Engineering Hydrology

Week-9

CHAPTER -4 FREQUENCY ANALYSIS

With practical problems

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Lecture contents of the week (Week-9)

CHAPTER-4 FREQUENCY ANALYSIS

Week-8

- 4.2 Flow Frequency
- 4.3 Flood Probability
- 4.4 Methods of Peak flood determination

CHAPTER-4 FREQUENCY ANALYSIS

Week-9

- 4.5 Regional Frequency Analysis
- 4.6 Low Flow Analysis
- 4.6.1 Definitions and Basic Concepts
- 4.6.2 Low Flow Frequency Analysis
- 4.6.3 Drought Analysis
- 4.7 Precipitation Probability
- 4.8 Risk, Reliability and Safety Factor

Lecture Learning Outcomes

Course Learning Outcomes: After completion of this Lecture, you will be able to:

CLO-1: Apply measurement techniques of the components of the hydrologic cycle, water balance and filling of missed data;

CLO-2: Examine rainfall-runoff relationship and hydrograph;

- Apply flood routing

CLO-3: **Examine the probability of occurrence;**

- **Define** flood frequency, probability and recurrence interval
- **List** methods of flood frequency analysis
- **Analyze** design flood using Gumbel's and log-pearson methods
- **Describe** regional frequency and low flow analysis
- **Explain** risk, reliability and safety factor

CLO-4: Analyze the water movement in to, over, and through the soil surface;

CLO-5: Design capacity of reservoir;

CLO-6: Design runoff volume and time of distribution of the runoff hydrograph from urbanization effect.

Gumbel's Equation for Practical use

- We have seen details of frequency analysis and Gumbel's distribution on week 8.
- We shall continue solving some problems of Gumbels distribution today by assisting spreadsheet tools as we have done for flood routing.
- Remember:

$$\text{P.D.F.:} \quad f(x) = \frac{1}{\beta}\exp\left[-\left(\frac{x-u}{\beta}\right) - \exp\left\{-\left(\frac{x-u}{\beta}\right)\right\}\right]$$

$$\text{C.D.F.:} \quad F(x) = e^{-\exp\left[-\left(\frac{x-u}{\beta}\right)\right]}$$

In terms of reduced variate: $z = \dfrac{x-u}{\beta}$

$$\text{P.D.F.:} \quad f(z) = e^{-z-(e^{-z})}$$

$$\text{C.D.F.:} \quad F(z) = e^{-e^{-z}}$$

Gumbel's Equation for Practical use

- According to his theory of extreme events, the probability of occurrence of an event **equal to or larger than** a value ***x_0*** is:

$$P\,(X \geq x_0) = 1 - e^{-e^{-y}}$$

- In which y is a dimensionless variable given by:

$$y = a\,(\bar{x} - \alpha\,), a = x - 0.45005\,\alpha_x, \alpha = \frac{1.2825}{\alpha_x}$$

- Then, $y = \frac{1.285(\,X - X)}{\delta_X} - + 0.577$

- The value of the variate X with a return period T is: $x_T = x + K\,s_x$

  and

- $K = \frac{(\,y_T - 0.577)}{1.2825}$

where,
K = frequency factor
y_T = reduced variate
S_n = reduced standard deviation

Gumbel's Equation for Practical use

- The original Gumbel equation describes the probability of non-excedence (q)

$$q = \exp[-\exp\{-y\}]$$

$$y = -1n(-1n(q)) = -1n(-1n(1-p))$$

$$y_T = -\left[\ln\left(\ln\left(\frac{T}{T-1}\right)\right)\right]$$

$$y_T = -\left[0.834 + 2.303\log\left(\log\left(\frac{T}{T-1}\right)\right)\right]$$

$$x_T = \bar{x} + K\sigma_x$$

applicable to an infinite sample size (i.e. N $\rightarrow \infty$).

$$K = \frac{(y_T - 0.577)}{1.2825}$$

Gumbel's Equation for Practical use

- The above equation giving the value of the variate x with recurrence interval T is used as:

$$x_T = \bar{x} + K\sigma_{n-1}$$

$$\sigma_{n-1} = \sqrt{\frac{(x-\bar{x})^2}{N-1}}$$

Where σ_{n-1} = standard deviation of the sample

K= frequency Factor expressed as: $K = \frac{y_T - \bar{y}_n}{s_n}$

In which y_T = reduced variate, a function of T and given as:

$$y_T = -[\ln.\ln \tfrac{T}{T-1}]$$

Gumbel's Equation for Practical use

Procedure to estimate the flood magnitude corresponding to a given **return period** based on an annual flood series.

