Engineering Hydrology

Week-8

CHAPTER -4 FREQUENCY ANALYSIS

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Lecture contents of the last weeks (Week-6-7)

CHAPTER -3 Flood Routing

3.1 General

3.2 Simple non-storage routing

3.3 Reservoir or Level Pool Routing

3.4 Channel Routing

Home assignment:

I hope you are sure you area able to exercise on:

- Flood routing
- Importance of flood routing
- Types of flood routing
- Techniques of flood routing
- Mathematical representation of flood routing
- Problem analysis using paper and excel

Lecture contents of the week (Week-8 & 9)

HAPTER -4 FREQUENCY ANALYSIS

4.1 Flow Frequency

4.2 Flood Probability

4.3 Methods of Peak flood determination

Lecture Learning Outcomes

Course Learning Outcomes: After completion of this Lecture, you will be able to:

CLO-1: Apply measurement techniques of the components of the hydrologic cycle, water balance and filling of missed data;

CLO-2: Examine rainfall-runoff relationship and hydrograph;

- Apply flood routing

CLO-3: Examine the probability of occurrence;

- **Define** flood frequency, probability and recurrence interval
- **List** methods of flood frequency analysis
- **Analyze** design flood using Gumbel's and log-pearson methods
- **Describe** regional frequency and low flow analysis
- **Explain** risk, reliability and safety factor

CLO-4: Analyze the water movement in to, over, and through the soil surface;
CLO-5: Design capacity of reservoir;
CLO-6: Design runoff volume and time of distribution of the runoff hydrograph from urbanization effect.

Frequency Analysis: General

- The annual **peak discharge** is considered to be a random variable.
- Probability and statistical methods are employed for analysis of random variables.
- A statement of the probability of the stream flows is **important for planning** water resources development
  - These probabilities are important to the **economic and social evaluation** of a project.
- The failure of which seriously threatens human life, a more **extreme event**, the **probable maximum flood**, has become the standard for designing the spillway.

Frequency Analysis: Importance

- The results of flood flow frequency analysis can be used for many engineering purposes:
  - for the design of dams, bridges, culverts, and flood control structures;
  - to determine the economic value of flood control projects; and
  - to delineate flood plains and determine the effect of encroachments on the flood plain.
- **Frequency** is the **number of times** a specific data value occurs in a dataset, while
- **Cumulative frequency** is the running **total of frequencies**, showing how many times a data value or a value **below it** has occurred.

Frequency Analysis: Importance

- Defining probability from a given set of hydrologic data employed for determining design flood for major hydraulic structures.
- Frequency analysis is the hydrologic term used to describe the **probability of occurrence** of a particular hydrologic event (e.g. rainfall, flood, drought, etc.).
- For planning and designing of water resources development projects, the important parameters are **river discharges** and related questions on the **frequency** & **duration** of normal flows and **extreme** flows.
- *The objective of frequency analysis of hydrologic data is to relate:*
  - *the magnitude of extreme events to their frequency of occurrence through the use of probability distributions*

Frequency Analysis: Terminologies

- **Univariate**: Uni means "one", so the data has only one variable (univariate). Univariate data requires to analyze each variable separately.
- **Random event/process/variable**: an event/process that is not and cannot be made exact and, consequently, whose outcome cannot be predicted, e.g., the sum of the numbers on two rolled dice.
- **Probability**: an estimate of the likelihood that a random event will produce a certain outcome.
- **A parameter** is a value or variable that influences the behavior or characteristics of a system, process, or function.

Frequency Analysis: Terminologies

- **A probability distribution**: is a statistical function that describes all the possible values and likelihoods that a random variable can take within a given range.
  - It depicts the expected outcomes of possible values for a given data-generating process.
  - It is defined by the ***mean, standard deviation, skewness***, and ***kurtosis***.
    - Read and do some exercise on these statistical distributions

