Advanced Power System Analysis
Lecture 2

Per-phase and Per unit analysis

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Lecture learning outcomes:

At the end of this lecture, you will be able to:

i. Understand the single phase power system

ii. Know the three phase power system

iii. Identify the importance of per phase analysis in balanced three phase system

iv. Define per unit analysis in power system.

Outlines

1. Introduction

- Power network consist of three-phase alternating current (AC) circuits, which is complex and equal in magnitude with 120 degree phase shift
- The three phase system analysis is very complex and takes time to compute different power network parameters.
- Simplifying these complex calculations is important in system analysis.
- Accordingly, per-phase analysis is defined as a tool used to simplify the analysis of multi/three-phase power systems while converting it into single phase form[1].
- Which is useful in fault analysis, system modeling, and load flow calculations.

Introduction Cont.…

- In a three-phase system, each phase can be analyzed independently, considering symmetrical compon ents.

Basic Steps in Per-Phase Analysis:

- Assume the system is balanced.
- Analyze one phase as a representative of the whole system.
- This reduces the complexity of analyzing three phases separately.
- Generally, per-phase analysis employed because of simplicity in understanding, easier for fault analysi s and CPD design.
- Reduces computation time in symmetrical load scenarios.

Introduction Cont.…

- Per-Unit is a normalization technique that simplifies calculations in power systems , specifically makes the system having different units into the same unit[2].
- All system quantities (voltage, current, power, impedance, etc.) are expressed as a fraction of a base

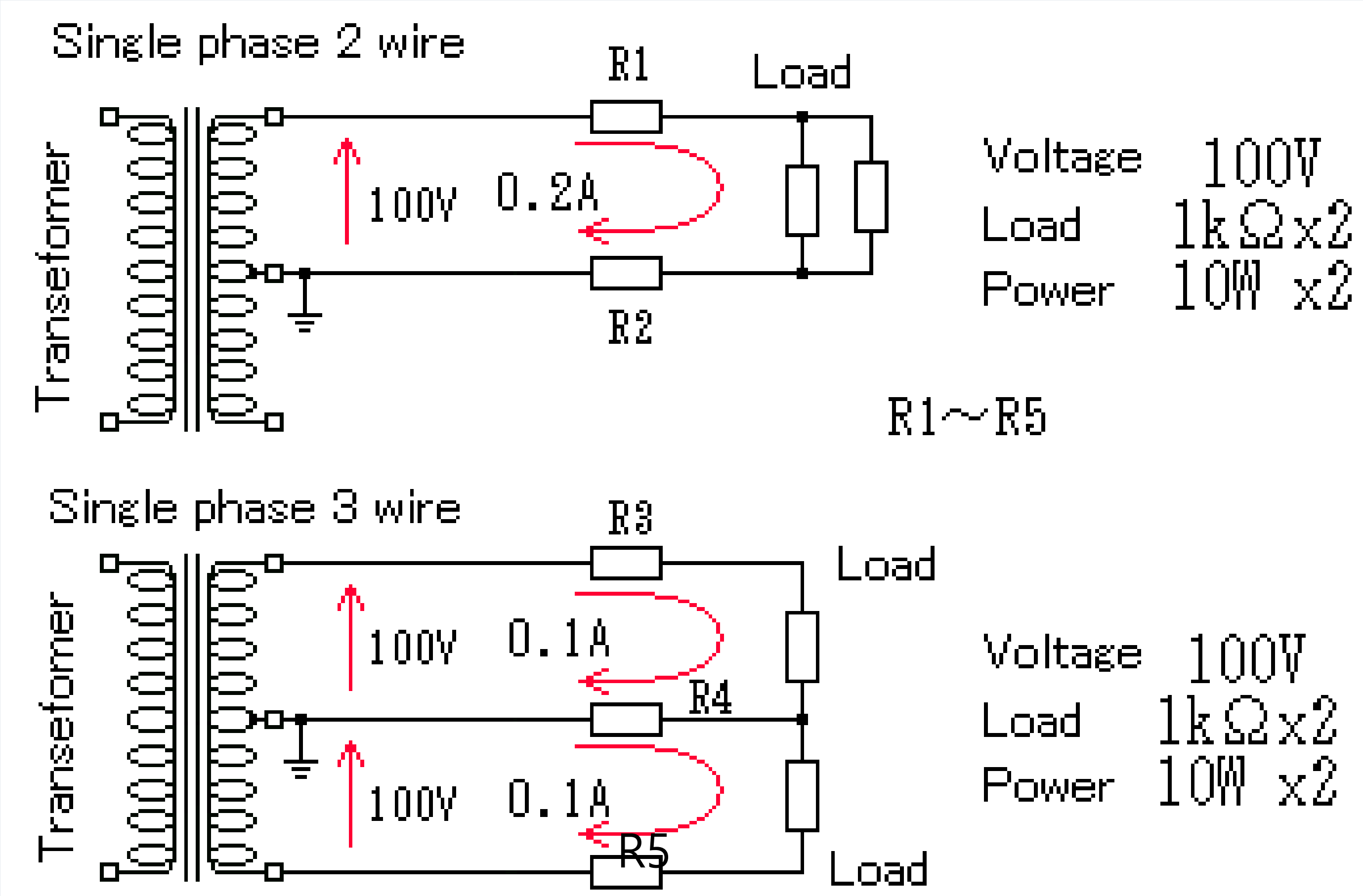
unit.