1. Assemble the discharge data and note the sample size N. Here the annual flood value is the variate $\boldsymbol{x}$. Find $\bar{x}$ and $\boldsymbol{\sigma}\_(\boldsymbol{n}-\mathbf{1})$ for the given data.
2. Using **a table**, determine $\overline{y_n}$ and S_n appropriate to given N.
3. Find y_T for a given ***T***.
4. Find ***K***.
5. Determine the required x_T
6. Plot as x_T *vs* T on a convenient paper such as a semi-log, log-log or Gumbel probability paper.

Gumbel's Equation for Practical use

Example 1

- Annual maximum recorded floods in a certain river, for the period 1951 to 1977 is given below.
- Verify whether the **Gumbel extreme-value I** distribution fit the recorded values.
- Estimate the flood discharge with return period of (i) **100 years** and (ii) **150 years** by graphical extrapolation.

Year	Max. Flood (m^3/s)	Year	Max. Flood (m^3/s)	Year	Max. Flood (m^3/s)
1951	2947	1960	4798	1969	6599
1952	3521	1961	4290	1970	3700
1953	2399	1962	4652	1971	4175
1954	4124	1963	5050	1972	2988
1955	3496	1964	6900	1973	2709
1956	2947	1965	4366	1974	3873
1957	5060	1966	3380	1975	4593
1958	4903	1967	7826	1976	6761
1959	3757	1968	3320	1977	1971

Gumbel's Equation for Practical use

Solutions: The flood discharge values are arranged in descending order and the **plotting position return period T_P** for each discharge is obtained as $T_p = (N+1)/m = (27+1)/m = 28/m$

- Where m = order number. The discharge magnitude Q can be plotted against the corresponding T_P on a Gumbel extreme probability paper.
- The statistics ***x(mean)*** *and* ***σ_{n-1}(Stan.Dev)*** *for the series are next calculated and are shown in* table below.

Gumbel's Equation for Practical use

Solutions: Reading y_n from table:

N	0	1	2	3	4	5	6	7	8	9
10	0.4952	0.4996	0.5035	0.507	0.51	0.5128	0.5157	0.5181	0.5202	0.522
20	0.5236	0.5252	0.5268	0.5283	0.5296	0.5309	0.532	**0.5332**	0.5343	0.5353
30	0.5362	0.5371	0.538	0.5388	0.5396	0.5402	0.541	0.5418	0.5424	0.543
40	0.5436	0.5442	0.5448	0.5453	0.5458	0.5463	0.5468	0.5473	0.5477	0.5481
50	0.5485	0.5489	0.5493	0.5497	0.5501	0.5504	0.5508	0.5511	0.5515	0.5518
60	0.5521	0.5524	0.5527	0.553	0.5533	0.5535	0.5538	0.554	0.5543	0.5545
70	0.5548	0.555	0.5552	0.5555	0.5557	0.5559	0.5561	0.5563	0.5565	0.5567
80	0.5569	0.557	0.5572	0.5574	0.5576	0.5578	0.558	0.5581	0.5583	0.5585
90	0.5586	0.5587	0.5589	0.5591	0.5592	0.5593	0.5595	0.5596	0.5598	0.5599
100	0.56									

Table : Reduced mean y_n in Gumbel's extreme value distribution,
N = sample size , reads as 10,0 for N=10 to 100,9 for N=109

Gumbel's Equation for Practical use

Solutions: Reading y_n from table:

N	0	1	2	3	4	5	6	7	8	9
10	0.4952	0.4996	0.5035	0.507	0.51	0.5128	0.5157	0.5181	0.5202	0.522
20	0.5236	0.5252	0.5268	0.5283	0.5296	0.5309	0.532	**0.5332**	0.5343	0.5353
30	0.5362	0.5371	0.538	0.5388	0.5396	0.5402	0.541	0.5418	0.5424	0.543
40	0.5436	0.5442	0.5448	0.5453	0.5458	0.5463	0.5468	0.5473	0.5477	0.5481
50	0.5485	0.5489	0.5493	0.5497	0.5501	0.5504	0.5508	0.5511	0.5515	0.5518
60	0.5521	0.5524	0.5527	0.553	0.5533	0.5535	0.5538	0.554	0.5543	0.5545
70	0.5548	0.555	0.5552	0.5555	0.5557	0.5559	0.5561	0.5563	0.5565	0.5567
80	0.5569	0.557	0.5572	0.5574	0.5576	0.5578	0.558	0.5581	0.5583	0.5585
90	0.5586	0.5587	0.5589	0.5591	0.5592	0.5593	0.5595	0.5596	0.5598	0.5599
100	0.56									