Frequency Analysis: Terminologies

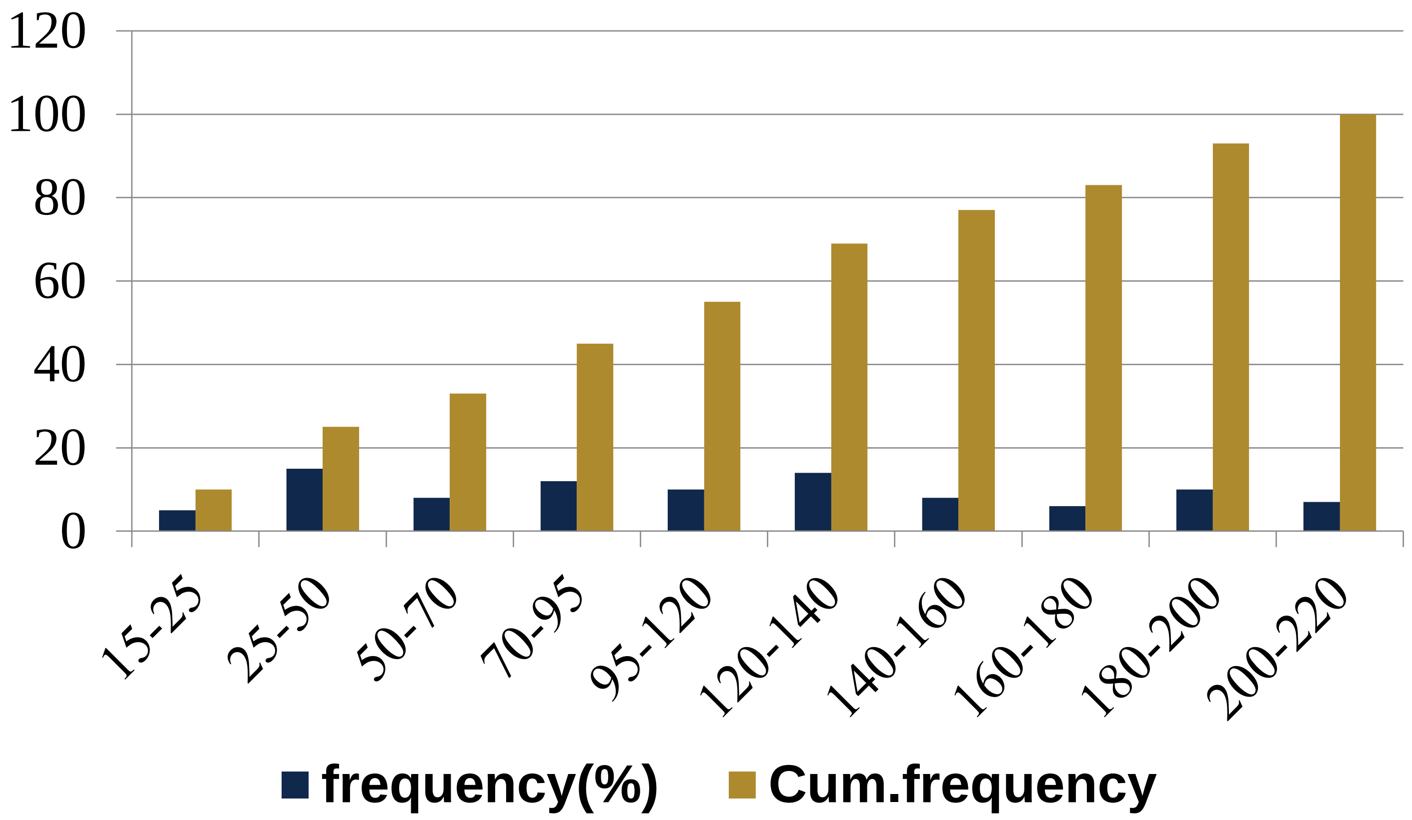
- **Plotting positions**: are numerical values that represent the probability of an observed event or value being exceeded within a dataset or sample.
- **A frequency distribution** is a way to organize and display the frequency of each value or category within a dataset.
  - It's presented in the form of a table or graph, that shows how many times each value appears.
  - **Frequency**: The number of times a specific value or category occurs in a dataset.
  - **Distribution**: The pattern or arrangement of frequencies across the different values or categories.

Frequency Analysis: Terminologies

- **A frequency distribution** is a way to organize and display the frequency of each value or category within a dataset.
  - **Ungrouped Frequency Distribution**: Each unique value in the dataset is listed with its frequency.
  - **Grouped Frequency Distribution**: Data is grouped into intervals or classes, and the frequency of observations within each class is shown.
  - **Relative Frequency**: The frequency of a value or category divided by the total number of observations, often expressed as a percentage.
  - **Cumulative Frequency**: The running total of frequencies as you move through the sorted data.

Frequency Analysis: Terminologies

Class	Frequency (%)	Cumm. frequency
15-25	10	10
25-50	15	25
50-70	8	33
70-95	12	45
95-120	10	55
120-140	14	69
140-160	8	77
160-180	6	83
180-200	10	93
200-220	7	100



Frequency Analysis: Hydrologic Statistics

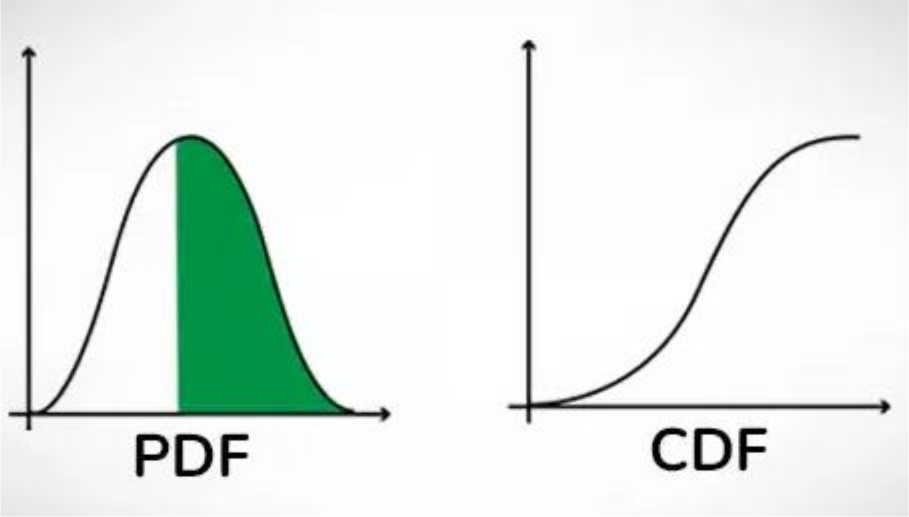
Remember from your past knowledge to:

1. define some of the important terms of **statistical and probabilistic hydrology**;
2. describe measures of **location**, **dispersion**, and **symmetry** of both grouped and ungrouped data;
3. describe **standard errors** of mean, **standard deviation**, and coefficient of **skewness**;

Frequency Analysis: Hydrologic Statistics

Remember from your past knowledge to:

1. **Sample data**: A sample consists of the data derived fi-om the observation of an event.
2. **Population**: A population can be defined as a combination of infinite number of discrete samples.
3. **Random events:** Those events whose occurrences are not affected by their earlier occurrences are called random events.
4. **Probability density function (PDF)**: It is the probability of occurrence of an event.
5. **Cumulative density function (CDF)**: It is the probability of non- of the event (less than or equal to).
6. **Parameters and statistics**



[CDF vs. PDF: What is the Difference? | GeeksforGeeks]

Frequency Analysis: Hydrologic Statistics

The commonly used sample characteristics include:

1. the **central tendency** or the value around which all other values are clustered, [mean, median]
2. the **spread** of sample values about the mean, [variance, SD, range]
3. the **asymmetry or skewness** of the frequency distribution, and
4. the **flatness** of the frequency distribution, [kurtosis].

Frequency Analysis: Hydrologic Statistics

The commonly used sample characteristics include:

1. **Variance (S^2):** It represents the dispersion about the mean.

$$S^2 = \frac{\sum_{i=1}^{N} (x_i - \bar{x})^2}{N}$$

2. **Standard deviation (S):** is the square root of the variance.

$$S = [1/(N-1) \sum_{i=1}^{N} (x_i - \bar{x})^2]^{1/2}$$

The commonly used sample characteristics include:

3. **Coefficient of variation (C_v)**: is a dimensionless dispersion parameter and is equal to the ratio of the standard deviation to the mean.

$$C_v = S / \overline{x}$$

4. **Symmetry**:

   1. **Third central moment (M_3)**: of sample data about the mean is given by:

$$M_3 = \frac{1}{N} \sum_{i=1}^{N} (x_i - \overline{x})^3$$

Frequency Analysis: Hydrologic Statistics

The commonly used sample characteristics include:

4. **Symmetry**:

   2. **Skewness coefficient (S_c)**: of sample data about the mean is given by:

$$C_s = \frac{N \sum_{i=1}^{N} (x_i - \bar{x})^3}{(N-1)(N-2)S^3}$$

5. **Peakedness** or **Flatness:** of the frequency distribution near its center is measured by **kurtosis coefficient** (C_k), which is expressed as:

$$C_k = \frac{N^2 \sum_{i=1}^{N} (x_i - \bar{x})^4}{(N-1)(N-2)(N-3)\,S^4}$$

Frequency Analysis: Hydrologic Statistics

The commonly used sample characteristics include:

6. **Standard Errors:** mean and standard deviation, derived from a short period of sample record, represent only an estimate of the population statistic.

- The reliability of such estimates can be evaluated from their standard errors (SE).
- The standard errors of commonly used mean, $\boldsymbol{SE(\bar{X})}$, standard deviation, $\boldsymbol{SE(S)}$;

$$SE(\bar{X}) = S/\sqrt{N}$$

$$SE(S) = S/\sqrt{2N}$$

Frequency Analysis: Hydrologic Statistics

The of the commonly probability distribution used in hydrology and water resources are**:**

1. **Normal**, ---- reduced variate: $z = \frac{x-\mu}{\delta}$ (μ, δ),
2. **Lognormal (two- and three-parameters)**, hydrologic variables act multiplicatively rather than additively. ----
3. **Extreme value type-I (Gumbel or EVI)**, ----
4. **Pearson type-III**, or **Gamma** -----
5. **Log-Pearson type-III**, -----
6. **General extreme value (GEV)**, -----