- Simplifies the comparison of different components in the system.
- Avoids dealing with large numerical values, making calculations easier
- Reduce the errors while referring the transformers from one side to the others in power system analysi s

2. Single Phase Power System (SPPS)

- Alternating current flowing via two wires in a power circuit is known as single-phase power[3].
- This is often set up as one live or phase wire that distributes the current to the load and one neutral wire that returns it to the source.
- Commonly used in residential, commercial, and small industrial applications.
- Basic components needed are; AC source, transmission lines and Loads
- Single-phase systems are simpler and cheaper to implement compared to three-phase systems.
- The circuit diagram for single phase two-wire and three wire system is given by Fig.1

SPPS Cont…..



- The total resistance = 500 ohm
- Power total=20 W

Figure 1. The circuit diagram representation of single phase system.
Url: https://e.as76.net/asn/gif/tan3.gif.

SPPS Cont.….

As presented in Fig.1, the single phase power system comprises:

- **Voltage Source with transformers:** This could be a generator or the utility grid providing alternating current.
- The voltage is typically sinusoidal with a frequency of 50 or 60 Hz, depending on the region.
- **Conductors** : There are two primary conductors:
- Live Wire (L): The wire that carries the current to the load.
- Neutral (N): The return path for the current, completing the circuit.
- **Load**: This could be any device or appliance that consumes electrical energy, such as lights, motors, or home appliances.

SPPS Cont.….

- Phasor Representation of single phase system: The voltage and current under normal conditions are sinusoidal, and their relationship can be described in terms of phasors, complex numbers as present ed in Fig.2
- The loads can be(resistance, inductance, and capacitance)
- Phasor diagrams present a graphical representation, plotted on a coordinate system, of the phase relationship between the voltages and currents within passive components or a whole circuit
- The generalized sinusoidal expression given as: $A(t) = A_m \sin(\omega t \pm \Phi)$ represents the sinusoid in the time-domain form.

SPPS Cont.….

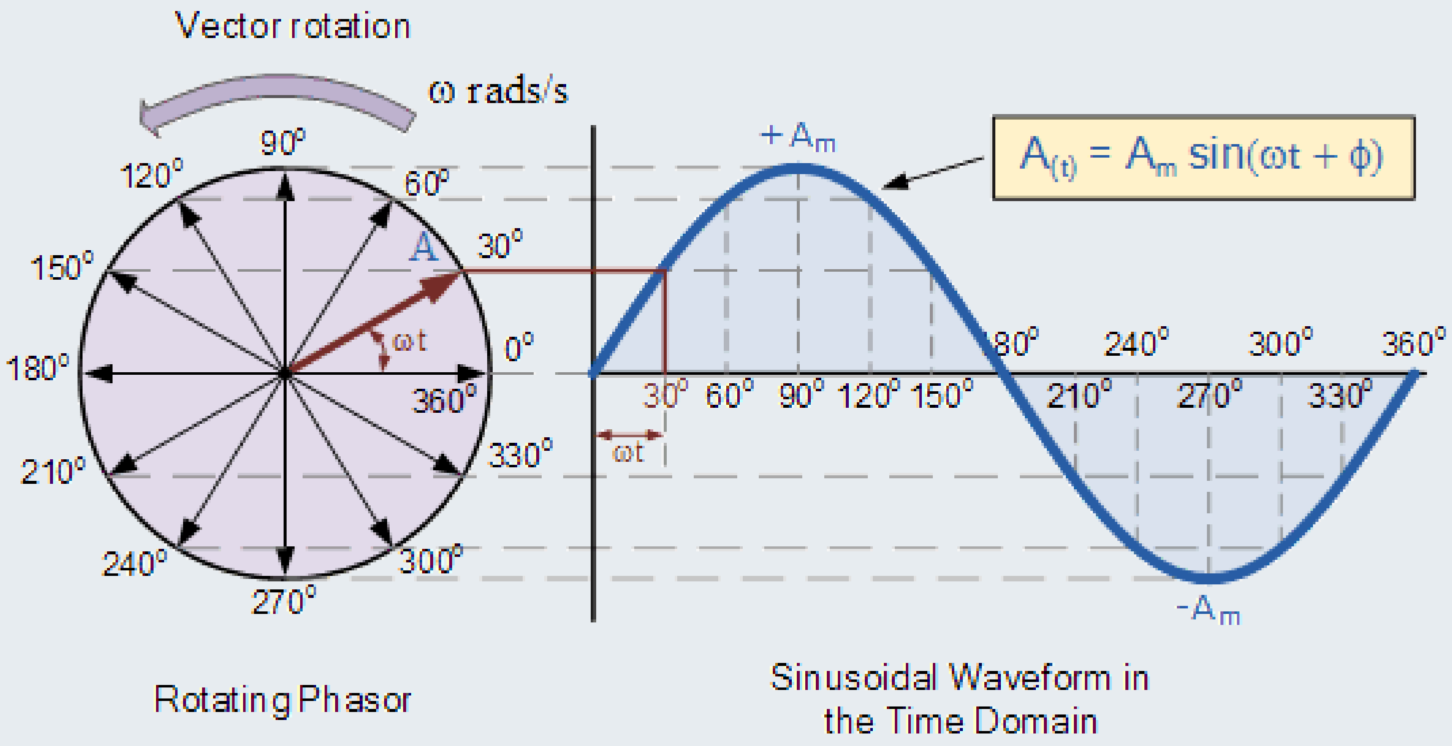


Figure 2. Phasor Diagrams for a Sinusoidal Waveform.