Table : Reduced mean y_n in Gumbel's extreme value distribution,
N = sample size , reads as 20,7 for N=27 as y_n =0.5332

Gumbel's Equation for Practical use

Solutions: Reading y_n from table:

N	0	1	2	3	4	5	6	7	8	9
10	0.9496	0.9676	0.9833	0.9971	1.0095	1.0206	1.0316	1.0411	1.0493	1.0565
20	1.0628	1.0696	1.0754	1.0811	1.0864	1.0915	1.0961	**1.1004**	1.1047	1.1086
30	1.1124	1.1159	1.1193	1.1226	1.1255	1.1285	1.1313	1.1339	1.1363	1.1388
40	1.1413	1.1436	1.1458	1.148	1.1499	1.1519	1.1538	1.1557	1.1574	1.159
50	1.1607	1.1623	1.1638	1.1658	1.1667	1.1681	1.1696	1.1708	1.1721	1.1734
60	1.1747	1.1759	1.177	1.1782	1.1793	1.1803	1.1814	1.1824	1.1834	1.1844
70	1.1854	1.1863	1.1873	1.1881	1.189	1.1898	1.1906	1.1915	1.1923	1.193
80	1.1938	1.1945	1.1953	1.1959	1.1967	1.1973	1.198	1.1987	1.1994	1.2001
90	1.2007	1.2013	1.202	1.2026	1.2032	1.2038	1.2044	1.2049	1.2055	1.206
100	1.2065									

Table : Reduced mean y_n in Gumbel's extreme value distribution,
N = sample size , reads as 20,7 for N=27 as S_n ***= 1.1004***

Gumbel's Equation for Practical use

A	B	C	D	E	F	G	H	I	J	K	L	M	N
		N	**27**		Y_{100}	4.600149							
		T	150		y_{150}	5.007293							
Year	**Max.Flood (m^3/s)**	**Ranked Max.Flood (m^3/s)**	**m=rank**	**P=m/N+1**	**p (%)**	**T=1/P**	**X_{100}**	**X_{150}**	**Y_T**	**S_n**	**y_n**	**k_{100}**	**k_{150}**
1951	2947	7826	1	0.04	3.57	28.00	9,557.80	10,087.85	5.007	**1.1004**	**0.5332**	3.6959	4.0659
1952	3521	6900	2	0.07	7.14	14.00							
1953	2399	6761	3	0.11	10.71	9.33							
1954	4124	6599	4	0.14	14.29	7.00							
1955	3496	5060	5	0.18	17.86	5.60							
1956	2947	5050	6	0.21	21.43	4.67							
1957	5060	4903	7	0.25	25.00	4.00							
1958	4903	4798	8	0.29	28.57	3.50							
1974	3873	2947	24	0.86	85.71	1.17							
1975	4593	2709	25	0.89	89.29	1.12							
1976	6761	2399	26	0.93	92.86	1.08							
1977	1971	1971	27	0.96	96.43	1.04							
Mean		**4263.15**											
Std.		**1432.58**											

Y_T: fx =-(LN(LN(D2/(D2-1))))

X_{100}: fx =C32+(M5*C33)

Gumbel's Equation for Practical use

Flood Discharge (m^3/s)	Rank (m)	P=m/(N+1)	Tp=(N+1)/m	Flood Discharge (m^3/s)	Rank (m)	P=M/(N+1)	Tp=(N+1)/m
7826	1	0.035714286	28	3873	15	0.535714286	1.86666667
6900	2	0.071428571	14	3757	16	0.571428571	1.75
6761	3	0.107142857	9.333333333	3700	17	0.607142857	1.64705882
6599	4	0.142857143	7	3521	18	0.642857143	1.55555556
5060	5	0.178571429	5.6	3496	19	0.678571429	1.47368421
5050	6	0.214285714	4.666666667	3380	20	0.714285714	1.4
4903	7	0.25	4	3320	21	0.75	1.33333333
4798	8	0.285714286	3.5	2988	22	0.785714286	1.27272727
4652	9	0.321428571	3.111111111	2947	23	0.821428571	1.2173913
4593	10	0.357142857	2.8	2947	24	0.857142857	1.16666667
4366	11	0.392857143	2.545454545	2709	25	0.892857143	1.12
4290	12	0.428571429	2.333333333	2399	26	0.928571429	1.07692308
4175	13	0.464285714	2.153846154	1971	27	0.964285714	1.03703704
4124	14	0.5	2				