Flood frequency distributions

Distribution	*Probability Density Function*	*Range*	*Equation of parameters in terms of the sample moments*
Exponential	$f(x)=\frac{1}{\beta}\exp\left[\frac{-(x)}{\beta}\right]$	$x \geq 0$	$\beta=\bar{x}$
Extreme value type-I	$f(x)=\frac{1}{\beta}\exp\left[-\frac{x-u}{\beta}-\exp\left(-\frac{x-u}{\beta}\right)\right]$	$(-\infty < x < \infty)$	$u=\bar{x}-0.5772\beta$ $\beta=\frac{\sqrt{6}\,s_x}{\pi}$
Gamma	$f(x)=\frac{(x)^{\gamma-1}e^{-x/\beta}}{\beta^{\gamma}\,\Gamma(\gamma)}$	$x \geq 0$	$\beta=\frac{(s_x)^2}{\bar{x}}$
Normal	$f(x)=\frac{1}{\sigma\sqrt{2\pi}}\exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)$	$(-\infty \leq x \leq \infty)$	$\mu=\bar{x}$, $\sigma=s_x$
Lognormal	$f(x)=\frac{1}{x\sigma_L\sqrt{2\pi}}\exp\left[-\frac{1}{2}\left(\frac{\ln x-\mu_L}{\sigma_L}\right)^2\right]$	$x > 0$	$\mu_y=\bar{y}$, $\sigma_y=s_y$
Pearson type-III	$f(x)=\frac{(x-u)^{\gamma-1}\exp\left[\frac{-(x-u)}{\beta}\right]}{\beta^{\gamma}\Gamma(\gamma)}$	$x \geq u$	$u=\bar{x}-s_x\sqrt{\gamma}$, $\beta=\frac{\sqrt{\gamma}}{s_x}$, $\gamma=\left(\frac{2}{g}\right)^2$
Log-Pearson type-III	$f(x)=\frac{(\ln x-u)^{\gamma-1}\exp[-(\ln x-u)/\beta]}{\lvert\beta\rvert\,\Gamma(\gamma)}$	$\ln x \geq u$	$u=\bar{y}-s_y\sqrt{\gamma}$ $\beta=\frac{\sqrt{\gamma}}{s_y}$, $\gamma=\left(\frac{2}{g_y}\right)^2$

- Probability distribution parameters in relation to sample moments

Frequency Analysis: Hydrologic Statistics

Example 1: Compute standard error of mean and standard deviation of the data given:

Month	Jan	Feb	Mar	Apr	May	June	July	Aug	Sept	Oct	Nov	Dec
Rainfall in mm	12	10	5	1	2	8	15	25	22	15	12	12

(a) Find the mean rainfall, variance, standard deviation and coefficient of variation.

(b) The standard error

(c) Skewness and kurtosis

Interpret your results.

Frequency Analysis: Hydrologic Statistics

Given:

Month	Jan	Feb	Mar	Apr	May	June	July	Aug	Sept	Oct	Nov	Dec
Rainfall in mm	12	10	5	1	2	8	15	25	22	15	12	12

Required: (a) $\bar{X}$ (mm), S^2, S, and Cv

(b) The standard error

Solution: **(a)**

$$Mean\ rainfall, \bar{X}$$

$$= \frac{12 + 10 + 5 + 1 + 2 + 8 + 15 + 25 + 22 + 15 + 12 + 12}{12}$$

$$= 11.58\ mm$$

$$\text{Variance } S^2 = \frac{\begin{bmatrix}(12-11.58)^2 + (10-11.58)^2 + (5-11.58)^2 + (1-11.58)^2 \\ +(2-11.58)^2 + (8-11.58)^2 + (15-11.58)^2 + (25-11.58)^2 \\ (22-11.58)^2 + (15-11.58)^2 + (12-11.58)^2 + (12-11.58)^2\end{bmatrix}}{12-1}$$

$$= \frac{\begin{bmatrix}0.1764 + 2.4964 + 43.2964 + 111.9364 \\ +91.7764 + 12.8164 + 11.6964 + 180.0964 \\ +108.5764 + 11.6964 + 0.1764 + 0.1764\end{bmatrix}}{11}$$

$$= 52.26$$

Standard deviation $S = 7.229$

Coefficient of variation = 7.229/11.58 = 0.624

Frequency Analysis: Hydrologic Statistics

Given: (b) N=12, and S=7.229 **Required:** (b) $\boldsymbol{SE}(\bar{\boldsymbol{X}})$, and, $\boldsymbol{SE}(\boldsymbol{S})$;

Solution:

$$\text{SE of mean} = \frac{7.229}{\sqrt{12}} = 2.086$$

$$\text{SE of standard deviation} = \frac{7.229}{\sqrt{2 \times 12}} = 1.475$$

Do the same for Skewness coefficient ($\boldsymbol{C_s}$) and kurtosis coefficient ($\boldsymbol{C_k}$):

$$C_s = 12 \times \frac{\begin{bmatrix} (12-11.58)^3 + (10-11.58)^3 + (5-11.58)^3 \\ + (1-11.58)^3 + (2-11.58)^3 + (8-11.58)^3 \\ + (15-11.58)^3 + (25-11.58)^3 + (22-11.58)^3 \\ + (15-11.58)^3 + (12-11.58)^3 + (12-11.58)^3 \end{bmatrix}}{(12-1)(12-2)(7.229)^3}$$