Url: https://www.electronics-tutorials.ws/wp-content/uploads/2018/05/accircuits-acp25.gif.

SPPS Cont.….

- The Phasor analysis is used to know the single phase current, voltage and power relations
- Its goal is to simplify the analysis of constant frequency AC systems. Using the phasor analysis, I, V & P are determined as

$$v(t) = V_{max} \cos(\omega t + \theta_v) \qquad \text{eqn.(1)}$$

$$i(t) = I_{max} \cos(\omega t + \theta_I)$$

- Root Mean Square (RMS) voltage of sinusoid

$$V_{rms} = \sqrt{\frac{1}{T}\int_0^T v(t)^2 dt} = \frac{V_{max}}{\sqrt{2}} \qquad \text{eqn.(2)}$$

SPPS Cont.....

- Based on the Euler's identity;

$$e^{j\theta} = \cos\theta + j\sin\theta$$

eqn.(3)

- Accordingly, the phase notation is rewritten as :

$$V_{(t)} = \sqrt{2}\left|V\right|\cos(\omega t + \theta_v)$$

$$= \sqrt{2}\left|V\right|\mathrm{Re}[e^{j(\omega t+\theta_v)}$$

eqn.(4)

Where, V is the r.m.s voltage.

- The r.m.s , cosine-referenced voltage is:

eqn.(5)

$$V = \left|V\right|e^{j\theta_V} = \left|V\right| < \theta_V$$

$$V_{(t)} = \mathrm{Re}\sqrt{2}Ve^{j\omega t}e^{\theta_V}$$

$$V = \left|V\right|\cos\theta_V + j\left|V\right|\sin\theta_V$$

$$I = \left|I\right|\cos\theta_I + j\left|I\right|\sin\theta_I$$

SPPS Cont.….

- Then, the power, P(t) is given as;

$$P_{(t)} = V_{(t)} * I_{(t)}$$
$$= \frac{1}{2} V_{\max} I_{\max} [\cos(\theta_V - \theta_I) + \cos(2\omega t + \theta_V + \theta_I)]$$

eqn.(6)

- Then, the average power is given by:

$$P_{avg} = {}^{1}\!/_{T} \int_0^T P(t) \partial t$$
$$= \frac{1}{2} V_{\max} I_{\max} \cos(\theta_V - \theta_I)$$
$$= |V| [I [\cos(\theta_V - \theta_I) \Rightarrow |V||I| \cos\phi$$

eqn.(7)

- Complex Power

$$S = |V||I|[\cos(\theta_V - \theta_I) + j\sin(\theta_V - \theta_I)]$$
$$= P + jQ$$
$$= VI^*$$

eqn.(8)

- Where, P is real power, Q is reactive power, S is apparent power and Φ is the power factor

3. Three Phase Power System(TPPS)

- Single phase is used only in low voltage, low power settings, such as residential and some commercial
- A balanced three-phase system which is presented in Fig.3 has [4]:
  - Three voltage sources with equal magnitude, but with an angle shift of 120°
  - Equal loads on each phase
  - Equal impedance on the lines connecting the generators to the loads
- Bulk power systems are almost exclusively three phase.

STPS Cont…..

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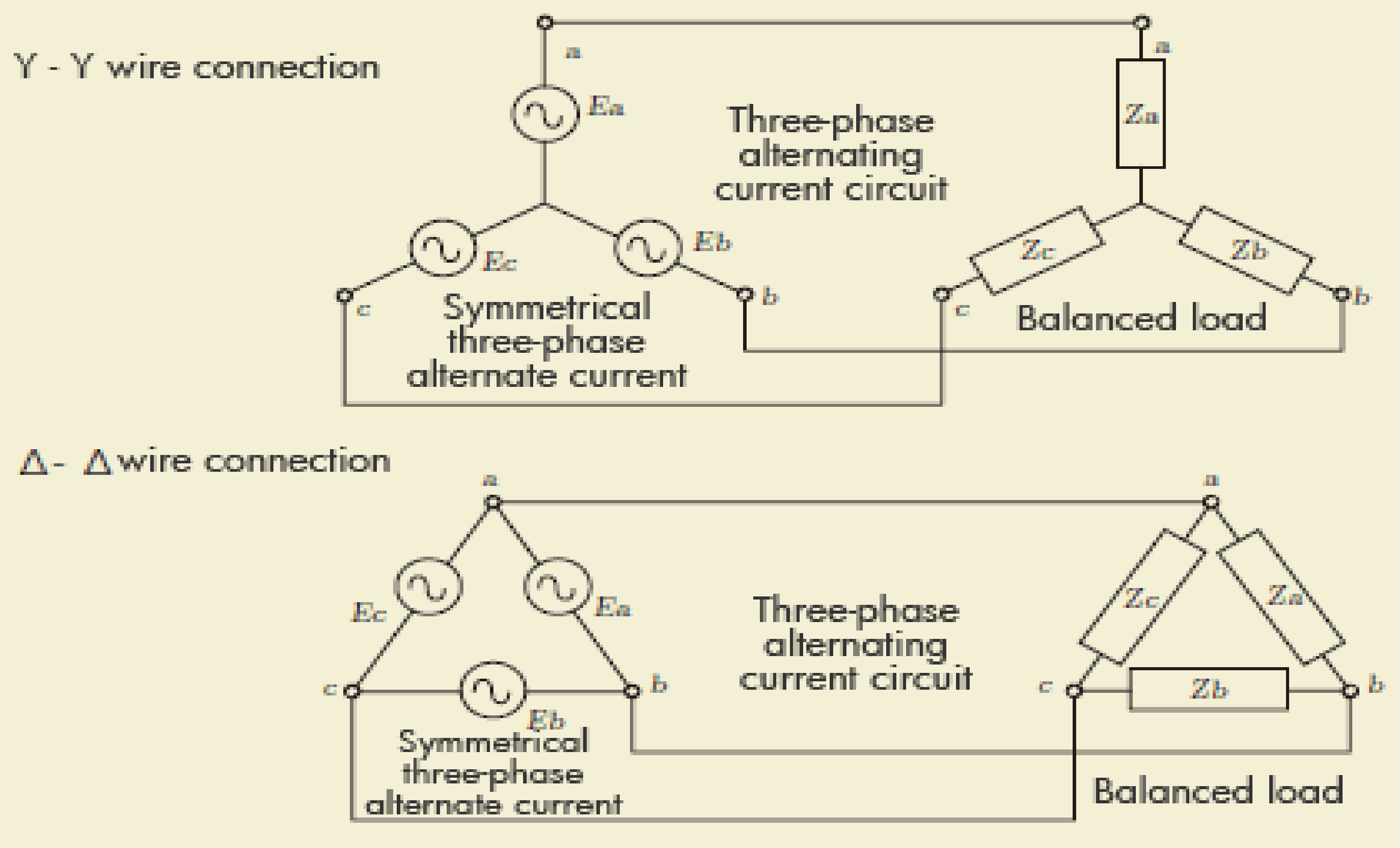


