$$C_1 = \frac{\Delta t + 2Kx}{\Delta t + 2K - 2Kx}$$

$$C_2 = \frac{\Delta t - 2Kx}{\Delta t + 2K - 2Kx}$$

$$C_3 = \frac{-\Delta t + 2k - 2KX}{\Delta t + 2k - 2kx}$$

Workout examples

- $$c_1 = (0.5 + 2 * 1.5 * 0.15)/(0.5 + 2 * 1.5 - 2 * 1.5 * 0.5) = 0.31$$

- $C_1 = \dfrac{\Delta t + 2Kx}{\Delta t + 2K - 2Kx}$
- $C_2 = \dfrac{\Delta t - 2Kx}{\Delta t + 2K - 2Kx}$

$C_3 = \dfrac{-\Delta t + 2k - 2KX}{\Delta t + 2k - 2kx}$

$$c_2 = (0.5 - 2 * 1.5 * 0.15)/(0.5 + 2 * 1.5 - 2 * 1.5 * 0.15) = 0.02$$

$$c_3 = (-0.5 + 2 * 1.5 - 2 * 1.5 * 0.15)/1.525 = 0.67$$

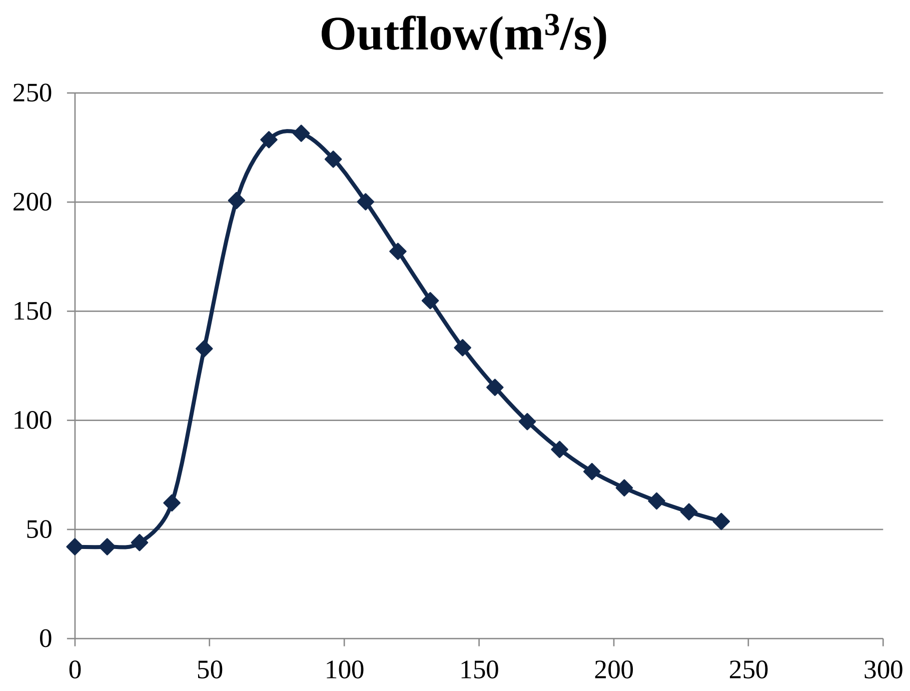
$$O_2 = c_1 I_1 + c_2 I_2 + c_3 O_1$$

$$O_2 = 0.31 I_1 + 0.02 I_2 + 0.67 O_1$$

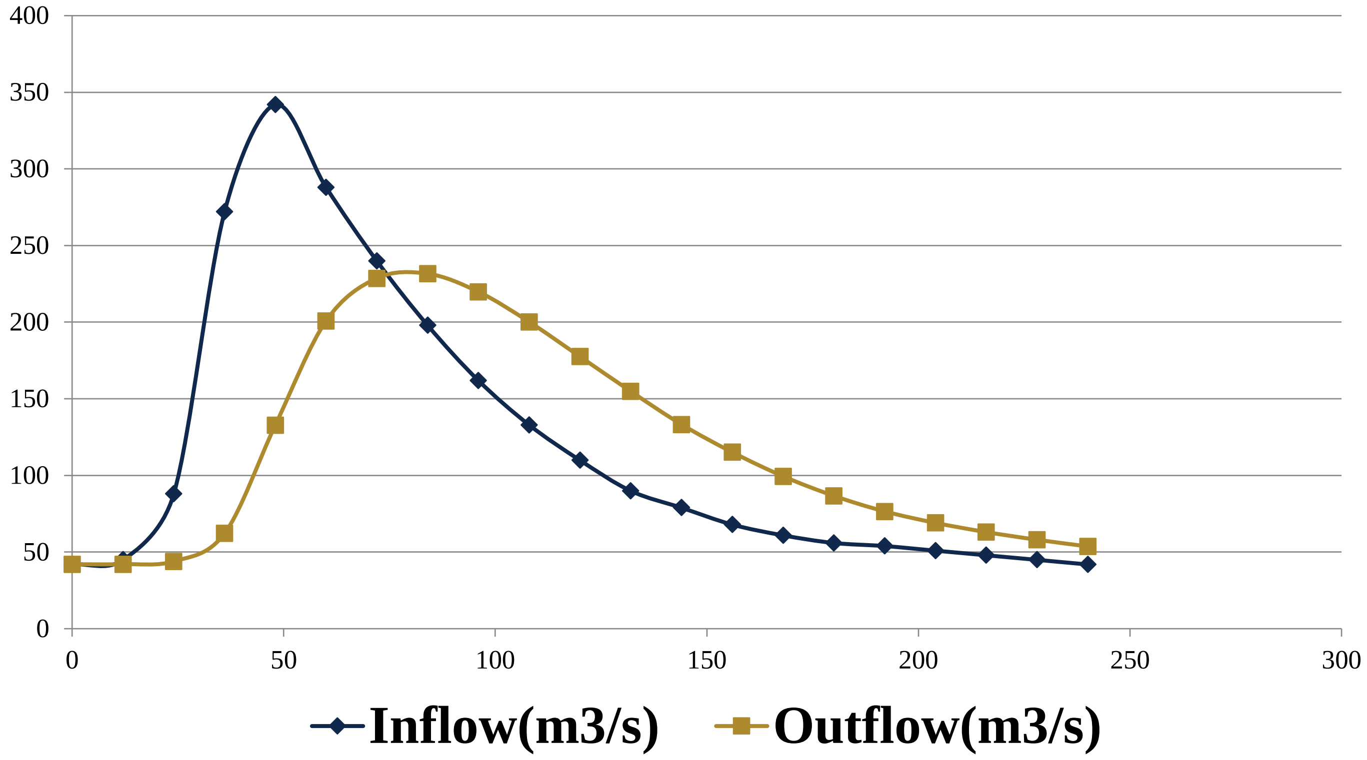
- $\mathbf{I_1}$, $\mathbf{I_2}$ are known from the inflow hydrograph, and $\mathbf{O_1}$ is taken as $\mathbf{I_1}$ at the beginning of the flood since the flow is almost steady.

Workout examples

Time (hr)	Inflow (m^3/s)	0.31I1 (m^3/s)	0.02I2 (m^3/s)	0.67O1 (m^3/s)	outflow (m^3/s)
0	42	_	_	_	42
12	45	13.02	0.9	28.14	42.06
24	88	13.95	1.76	28.18	43.89
36	272	27.28	5.44	29.40	62.12
48	**342**	84.32	6.84	41.62	132.78
60	288	106.02	5.76	88.96	200.74
72	240	89.28	4.8	134.49	228.57