$$= 12 \times \frac{\begin{bmatrix} 0.074 - 3.9443 - 284.8903 - 1184.2871 \\ -879.2179 - 45.8827 + 40.00 + 2416.8936 \\ +1131.366 + 40.00 + 0.074 + 0.074 \end{bmatrix}}{11 \times 10 \times 377.77}$$

$$= 0.355$$

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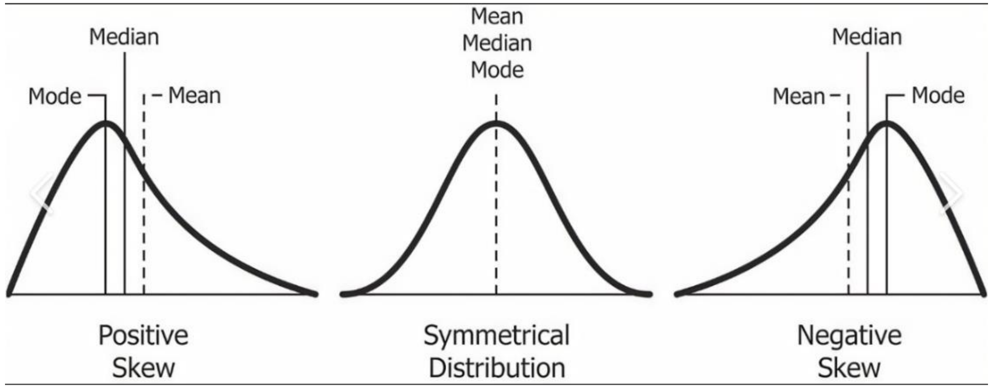
Frequency Analysis: Hydrologic Statistics

Interpretations:

- A **high standard error** suggests that sample means are **widely spread** around the population mean, meaning the sample may not be a good representation of the population.
- **The CV** provides a standardized measure of variability, making it useful for comparing variability across different datasets or variables with different scales.
- A **lower CV** suggests **less relative variability**, with data points clustered closer to the mean.
- A symmetric distribution has a **skewness of zero**, while **positive** skewness indicates a longer tail to the **right**, and negative skewness indicates a longer tail to the left.



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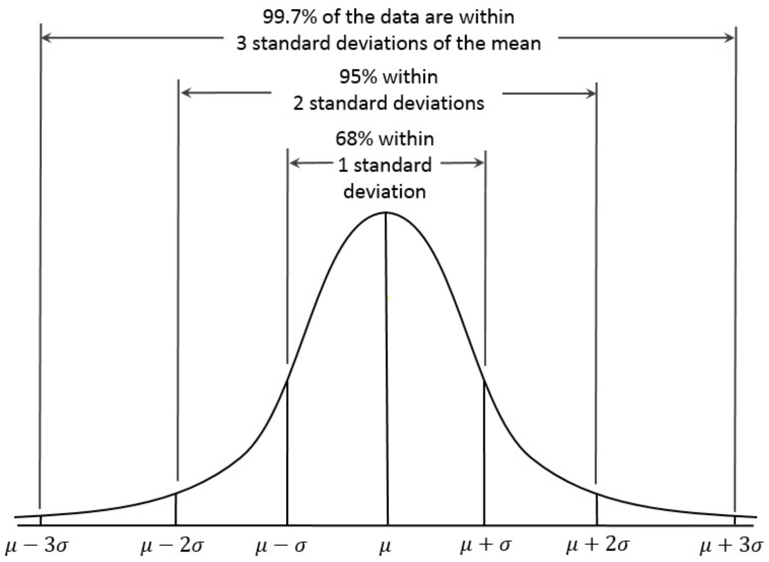

Frequency Analysis: Hydrologic Statistics

Probability of normal distribution:

- The normal distribution is one of the most important distributions.
- This is a bell-shaped symmetrical distribution having coefficient of skewness equal to zero.
- In terms of reduced variate: $z = (\frac{x-\mu}{\delta})$

$$\text{P.D.F.: } f(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right]$$

$$\text{C.D.F.: } F(x) = \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{x} \exp\left[-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right] dx$$

$$\text{P.D.F.: } f(z) = \frac{1}{\sqrt{2\pi}} \exp(-z^2/2)$$

$$\text{C.D.F.: } F(z) = -\int_{\infty}^{z} \frac{1}{\sqrt{2\pi}} \exp(-z^2/2)\, dz$$