Figure 3. Balanced Three Phase system with (Y-Y) and (Δ-Δ), No-Neutral current.
url: https://www.chip1stop.com/sp/wp-content/uploads/tutorials026_001_en.gif

STPS Cont…..

- Assume Y-Y connection as presented in Fig.4 and determine the three phase power

$$I_n = I_a + I_b + I_c$$
$$I_n = \frac{V}{Z}(1\angle 0° + 1\angle -120° + 1\angle 120°) = 0$$
$$S = V_{an}I_{an}^{*} + V_{bn}I_{bn}^{*} + V_{cn}I_{cn}^{*} = 3V_{an}I_{an}^{*}$$

Eqn.(9)

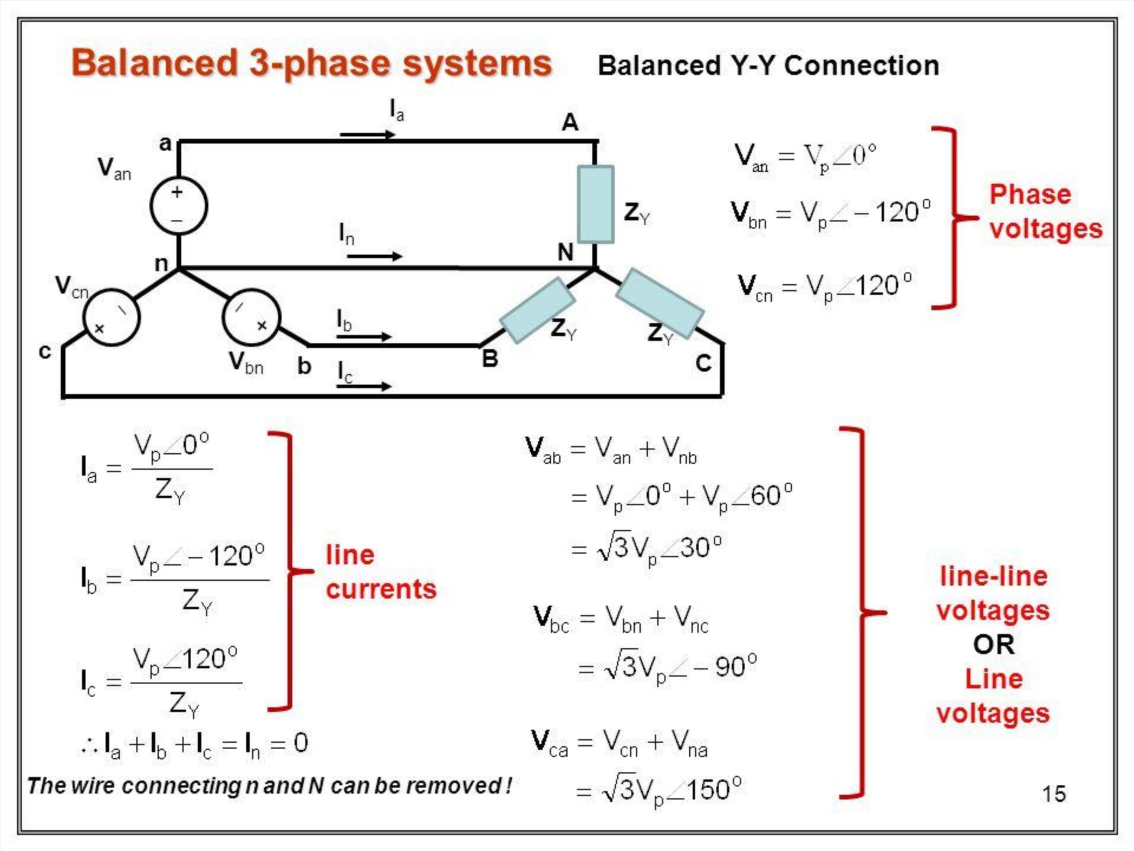


Figure 4. The Balanced (Y-Y) three Phase system relationship.