Gumbel's Equation for Practical use

N	27
mean	4263
Stan. Dev.	1433

Using these the discharge x_T for some chosen return interval is calculated by using Gumbel's formulae . From Table for N = 27, $\mathbf{y_n}$ = 0.5332 and $\mathbf{S_n}$ = 1.1004.

$$y_T = -[\ln.\ln\frac{T}{T-1}]$$

$$K = \frac{y_T - \overline{y_n}}{s_n}$$

Choosing T = 10 years,

$$x_T = \overline{x} + K\sigma_{n-1}$$

$\boldsymbol{y}_T = -[\ln(\ln(10/9))] = \mathbf{2.25037}$ and

$\boldsymbol{K} = (2.25307 - 0.5332)/1.1004$ = **1.56**

$\boldsymbol{x}_T = 4263 + (1.56 * 1432.6) = \mathbf{6499 m^3/s.}$

Similarly, values of $\boldsymbol{x}_T$ are calculated for two more T values as shown below.

Gumbel's Equation for Practical use

When these values are plotted on **Gumbel probability paper,** it is seen that these points lie on a **straight line** according to the property of the Gumbel's extreme probability paper. Then by **extrapolation** of the theoretical $\boldsymbol{x_T}$ Vs T relationship, from this plot, *at*

$T = 100\,years, \boldsymbol{x_T} = \mathbf{9600\,m^3/s}$, *and at*

$T = 150\,years, \boldsymbol{x_T} = \mathbf{10700\,m^3/s}$.

By using above Eq., $\boldsymbol{x_{100}} = \mathbf{9558\,m^3/s}$ *and*

$\boldsymbol{x_{150}} = \mathbf{10088\,m^3/s}$.

T (Years)	X_T [obtained by above Eq. (m^3/s)
5	5522
10	6499
20	7436

Confidence limits for the fitted data

- Indicates the limits about the calculated value between which the true value can be said to lie with a specific probability based on sampling errors only.
- For a confidence probability c, the confidence interval of the variate x_T is bound by value x_1 and x_2 given by: $$x_{1/2} = x_T \pm f(c)\, S_e$$

where ***f(c)*** = function of the confidence probability ***c*** determined by using the ***table*** of **normal variate** as:

C in per cent	50	68	80	90	95	99
f(c)	0.674	1.00	1.282	1.645	1.96	2.58

S_e = probable error = $b \dfrac{\sigma_{n-1}}{\sqrt{N}}$

$b = \sqrt{1 + 1.3K + 1.1K^2}$

K = frequency factor

σ_{n-1} = standard deviation of the sample

N = sample size.

Gumbel's Equation for Practical use

Example 2:

Data covering a period of 92 years for a certain river yielded the **mean** and **standard deviation** of the annual flood series as **6,437** and **2,951** m^3/s, respectively. Using Gumbel's method, estimate the flood discharge with a return period of **500 years**.

What are the:

(a) 95% and

(b) 80% confidence limits for this estimate?

Gumbel's Equation for Practical use

- ***Solution:*** From Table for N = 92 years,

  $\boldsymbol{y_n} = \mathbf{0.5589}$, and $\boldsymbol{S_n} = \mathbf{1.2020}$.

Then,

- $y_{\mathbf{500}} = -[\ln((\ln(500/499))] = \mathbf{6.21361}$
- $K_{\mathbf{500}} = (6.21361 - 0.5589)/1.2020 = \mathbf{4.7044},$
- $\boldsymbol{X_{500}} = 6437 + 4.7044 * 2951 = \mathbf{20,320} \boldsymbol{m^3/s}.$
- From equations of confidence limits:

  $\boldsymbol{b} = \sqrt{}(1 + 1.3 * 4.7044 + 1.1 * 4.70442) = \mathbf{5.61}$

  $\boldsymbol{S_e} = 5.61 * (2951/\sqrt{}92) = \mathbf{1726}$

$$s_e = probable.error = b\frac{\sigma_{n-1}}{\sqrt{N}}$$

$$b = \sqrt{(1 + 1.3K + 1.1K^2)}$$

Gumbel's Equation for Practical use

(a) For the 95% confidence probability $\boldsymbol{f(c)} = 1.96$ *and*

$$x_{1/2} = 20320\boldsymbol{x_T} \pm (1.96 * 1726)$$

which results in $\boldsymbol{x_1 = 23,703 m^3/s}$ and

$$\boldsymbol{x_2 = 16,937 m^3/s}.$$

Thus the estimated discharge of **20,320m³/s** has a **95%** probability of lying between **23,700** and **16,940**m³/s.

