Advanced Power System Analysis
Lecture 5
Load Flow Methods

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Lecture learning outcomes:

At the end of this lecture, you will be able to:

i. Differentiate the Types of Load Flow Solution Techniques

ii. Formulate the Gauss–Seidel load Flow Method

iii. Develop Newton Raphson and Fast Decoupled Load Flow Methods

iv. Know the effect of PQ and PV buses in Load flow analysis.

Outlines

1. Introduction
2. Gauss-Seidel power flow equations
3. Newton-Raphson Power Flow
4. Fast Decoupled Power Flow

Summary

References

1. Introduction

- Iterative power flow solution techniques like Gauss-Seidel, Newton-Raphson, and Fast Decoupled Load flow form the basis of power system analysis [1].

- The following are the primary advantages of using iterative power flow solution approaches:

1. **Capable of Handling Non-Linear Systems:**

- Phase angles and voltage magnitudes are the main causes of power flow equations' nonlinearity.

- Iterative techniques are capable of accurately solving these non-linear equations, in contrast to direct approaches, which perform better for linear systems.

Introduction Cont.…

2. Better Scalability

- Suitable for large-scale power systems with thousands of buses.
- Techniques like Fast Decoupled Load Flow are designed to work efficiently on very large grids.

3. Increased Accuracy

- Methods like Newton-Raphson provide very accurate results, especially when the system is well-conditioned and a good initial guess is used

Introduction Cont.…

4. Efficient Memory

- Iterative methods can be optimized to use less memory compared to solving large matrices directly.
- Important for large power systems with limited computational resources.

5. Flexibility: easy to modify for:

- Different types of buses (PQ, PV, slack)
- Control variables (Tap changers, reactive power limits and AVR) are easily incorporated to the iterative solutions.

Introduction Cont.…

6. Modification Speed

- Newton-Raphson methods that have been optimized to converge more quickly and use less calculate each iteration include Fast Decoupled Load Flow.

7. Suitable for Real-Time Uses

- Iterative approaches are employed in real-time power system monitoring and control tools due of their speed and flexibility.
- In general, it’s very suitable for the variable load condition that leads to power system parameter fluctuations from the stated values.

Introduction Cont.…

- However, the number of steps needed to obtain actual solution's depends on the initial guess's , problem size and the iterative technique used[2].
- Thus, based on the named constraints, the approximate solutions can be either converge, diverge or oscillate
- But, the round-off error in all case goes on getting corrected in each step.
- The iterative process is then repeated till the bus voltage converges with in prescribed accuracy

2. Gauss-Seidel Power Flow (G-S)

- The gauss-seidel method is an iterative algorithm for solving a set of non linear load flow equations.
- It is an iterative solution of power flow equations and simplest load flow solution
- There are two approaches for G-S power flow analysis

i. When PV bus is absent

ii. Considering all buses

Case-1: Consider G-S power flow when PV-bus is absent and algorithm of Power flow for PQ bus only[3].

- PV buses are absent means there is $n - 1$ buses or PQ buses, and slack bus only.

G-S Cont….

Steps in computing G-S methods for PQ bus only is:

- Step 1. Form the bus admittance matrix of the network by direct inspection method, formulate Y_{bus} matrix.
- Step 2. If the slack bus is not specified, select one of the generator buses as the slack bus. The voltage at the slack bus is assumed as V_i = 1 + j0.0
- Step 3. Assume initial values of voltages for all buses except the slack bus. Vi (0) = 1 + j0.0
- Step 4. Set convergence criterion, ε. If the largest of absolute of the residues exceeds the convergence criterion, the process is repeated, otherwise it is terminated.

G-S Cont….