Url:https://slideplayer.com/slide/5867350/19/images/15/Balanced+3-phase+systems.jpg

STPS Cont.….

- As observed in Fig.4, in electrical power systems, there are two ways to connect 3ϕ systems(generators, transformers and load)

  i. Wye (Y)

  ii. Delta (Δ)

  - For Y-connected system where phase A is reference voltage and then, each phase voltage based on Fig.5 is given by:

$$V_{an} = |V| < \alpha^0$$
$$V_{bn} = |V| < (\alpha^0 - 120^0)$$
$$V_{cn} = |V| < (\alpha^0 + 120^0)$$

eqn.(10)

STPS Cont…..

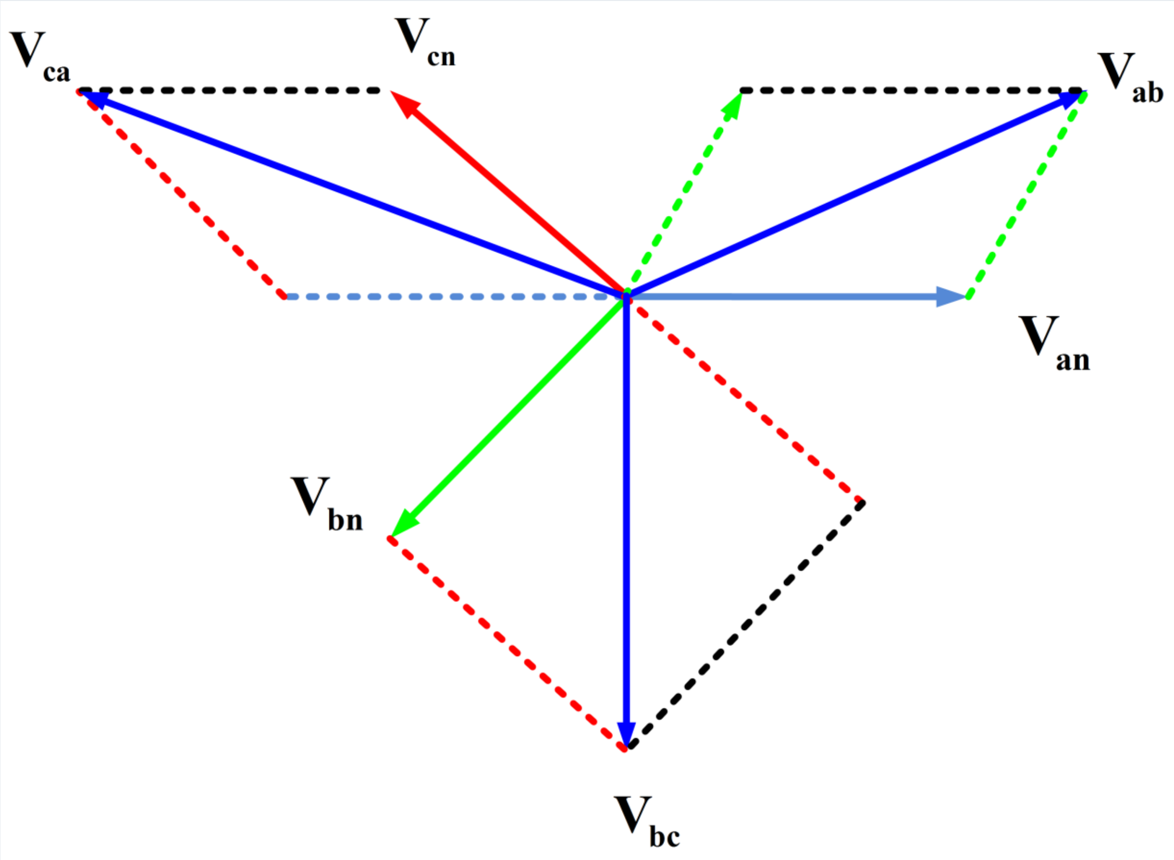


Figure 5. Three Phase star (Y)connected current voltage relationship.
Url: https://electricalacademia.com/electric-power/star-wye-connection-and-three-phase-power-voltage-and-current/

STPS Cont.….

- The Line Voltages based on the Y-connected system is given as:

If α = 0

$$V_{ab} = V_{an} - V_{bn} = |V|(1 < \alpha - 1 < (\alpha + 120)$$
$$= \sqrt{3}|V| < (\alpha + 30^0)$$
$$V_{bc} = \sqrt{3}|V| < (\alpha - 90^0)$$
$$V_{ca} = \sqrt{3}|V| < (\alpha + 150^0)$$

eqn.(11)

- It implies that both the phase and Line to line voltages of three phase system are balanced.

STPS Cont…..