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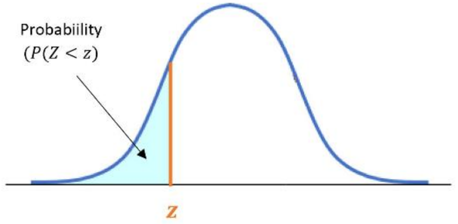


z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.00	0.5000	0.4960	0.4920	0.4880	0.4841	0.4801	0.4761	0.4721	0.4681	0.4641
-0.10	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
-0.20	0.4207	0.4168	0.4129	0.4091	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
-0.30	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
-0.40	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
-0.50	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
-0.60	0.2743	0.2709	0.2676	0.2644	0.2611	0.2579	0.2546	0.2514	0.2483	0.2451
-0.70	0.2420	0.2389	0.2358	0.2327	0.2297	0.2266	0.2236	0.2207	0.2177	0.2148
-0.80	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
-0.90	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
-1.00	0.1587	0.1563	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
-1.10	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
-1.20	0.1151	0.1131	0.1112	0.1094	0.1075	0.1057	0.1038	0.1020	0.1003	0.0985
-1.30	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
-1.40	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0722	0.0708	0.0694	0.0681
-1.50	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
-1.60	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
-1.70	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
-1.80	0.0359	0.0352	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
-1.90	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
-2.00	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
-2.10	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
-2.20	0.0139	0.0136	0.0132	0.0129	0.0126	0.0122	0.0119	0.0116	0.0113	0.0110
-2.30	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
-2.40	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0070	0.0068	0.0066	0.0064
-2.50	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
-2.60	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
-2.70	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
-2.80	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
-2.90	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
-3.00	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
-3.10	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
-3.20	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
-3.30	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
-3.40	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002	0.0002

Frequency Analysis: Hydrologic Statistics

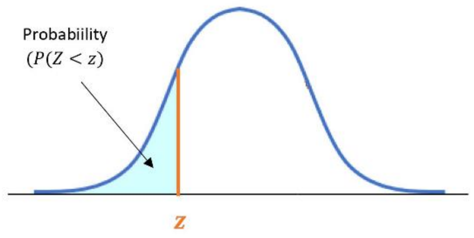
Example 2: From a mean monthly rainfall data of 10 years, the mean is 65mm and standard deviation is 7.0. Find the percentage of magnitude of rainfall which is:

(a) between 70 mm and 80 mm

(b) at least 90 mm

Assume that the rainfall data are ***normally distributed***.

Use Table to find the value of variate, z.



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0.00	0.5000	0.4960	0.4920	0.4880	0.4841	0.4801	0.4761	0.4721	0.4681	0.4641
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-0.40	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
-0.50	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
-0.60	0.2743	0.2709	0.2676	0.2644	0.2611	0.2579	0.2546	0.2514	0.2483	0.2451
-0.70	0.2420	0.2389	0.2358	0.2327	0.2297	0.2266	0.2236	0.2207	0.2177	0.2148
-0.80	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
-0.90	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
-1.00	0.1587	0.1563	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
-1.10	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
-1.20	0.1151	0.1131	0.1112	0.1094	0.1075	0.1057	0.1038	0.1020	0.1003	0.0985
-1.30	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
-1.40	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0722	0.0708	0.0694	0.0681
-1.50	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
-1.60	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
-1.70	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
-1.80	0.0359	0.0352	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
-1.90	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
-2.00	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
-2.10	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
-2.20	0.0139	0.0136	0.0132	0.0129	0.0126	0.0122	0.0119	0.0116	0.0113	0.0110
-2.30	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
-2.40	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0070	0.0068	0.0066	0.0064
-2.50	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
-2.60	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
-2.70	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
-2.80	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
-2.90	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
-3.00	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
-3.10	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
-3.20	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
-3.30	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
-3.40	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002	0.0002

Frequency Analysis: Hydrologic Statistics

Given: N= 10 years, μ = 65mm, and δ=7.0.

Required: Find the percentage of magnitude of rainfall which is:

(a) $P(70 \leq x \leq 80) =?$

(b) $P(x \geq 90) =?$

$z_1 = (70 - 65)/7 = -0.71$

$z_2 = (80 - 65)/7 = 2.14$

Solution: (a)

$$P(70 \leq x \leq 80) = P(0.71 \leq z \leq 2.14)$$

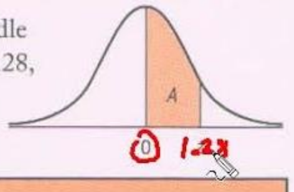
$$= F(2.14) - F(0.71) = 0.9838 - 0.7612$$

$$= 0.2226$$

- Approximately 22.26% of rainfall occurred having a magnitude range of 70 mm to 80 mm.

Normal Curve (z) Table

The normal curve table gives only the percentage of data starting from the middle ($z = 0$), out to whatever z score you look up. For instance, if you look up $z = 1.28$, you get .3997. This means .3997 or 39.97% of the data in the normal curve is found between $z = 0$ and $z = 1.28$.



	Normal									
z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
0.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
0.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
0.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2517	.2549
0.7	.2580	.2611	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
0.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706

Frequency Analysis: Hydrologic Statistics

Solution: (b)

$$z_1 = (90 - 65)/7 = 3.57$$

$$P(x \geq 90) = P(z \geq 3.57) = 1 - F(3.57) = 1 - 0.9998$$

$$= 0.0002$$

- Approximately 0.02% of rainfall occurred having a magnitude of at least 90 mm.
- **List and discuss the engineering importance of the above results**.