(b) For 80% confidence probability, $\boldsymbol{f(c)} = 1.282$ and

$$x_{1/2} = 20{,}320\boldsymbol{x_T} \pm (1.282 * 1726),$$ which results in

$$\boldsymbol{x_1} = 22{,}533 m^3/s$$ and

$$\boldsymbol{x_2} = 18{,}107 m^3/s.$$

Thus the estimated discharge of **20,320 m³/s** has an 80% probability of lying between **22,533** and **18,107m³/s**.

II. Log-Pearson Type III Distribution (LPT-III)

- In this distribution the variate *X* is first transformed into logarithmic form and the transformed data is then analyzed. $\boldsymbol{Z} = \log \boldsymbol{x}$
- For this **z** series, for any recurrence interval $\boldsymbol{T}$, $\boldsymbol{z_T} = \bar{\boldsymbol{z}} + \boldsymbol{K_z}\,\boldsymbol{\sigma_z}$

Where, σ_z = standard deviation of the Z variate sample, and $\boldsymbol{K_z} = a$ frequency factor which is a function of recurrence interval $\boldsymbol{T}$ and the coefficient of skew $\boldsymbol{C_s}$,

and C_s = coefficient of skew of variate Z = $$\frac{N\sum(z-\bar{z})^3}{(N-1)(N-2)(\sigma_z)^3}$$

LPT-III

- the corresponding value of x_T is obtained by:

$$x_T = antilog(z_T)$$

- Generally a ***minimum of 30*** years of data is considered as essential in order to use flood frequency analysis
- $K_z = F(Cs, T)$ for use in Log-Pearson Type III Distribution

Table 4.6: $K_z = F(C_s, T)$ for use in Log-Pearson Type III Distribution

Coef. of skew, C_s	Return Period T in years						
	2	10	25	50	100	200	1000
3.0	-0.396	1.180	2.278	3.152	4.051	4.970	7.250
2.5	-0.360	1.250	2.262	3.048	3.845	4.652	6.600
2.2	-0.330	1.284	2.240	2.970	3.705	4.444	6.200
2.0	-0.307	1.302	2.219	2.912	3.605	4.298	5.910
1.8	-0.282	1.318	2.193	2.848	3.499	4.147	5.660
1.6	-0.254	1.329	2.163	2.780	3.388	3.990	5.390
1.4	-0.225	1.337	2.128	2.706	3.271	3.828	5.110
1.2	-0.195	1.340	2.087	2.626	3.149	3.661	4.820

LPT-III

Coef. of skew, C_s	Return Period T in years						
	2	10	25	50	100	200	1000
1.0	-0.164	1.340	2.043	2.542	3.022	3.489	4.540
0.9	-0.148	1.339	2.018	2.498	2.957	3.401	4.395
0.8	-0.132	1.336	1.998	2.453	2.891	3.312	4.250
0.7	-0.116	1.333	1.967	2.407	2.824	3.223	4.105
0.6	-0.099	1.328	1.939	2.359	2.755	3.132	3.960
0.5	-0.083	1.323	1.910	2.311	2.686	3.041	3.815
0.4	-0.066	1.317	1.880	2.261	2.615	2.949	3.670
0.3	-0.050	1.309	1.849	2.211	2.544	2.856	3.525
0.2	-0.033	1.301	1.818	2.159	2.472	2.763	3.380
0.1	-0.017	1.292	1.785	2.107	2.400	2.670	3.235
0.0	**0.000**	**1.282**	**1.751**	**2.054**	**2.326**	**2.576**	**3.090**
-0.1	0.017	1.270	1.716	2.000	2.252	2.482	2.950
-0.2	0.033	1.258	1.680	1.945	2.178	2.388	2.810
-0.3	0.050	1.245	1.643	1.890	2.104	2.294	2.675
-0.4	0.066	1.231	1.606	1.834	2.029	2.201	2.540
-0.5	0.083	1.216	1.567	1.777	1.955	2.108	2.400
-0.6	0.099	1.200	1.528	1.720	1.880	2.016	2.275
-0.7	0.116	1.183	1.488	1.663	1.806	1.926	2.150
-0.8	0.132	1.166	1.448	1.606	1.733	1.837	2.035
-0.9	0.148	1.147	1.407	1.549	1.660	1.749	1.910
-1.0	0.164	1.128	1.366	1.492	1.588	1.664	1.880
-1.4	0.225	1.041	1.198	1.270	1.318	1.351	1.465
-1.8	0.282	0.945	1.035	1.069	1.087	1.097	1.130
-2.2	0.330	0.844	0.888	0.900	0.905	0.907	0.910
-3.0	0.396	0.660	0.666	0.666	0.667	0.667	0.668