- To determine the total power in three phase Y-connected system both in phase and Line are given by:

- The power while considering line:

$$S_{3-\phi} = \sqrt{3} * V_{L-L} * I_{L-L}$$

eqn.(12)

- By substitution, for star connected system is:

$$V_L = \sqrt{3} * V_{Phas} = \sqrt{3} * V_{Phas} < \phi^0$$

$$and, I_L = I_{Phas}$$

$$\Leftrightarrow S_{3-\phi} = \sqrt{3} * (\sqrt{3} * V_{Phas} < \phi^0 * I_{Phas})$$

$$= 3V_{Phas} * I_{Phas} < \phi^0$$

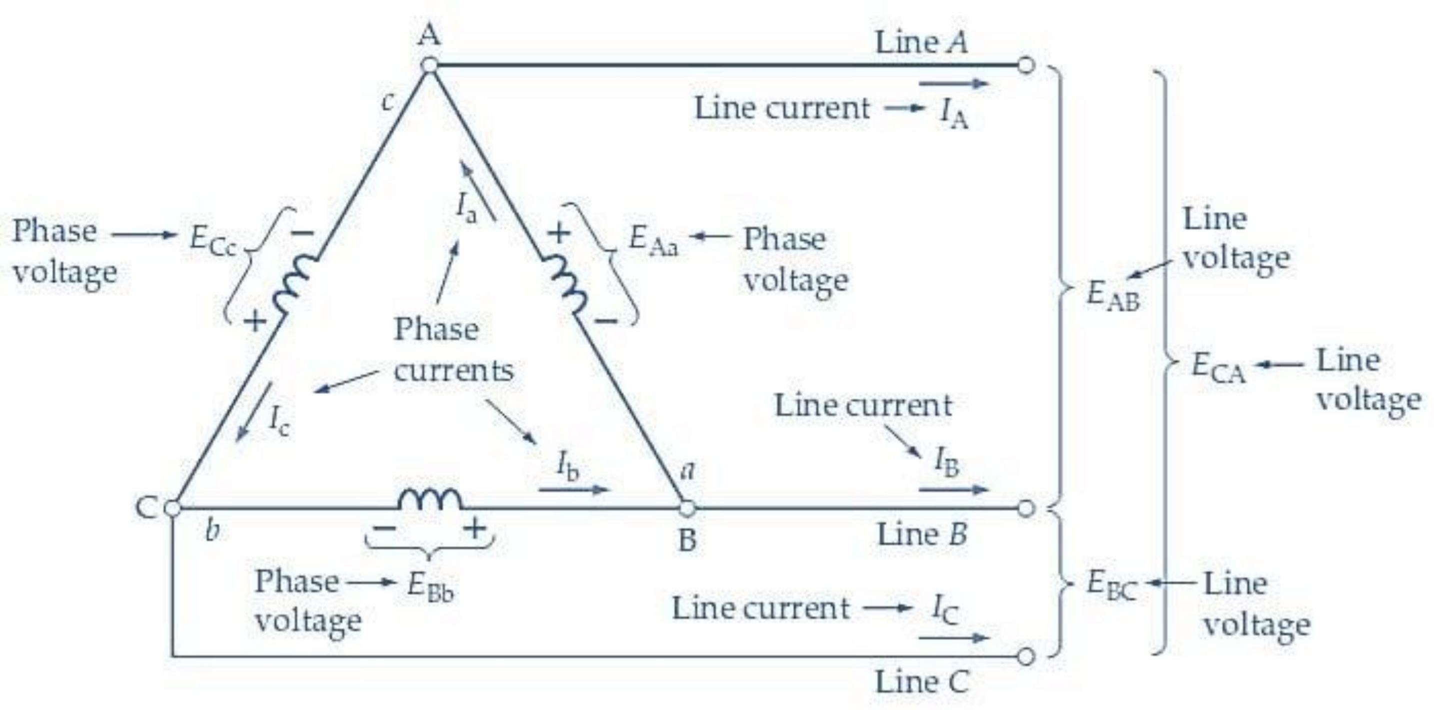
$$= 3V_{Phas} * I_{Phas}{}^*$$

$$P_{3-\phi} = 3V_{Phas} * I_{Phas} * \cos(\phi)$$

$$Q_{3-\phi} = 3V_{Phas} * I_{Phas} * Sin(\phi)$$

eqn.(13)

- Similarly the power for Delta connected system based on Fig.6 is determined as:
- In this type of connection where the three phases are connected in a closed loop, forming a triangle:
- The the line voltage equal to the phase voltage, and the line current being √3*times the phase current.



$$V_{Phas} = V_{L-L}$$

$$I_a = I_{bc} - I_{ca}$$

$$= \sqrt{3} * I_{ab} < -30^0$$

$$I_b = I_{bc} - I_{ab}$$

$$I_c = I_{ca} - I_{bc}$$

Eqn.(14)

$$S_{3-\phi} = 3V_{Phase} * I^*_{Phase}$$

Eqn.(15)

Figure 6. Delta connected system current-voltage relationship.

Url: https://eepower.com/uploads/articles/determining-phase-and-line-voltages-and-currents-in-delta-connected-generators-fig1.jpg

STPS Cont…..

Example 1: Assume a Δ-connected load is supplied from a 3ϕ 11 kV (L-L) source with Z = 50∠30° Ω balanced load. Determine the line currents and voltage considering reference voltage V_{ab} with 0 degree reference angle.

Given: $V_{ab} = 11 < 0^0\, KV$

- Which means

$$V_{bc} = 11 < -120^0\, KV$$
$$V_{ca} = 11 < 120^0\, KV$$

- Then, the line currents are

$$I_{ab} = \frac{V_{ab}}{Z} = \frac{11 < 0KV}{50 < 30^0\, Ohm} = 220 < -30^0\, amp$$
$$I_{bc} = 220 < -150^0\, amp$$
$$I_{ca} = 220 < 90^0\, amp$$

Per-phase analysis

Then, the phase currents are determined as:

STPS Cont.….