84	198	74.4	3.96	153.14	**231.50**
96	162	61.38	3.24	155.11	219.73
108	133	50.22	2.66	147.21	200.09
120	110	41.23	2.2	134.06	177.49
132	90	34.1	1.8	118.92	154.82
144	79	27.9	1.58	103.73	133.21
156	68	24.49	1.36	89.25	115.10
168	61	21.08	1.22	77.11	99.41
180	56	18.91	1.12	66.61	86.64
192	54	17.36	1.08	58.04	76.48
204	51	16.74	1.02	51.24	69.00
216	48	15.81	0.96	46.23	63.00
228	45	14.88	0.9	42.21	57.99
240	42	13.95	0.84	38.85	53.64



Workout examples



The reduction in peak **is 110.5 m^3/s** and the lag time is **36 hr**, *i.e., the peak* outflow is after 84 hr (= 3 .5 days) after the commencement of the flood through the reach.

Home assignment:

Make sure that you can write, select, compare on the following topics:

- Flood routing
- Importance of flood routing
- Types of flood routing
- Techniques of flood routing
- Mathematical representation of flood routing
- Calculation for K, X, Q

References

Ojha, C.S.P., R. Berndtsson, and P. Bhunya, 2008. Engineering Hydrology, Oxford University Press ISBN: 978-0-19-569461-1

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Thank you very much for your active attendance!!

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