- When the skew is zero, i.e. Cs = 0, the **log-Pearson Type III** distribution reduces to **log normal distribution**.
- The log-normal distribution plots as a straight line on logarithmic probability paper.

LPT-III

Example 3: For the annual flood series data given in the table (example-1),

- estimate the ***flood discharge*** for a return period of:

(a) 100 years

(b) 200 years and

(c) 1000 years by using Log-Pearson Type III distribution.

Year	Max. Flood (m^3/s)	Year	Max. Flood (m^3/s)	Year	Max. Flood (m^3/s)
1951	2947	1960	4798	1969	6599
1952	3521	1961	4290	1970	3700
1953	2399	1962	4652	1971	4175
1954	4124	1963	5050	1972	2988
1955	3496	1964	6900	1973	2709
1956	2947	1965	4366	1974	3873
1957	5060	1966	3380	1975	4593
1958	4903	1967	7826	1976	6761
1959	3757	1968	3320	1977	1971

- ***Solution:***
- *The variate* $\boldsymbol{z} = \log \boldsymbol{x}$ *is* first calculated for all the discharges in table below. Then the statistics $\bar{z}$, σ_z *and* C_s *are calculated from the data to obtain:*

$$\bar{z} = 3.6071$$

$$\sigma_z = 0.1427$$

$$C_s = \frac{(27*0.0030)}{(26)(25)(0.1427)^3} = \mathbf{0.043}$$

- The flood discharge for a given ***T*** is calculated as below. Here, values of $\boldsymbol{K_z}$ for given ***T*** and

  $\mathbf{C_s}$ = **0.043** are read from table .

LPT-III

Variate Z					
Year	Flood x (m^3 /s)	z = log x	Year	Flood x (m^3/s)	z = log x
1951	2947	3.4694	1965	4366	3.6401
1952	3521	3.5467	1966	3380	3.5289
1953	2399	3.38	1967	7826	3.8935
1954	4124	3.6153	1968	3320	3.5211
1955	3496	3.5436	1969	6599	3.8195
1956	2947	3.4694	1970	3700	3.5682
1957	5060	3.7042	1971	4175	3.6207
1958	4903	3.6905	1972	2988	3.4754
1959	3751	3.5748	1973	2709	3.4328
1960	4798	3.6811	1974	3873	3.588
1961	4290	3.6325	1975	4593	3.6621
1962	4652	3.6676	1976	6761	3.83
1963	5050	3.7033	1977	1971	3.2947
1964	6900	3.8388			
$z = 3.6071, \quad \sigma_z = 0.1427, C_s = \mathbf{0.043}$					

T (years)	Kz (from table for C_s = 0.043)	$K_z\sigma_z$	$Z_T = z + K_z\sigma_z$	X_T = antilog Z_T (m^3/s)
100	2.358	0.3365	3.9436	8,782
200	2.616	0.3733	3.9804	9,559
1000	3.152	0.4498	4.0569	11,400

Drought analysis

- Hydrological drought refers to a lack of water in the ***hydrological system***, leading to decreased water levels in rivers, lakes, reservoirs, and groundwater.
- Hydrological drought is linked to:
  - meteorological drought (reduced precipitation) and
  - agricultural drought (impacts on agriculture).
- Meteorological drought is the ***initial cause*** of the problem, and agricultural drought is the ***final impact***.
- Monitoring and Forecasting: it is studied through indices:
  - Standardized Streamflow Index (SSI) and
  - using anomalies in hydrometeorological variables.
- There's a push to develop better early warning systems for hydrological droughts.