- Which is,

$$I_a = I_{ab} - I_{ca}$$
$$= 220 < -30 - 220 < 90$$
$$= 220\cos(-30) + j220\sin(-30) - (220\cos(90) + j220\sin(90))$$
$$= -110 - j120 - 0 - j220 \Rightarrow -110 - j340$$
$$= (\sqrt{(-110^2) + (-340^2)} * \tan^{-1}(\frac{-340}{-110})$$
$$= 357.351 < 1.26$$

- Then, I_b and I_c can be determined as:

$$I_b = 357.351 < 1.26 - 120$$
$$I_b = 357.351 < -118.74$$
$$I_c = 357.351 < 121.26$$

Per-phase analysis

- **Delta-wye transformation:** sometimes referred to as a pi-to-tee transformation, makes it easier to analyze and compute circuits in three-phase power systems
- Which is used by transforming a delta (Δ) network to an equivalent wye (Y) network or vice versa.
- This is especially useful for unbalanced loads as presented in Fig.7

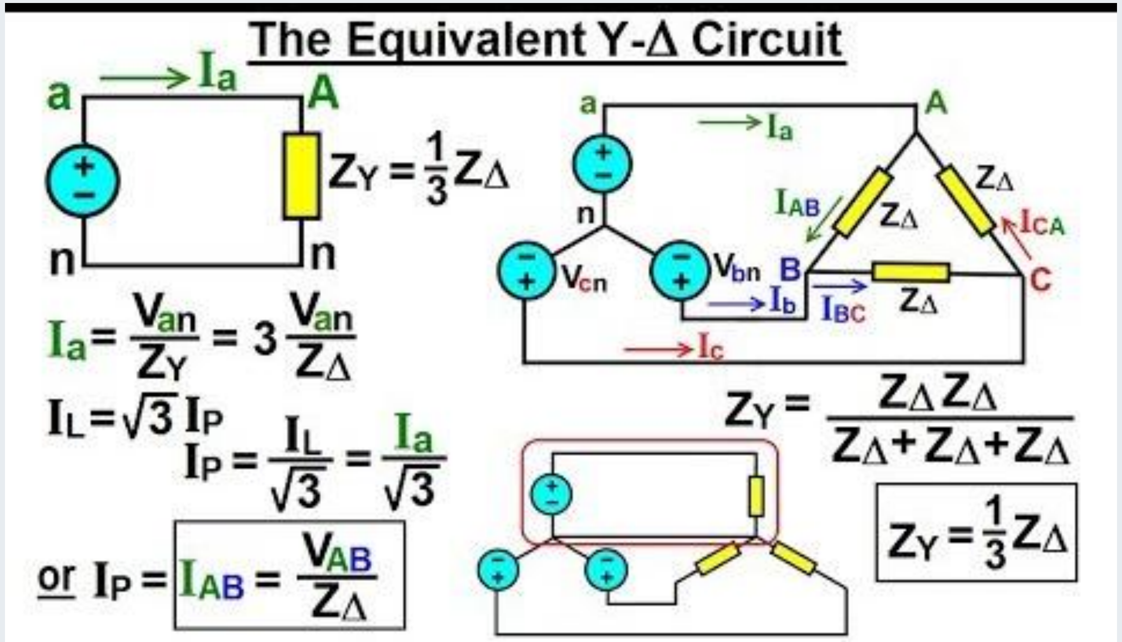


Figure 7. Star-Delta conversion.
Url:https://i.ytimg.com/vi/k7_SMjgVsUo/hqdefault.jpg

STPS Cont…..

- Thus, to simplify and use balanced three phase system analysis, the delta connected loads is convert ed into star as follows:
- The delta-connected loads can be replaced by Y-connected load with :

$$Z_Y = Z_\Delta / 3 \qquad \text{eqn.(16)}$$

- The delta-connected source can be replaced by Y-connected source with :

$$V_{Phase} = V_{Line} / \sqrt{3} < \alpha \qquad \text{eqn.(17)}$$

- Which means, from Delta side :

$$I_a = \frac{V_{ab}}{Z_\Delta} - \frac{V_{ca}}{Z_\Delta} = \frac{V_{ab} - V_{ca}}{Z_\Delta} \qquad \text{eqn.(18)}$$

$$\Leftrightarrow Z_\Delta = \frac{V_{ab} - V_{ca}}{I_A}$$

STPS Cont.….