Probability and return period

- **Probability** is the **chance** that an event will occur in any given year.
- Probability analysis seeks to define the flood flow with probability of ***p*** being equaled or exceed in any year.
- The probability of each events can be calculated as follow:

$$P = \frac{m}{N+1}$$

- The recurrence interval ***T***, (also called return period) of frequency is calculated as follow

$$T = \frac{1}{P}$$

- **Return period or recurrence interval**, is the **average time between occurrences** of an event, like a flood, of a certain magnitude or greater.
  - It's the **inverse** of the annual probability of exceedance
  - For example, **a 100-year flood** means that there's a 1% chance/probability of a flood of that magnitude occurring in **any given year**.

Probability and return period

- Consider, for example, a list of flood magnitudes of a river arranged in **descending order** as shown in Table.
- The length of record is 50 years.

Table 4.3: Calculation of Frequency T

Order No. m	Flood magnitude Q (m^3/s)	T in years = 51/m
1	160	51.00
2	135	25.50
3	128	17.00
4	116	12.75
.	.	.
.	.	.
.	.	.
49	65	1.04
50	63	1.02

Source: ravan …

Flood frequency distributions

- A plot of ***Q*** Vs ***T*** yields the probability distribution.
- For small return periods (i.e. for interpolation) or where limited extrapolation is required, a simple best-fitting curve through plotted points can be used as the probability distribution.
- However, when larger extrapolations of ***T*** are involved,
  - **Gumbel extreme-value**,
  - **Log-Pearson Type III**, and
  - **log normal distributions** have to be used.

Theoretical Distributions

- **Most frequency-distribution** functions applicable in hydrologic studies can be expressed by the following equation known as the **general equation** of hydrologic frequency analysis:

$$x_T = \bar{x} + K\sigma$$

- Where $\boldsymbol{x_T}$ = value of the variate $\boldsymbol{X}$ of a random hydrologic series with a return period $\boldsymbol{T}$,
  - $\bar{x}$ = mean of the variate, location parameter
  - $\boldsymbol{\sigma}$ = standard deviation of the variate,
  - $\boldsymbol{K}$ = **frequency factor** which depends upon the return period, ***T*,** and the assumed frequency distribution.

Theoretical Distributions

- **Or,** Let the relation among x_T (discharge corresponding to a return period T), location parameter u, and scale parameter β be of the type: $X_T = u + \beta Y_T$

I. Extreme-Value Type I Distribution (Gumbel's Method)

- This extreme value theory of Gumbel is only applicable to **annual extremes.**
- Data are ranked in **ascending order** and it makes use of the **probability of non-excedence, *q***.

$$q = 1 - P$$

- The return period **T** is therefore given by

$$T = 1 / P_{ne} = 1 / (1 - q)$$

I. Extreme-Value Type I Distribution (Gumbel's Method)

- It is one of the most commonly used distributions in flood frequency analysis.
- It is the double exponential distribution also known as:
  - **the double exponential distribution**
  - **Gumbel distribution** or
  - **Extreme value type-I** or
  - **Gumbel EV1 distribution**.
- The PDF and CDF of the distribution (the probability of occurrence of an event equal to or larger than a value $x_0, P(x \geq x_0)$) is:

P.D.F.: $$f(x) = \frac{1}{\beta}\exp\left[-\left(\frac{x-u}{\beta}\right) - \exp\left\{-\left(\frac{x-u}{\beta}\right)\right\}\right]$$

C.D.F.: $$F(x) = e^{-\exp\left[-\left(\frac{x-u}{\beta}\right)\right]}$$

In terms of reduced variate: $z = \dfrac{x-u}{\beta}$

P.D.F.: $$f(z) = e^{-z-(e^{-z})}$$

C.D.F.: $$F(z) = e^{-e^{-z}}$$

Gumbel's Method

- for prediction of flood peaks, maximum rainfalls, maximum wind speed, etc.
- Gumbel makes use of a **reduced variate *y*** as a function of ***q*,**
- According to his theory of extreme events, the probability of occurrence of an event **equal to or larger than** a value $\boldsymbol{x_0}$ is:

$$P\,(X \geq x_0) = 1 - e^{-e^{-y}}$$

- In which y is a dimensionless variable given by:

$$y = a\,(\bar{x} - \alpha\,), a = x - 0.45005\,\alpha_x, \alpha = \frac{1.2825}{\alpha_x}$$

- Then, $y = \frac{1.285(X - X)}{\delta_X} - + 0.577$
- The value of the variate X with a return period T is: $x_T = x + K\,s_x$ and
- $K = \frac{(y_T - 0.577)}{1.2825}$

where,
K = frequency factor
$\boldsymbol{y_T}$ = reduced variate
$\boldsymbol{S_n}$ = reduced standard deviation

Theoretical Distributions of Floods

- The original Gumbel equation describes the probability of non-excedence (q)

$$\boldsymbol{q} = \exp[-\exp\{-\boldsymbol{y}\}]$$

y = -1n(-1n(q)) = -1n(-1n(1-p))

$$y_T = -\left[\ln\left(\ln\left(\frac{T}{T-1}\right)\right)\right]$$

$$y_T = -\left[0.834 + 2.303\log\left(\log\left(\frac{T}{T-1}\right)\right)\right]$$

$$x_T = \bar{x} + K\sigma_x$$

applicable to an infinite sample size (i.e. N $\rightarrow \infty$).