Low flow analysis

Methods:

- **Low Flow Frequency Analysis (LFFA)**: A stochastic approach to characterize the likelihood of flow persisting ***below a certain level*** for a given duration.
- **Frequency Distribution Curve (FDC)**: A common method that displays the ***range*** of streamflow from low flows to floods.
- **Low flow indices**: Such as ***7Q10*** (lowest 7-day average flow in 10 years) or **1Q10** (lowest 1-day average flow in 10 years), commonly used in the US.
- **Flow duration curve analysis**: Examines the proportion of time a specific flow is ***exceeded*** or ***not exceeded***.

Low flow analysis

Applications:

- Water resource ***development*** and ***management***.
- Determining ***environmental flow*** requirements, EFR.
  - EFR is the quantity, timing, and quality of water flow needed to maintain healthy freshwater
- Assessing the impact of ***climate change*** on low flows.
- Developing water ***quality standards***.
- Understanding the effects of ***land use changes*** on low flows.

FLOW DURATION CURVES

It is the graphical representation of the probability of the variate ($P_{X_T \geq X}$) against the magnitude of the variate (X_T).

- E.g

Discharge (m3/s)	Descending Order	Rank	%Exceeded or Equaled (m / (N+1))
(a)	(b)	(c)	(d)
106.70	1200	1	8.33%
107.10	964.7	2	16.67%
148.20	497	3	25.00%
497.00	338.6	4	33.33%
1200.00	177.6	5	41.67%
964.70	148.2	6	50.00%
338.60	142.7	7	58.33%
177.60	141	8	66.67%
141.00	141	9	75.00%
141.00	126.6	10	83.33%
142.70	107.1	11	91.67%
126.60	106.7	12	100.00%

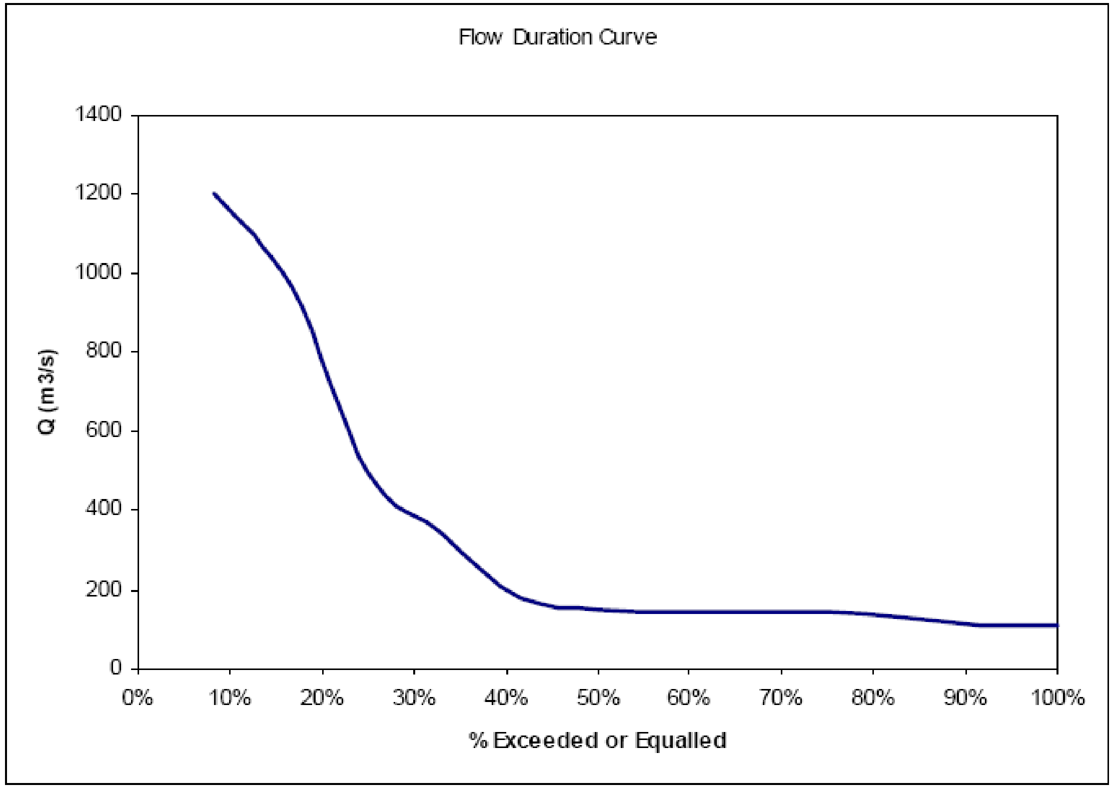

Figure 4.1: Flow Duration Curve

REGIONAL FLOOD FREQUENCY ANALYSIS

- The available data at a catchment is too short to conduct frequency analysis.
- Available long time data from neighboring catchments are tested for homogeneity and a group of stations satisfying the test.
- **Regional flood frequency analysis (RFFA)** is a statistical method used to estimate flood characteristics, like design floods, in ungauged or partially gauged areas by leveraging data from surrounding gauged sites.