- From star connected side:

$$V_{ab} = Z_Y(I_a - I_b)$$
$$V_{ca} = Z_Y(I_c - I_a)$$
$$Then,$$
$$V_{ab} - V_{ca} = Z_Y(2I_a - I_b - I_c)$$

eqn.(19)

- For balanced three phase system without having neutral point, the summation of three phase current

is zero, means :

$$I_a + I_b + I_c = 0$$
$$Ia = -I_b - I_c$$

eqn.(20)

Then, by substitution eqn.(19) is rewritten as:

$$V_{ab} - V_{ca} = 3Z_Y I_a$$
$$\Rightarrow 3Z_Y = \frac{V_{ab} - V_{ca}}{I_a} = Z_\Delta$$
$$\Leftrightarrow Z_Y = \frac{Z_\Delta}{3}$$

eqn.(21)

STPS Cont.…

Advantage of balanced three phase system analysis:

- Offers higher power transmission efficiency, smoother power delivery, and reduced vibration in rotating machinery compared to single-phase systems.
- Balanced three phase system can transmit more power for same amount of wire, twice as much as s ingle phase
- Torque produced by three-phase machines is constant
- Three phase machines use less material for same power rating
- Three phase machines start more easily than single phase machines

4. Per Phase Analysis

Disadvantage of Balanced three phase system:

- Require more conductors and transformers compared to single-phase systems
- increasing infrastructure costs
- Complex and difficult to find out each parameter during calculations.
- As a solution the per-phase analysis is used.

- Per phase analysis allows analysis of three phase systems with the same effort as for a single phase system

Per Phase Analysis Cont.….

For per-phase analysis the following points are used:

- All loads and sources Y connected
- No mutual Inductance between phases
- All neutrals are at the same potential
- All phases are completely decoupled
- All system values are the same sequence as sources.
- The sequence order are phase b lags phase-a and phase-c lags phase-b, known as “positive” seque nce;

Per Phase Analysis Cont.…

Steps for per phase analysis

- Convert all Δ load/sources to equivalent Y's
- Solve phase "a" independent of the other phases
- Total system power $S = 3\, V_a\, I_a^*$
- If desired, phase "b" and "c" values can be determined by inspection (i.e., ±120° degree phase shifts)
- If necessary, go back to original circuit to determine line-line values or internal Δ values.

5. Per Unit Analysis

- In power systems, variables are normalized using per unit analysis to prevent issues with referencing impedances across transformers.
- It entails selecting basis values for current, voltage, power, and impedance, and then expressing each quantity as a ratio of its real to base values
- **Accordingly,** the ratio of the actual value in any value to the chosen base value of the same unit is known as; per unit value, it has a unit less quantities
- A key problem in analyzing power systems is the large number of transformers.
- It would be very difficult to continually have to refer impedances to the different sides of the transformers.

Per Unit Analysis Cont…..

- Referring quantities from one side of the transformer to the other side leads serious errors generation, which can be avoided using per unit system.
- Voltages, currents and impedances expressed in per unit do not change when they are referred
- Per unit impedances of electrical equipment of similar type usually lie within a narrow range, when the equipment ratings are used as base values.
- Transformer connections do not affect the per unit values.

Per Unit Analysis Cont.….

- In power systems the basic quantities which are very important are voltage, current, impedance and power.
- For all per unit calculations a base KVA or MVA and a base KV are chosen.

The following are some of the steps for selection of base values.

- Rating of the largest plant or unit for base MVA or KVA.
- The total capacity of a plant or system for base MVA or KVA.
- Any arbitrary value.

Per Unit Analysis Cont…..

- Once the base values or reference values are chosen. the other quantities can be obtained as;

1. Pick a 1-Φ if VA base for the entire system, S_B
2. Pick a voltage base for each different voltage level, V_B. Voltage bases are related by transformer turns ratios. V is phase voltage for 1-Φ
3. Calculate the current base, $I_B = S_B/V_B$
4. Calculate the impedance base, $Z_B = (V_B)^2/S_B$
5. Convert actual values to per unit
6. Convert back to actual as necessary

Per Unit Analysis Cont.….

I. Pick a 3-Φ VA base for the entire system, $S_B^{3\phi}$

II. Pick a voltage base for each different voltage level, V_B. Voltages are line to line.

III. Calculate the current base, I_B

$$I_B^{3\phi} = \frac{S_B^{3\phi}}{\sqrt{3}\, V_{B,LL}} = \frac{3\, S_B^{1\phi}}{\sqrt{3}\,\sqrt{3}\, V_{B,LN}} = \frac{S_B^{1\phi}}{V_{B,LN}} = I_B^{1\phi}$$ eqn.(22)

IV. Calculate the impedance base

$$Z_B = \frac{V_{B,LL}^2}{S_B^{3\phi}} = \frac{(\sqrt{3}\, V_{B,LN})^2}{3S_B^{1\phi}} = \frac{V_{B,LN}^2}{S_B^{1\phi}}$$ eqn.(23)

Per Unit Analysis Cont.….

Per Unit Change to Common MVA Base

- Parameters for equipment are often given using power rating of equipment as the MVA base
- To analyze a system all per unit data must be on a common power base

$$Z_{Pu}^{Newbase} = Z_{Pu}^{Originalbase} * \frac{\dfrac{V_{base}^{2}}{S_{Base}^{Originalbase}}}{\dfrac{V_{base}^{2}}{S_{Base}^{Newbase}}} \qquad \text{eqn.(24)}$$

$$= Z_{Pu}^{Originalbase} * \frac{S_{Base}^{Newbase}}{S_{Base}^{Originalbase}}$$

Summary

- This lecture provides clear presentation of per-phase and per unit analysis fundamental and their impor tance in power system analysis, specifically in load flow, fault and power stability analysis
- Thus, the voltage, current and power relationship in single-phase, and three phase power system is pre sented
- Then, the relationship between three phase balanced system with single phase system using per phas e analysis is dictated
- In addition, the advantage of three phase system over single phase system is also incorporated
- The per unit importance of per unit analysis in terms of reducing the transformer effect in power system analysis also discussed.

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Thank you !