$$K = \frac{(y_T - 0.577)}{1.2825}$$

Gumbel's Equation for Practical use

- The above equation giving the value of the variate x with recurrence interval T is used as:

$$x_T = \bar{x} + K\sigma_{n-1}$$

$$\sigma_{n-1} = \sqrt{\frac{(x-\bar{x})^2}{N-1}}$$

Where σ_{n-1} = standard deviation of the sample

K= frequency Factor expressed as: $K = \frac{y_T - \bar{y}_n}{s_n}$

In which y_T = reduced variate, a function of T and given as:

$$y_T = -[\ln.\ln \tfrac{T}{T-1}]$$

Gumbel's Equation for Practical use

- $\overline{y_n}$ is a reduced mean, a function of sample size N and is given in table

  (For N $\rightarrow \infty$,) $\overline{y_n} \rightarrow 0.577$

- S_n is a reduced standard deviation, which is a function sample size N and is given in table.

  (For N $\rightarrow \infty$,) $\overline{y} \rightarrow 1.2825$

Gumbel's Equation for Practical use

Procedure to estimate the flood magnitude corresponding to a given **return period** based on an annual flood series.

1. Assemble the discharge data and note the sample size N. Here the annual flood value is the variate $\boldsymbol{x}$. Find $\bar{x}$ and $\sigma\_(n-1)$ for the given data.
2. Using **a table**, determine $\overline{y_n}$ and S_n appropriate to given N.
3. Find y_T for a given ***T***.
4. Find ***K***.
5. Determine the required x_T
6. Plot as x_T *vs* T on a convenient paper such as a semi-log, log-log or Gumbel probability paper.

Gumbel's Equation for Practical use

N	0	1	2	3	4	5	6	7	8	9
10	0.4952	0.4996	0.5035	0.507	0.51	0.5128	0.5157	0.5181	0.5202	0.522
20	0.5236	0.5252	0.5268	0.5283	0.5296	0.5309	0.532	**0.5332**	0.5343	0.5353
30	0.5362	0.5371	0.538	0.5388	0.5396	0.5402	0.541	0.5418	0.5424	0.543
40	0.5436	0.5442	0.5448	0.5453	0.5458	0.5463	0.5468	0.5473	0.5477	0.5481
50	0.5485	0.5489	0.5493	0.5497	0.5501	0.5504	0.5508	0.5511	0.5515	0.5518
60	0.5521	0.5524	0.5527	0.553	0.5533	0.5535	0.5538	0.554	0.5543	0.5545
70	0.5548	0.555	0.5552	0.5555	0.5557	0.5559	0.5561	0.5563	0.5565	0.5567
80	0.5569	0.557	0.5572	0.5574	0.5576	0.5578	0.558	0.5581	0.5583	0.5585
90	0.5586	0.5587	0.5589	0.5591	0.5592	0.5593	0.5595	0.5596	0.5598	0.5599
100	0.56									

Table : Reduced mean y_n in Gumbel's extreme value distribution,
N = sample size , reads as 10,0 for N=10 to 100,9 for N=109

Gumbel’s Equation for Practical use

N	0	1	2	3	4	5	6	7	8	9
10	0.9496	0.9676	0.9833	0.9971	1.0095	1.0206	1.0316	1.0411	1.0493	1.0565
20	1.0628	1.0696	1.0754	1.0811	1.0864	1.0915	1.0961	**1.1004**	1.1047	1.1086
30	1.1124	1.1159	1.1193	1.1226	1.1255	1.1285	1.1313	1.1339	1.1363	1.1388
40	1.1413	1.1436	1.1458	1.148	1.1499	1.1519	1.1538	1.1557	1.1574	1.159
50	1.1607	1.1623	1.1638	1.1658	1.1667	1.1681	1.1696	1.1708	1.1721	1.1734
60	1.1747	1.1759	1.177	1.1782	1.1793	1.1803	1.1814	1.1824	1.1834	1.1844
70	1.1854	1.1863	1.1873	1.1881	1.189	1.1898	1.1906	1.1915	1.1923	1.193
80	1.1938	1.1945	1.1953	1.1959	1.1967	1.1973	1.198	1.1987	1.1994	1.2001
90	1.2007	1.2013	1.202	1.2026	1.2032	1.2038	1.2044	1.2049	1.2055	1.206
100	1.2065									

Table : Reduced standard deviation S_n in Gumbel's extreme value distribution, N = sample size, reads as 10,0 for N=10 to 100,9 for N=109

Home assignment:

Make sure that you can:

- **Define** flood frequency, probability and recurrence interval
- **List** methods of flood frequency analysis
- **Analyze** design flood using Gumbel's and log-pearson methods

References

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