Design Flood and Design Storm

- Small structures such as culverts and storm drainages can be designed for less severe floods.
  - It can cause temporary inconvenience like the disruption of traffic
- Storage structures such as dams demand greater attention to the magnitude of floods.
  - The failure of these structures causes large loss of life and great property.
- **Design Flood** is a flood size adopted for the design of a structure.
- **Design storm:** he storm-producing probable maximum precipitation (PMP) for deriving PMF or a standard project storm (SPS) for SPF calculations.

Risk, Reliability and Safety factor

- **Risk and Reliability:** The designer of a hydraulic structure always faces a nagging **doubt** about the risk of failure of his structure.
  - Due to uncertainty of discharge or stage measurements
- example,
  - consider a weir with an expected life of 50 years and designed for a flood magnitude of return period T = 100 years.
  - This weir may fail if a flood magnitude greater than the design flood occurs within the life period (50 years) of the weir.

- a structure should be designed depends upon the acceptable level of risk.
- This acceptable risk is governed by **economic** and **policy considerations**.

Risk, Reliability and Safety factor

- **Risk, $\overline{\boldsymbol{R}}$** *is* t*he* probability of occurrence of an event $(\boldsymbol{x} \geq \boldsymbol{xT})$ at least once over a period of n successive years.

- $\bar{R} = 1 - (Probability\ of\ non - occurrence\ of\ the\ event\ in\ n - years)$.

$$\overline{R} = 1 - (1 - P)^n = 1 - \left(1 - \frac{1}{T}\right)^n$$

where $P = probability\ P\ (x \geq x_T) = 1/T$

T = return period

- The reliability R_e, is defined as:

$$R_e = 1 - \overline{R} = \left(1 - \frac{1}{T}\right)^n$$

$$P = \text{probability } P\ (x \geq x_T) = \frac{1}{T}$$

Risk, Reliability and Safety factor

Safety Factor:

- *Additional uncertainties than meteorological and hydrological comes from:*
  - structural, constructional, operational and environmental causes as well as from non-technological considerations such as economic, sociological and political situations.
- any water resource development project will have a safety factor for a given hydrological parameter M as defined as:

- $$Safety\ factor\ (for\ the\ parameter\ M) = (SF)m = \frac{Actual\ value\ of\ the\ parameter\ adopted\ in\ the\ design\ of\ the\ project}{Value\ of\ the\ parameter\ obtained\ from\ hydrological\ considerations\ only} = \frac{C_{am}}{C_{hm}}$$

- M includes such items as flood discharge magnitude, maximum river stage, reservoir capacity and free board.
- The difference (C_{am} and C_{hm}) is known as ***safety margin***.

Home assignment:

Assignment 2 Q no **2**

Annual maximum recorded floods in a certain river, for the period 1951 to 1977 is given below.

- Verify whether the Gumbel extreme-value I and log person type III distribution fit the recorded values.
- Estimate the flood discharge with return period of:
  - (i) 100 years and
  - (ii) 150 years using both flood frequency analysis methods and compare the result?

Year	Max.Flood(m3/s)	Year	Max.Flood(m3/s)	Year	Max.Flood(m3/s)
1951	2947	1960	4798	1969	6599
1952	3521	1961	4290	1970	3700
1953	2399	1962	4652	1971	4175
1954	4124	1963	5050	1972	2988
1955	3496	1964	6900	1973	2709
1956	2947	1965	4366	1974	3873
1957	5060	1966	3380	1975	4593
1958	4903	1967	7826	1976	6761
1959	3757	1968	3320	1977	1971

Home assignment:

Make sure that you can write, select, compare on the following topics:

- Define flood frequency, probability and recurrence interval
- List methods of flood frequency analysis
- Analyze design flood using Gumbel's and log-pearson methods
- Describe regional frequency and low flow analysis
- Explain risk, reliability and safety factor

References

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Subramanya, K. 2008. Engineering Hydrology, Tata McGraw-Hill Publishing Company Limited, New Delhi

Thank you very much for your active attendance!!