- **Step** 5. Set iteration count k = 0.
- **Step** 6. Bus count i = 1. If ‘I’ is the slack bus, then there will be an increment in the bus count.
- **Step** 7. Solve the voltage equation for bus i as we know that.

$$P_i - jQ = V_i^* I_i \qquad \text{eqn.(1)}$$

$$\Rightarrow P_i - jQ = V_i^* (\sum_{i=1}^{n} Y_{i,j} * V_j), \quad j = 1,2....n$$

- **Which is similar that:**

$$P_i - jQ = V_i^* (\sum_{i=1}^{n} Y_{i,j} * V_j), \quad j = 1,2....n \qquad \text{eqn.(2)}$$

$$\Leftrightarrow P_i - jQ = V_i^* Y_{i,i} V_i + (\sum_{i=1, j \neq i}^{n} Y_{i,j} * V_j) \quad j = 1,2....n$$

$$V_i = \frac{P_i - jQ}{Y_{i,i} * V_i^*} - (\sum_{i=1, j \neq i}^{n} Y_{i,j} * V_j) \quad j = 1,2....n$$

G-S Cont….

Thus,

$$V_i^{(k+1)} = \frac{\dfrac{P_i - jQ_i}{V_i^{*[k]}} + \sum_{j=1}^{n} Y_{ij} V_j^k}{\sum_{j=0}^{n} Y_{ij}}, i \neq j$$

eqn.(3)

- Accordingly, the iterative solution of power flow are given by:

$$V_i^{k+1} = \frac{\dfrac{P_i - jQ}{(V_i^k)^*} - \left(\sum_{j=1, j\neq i}^{n} Y_{i,j} * V_j^k\right)}{Y_{i,i}}$$

eqn.(4)

$$P_i^{[k+1]} = \mathrm{Re}[(V_i^k)^*[V_i^{k+1} Y_{i,i} + (\sum_{j=1, j\neq i}^{n} Y_{i,j} * V_j^k)]$$

$$P_i^{[k+1]} = \mathrm{Im}[(V_i^k)^*[V_i^{k+1} Y_{i,i} + (\sum_{j=1, j\neq i}^{n} Y_{i,j} * V_j^k)]$$

- **Step 8:** Calculate the change in bus voltage ΔV_i:

$\Delta Vi = V_i^{(k+1)} - V_i^k$ eqn.(5)

G-S Cont….

Step 9: The process of convergence in G-S method is slow as it requires large number of iterations to obtain the solution.

- It can be speeded up using acceleration factor.

$$V_{i(accel)}^{k+1} = V_i^k + \alpha[V_i^{k+1} - V_i^k] \quad \text{eqn.(6)}$$

- Step 10: Calculate the bus voltages V_i^{k+1} for all the buses except the slack bus, i = 2,3, ….,n.
- Step 11: Repeat the iteration until the required solution achieved.
- Step 12: Finally calculate the power flow and power losses at slack bus using the load flow equations.

G-S Cont….

Case 2: when PV bus is considered, the steps used are

- Step 1: Calculate the imaginary part or reactive power of PV bus 1st using the eqn.7 for Q value.

$$Q_i^{[k+1]} = \mathrm{Im}[(V_i^k)^*[V_i^{k+1}Y_{i,i} + (\sum_{j=1, j\neq i}^{n} Y_{i,j} * V_j^k)]] \quad \text{eqn.(7)}$$

- For the given value of reactive power minimum and maximum limit:

$$Q_{\min} \leq Q_i \leq Q_{\max} \quad \text{eqn.(8)}$$

- If Q_i is within the limits indicates that with available reactive power generation, it is possible to maintain the voltage magnitude at the named bus and only angle δ_i is calculated

G-S Cont….

- δ_i is calculated using the expression for voltage as presented before.
- Thus, the presented attempts from Step-1 to Step-12 in PQ bus also works for PV buses.
- However, if Q is less than the minimum limit and greater than the maximum lit , consider the PV bus as a PQ bus.
- Thus, reactive power demand is equal to the Q_{min} if the calculated value is less than minimum demand
- The reactive power demand is equal to the Q_{max} if the calculated value is greater than maximum demand

G-S Cont….

- Then, the steps from Step-1 to Step-12 is applied at the same manner to determine the voltage angle of PQ bus.
- It means that the voltage controlled bus is treated as a load bus with the known quantities as P_D and Q_D and unknown quantities as |V| and δ
- Qi outside the limits, if the reactive power limits are not satisfied indicates that the voltage level at that bus can not be maintained at the specified value.
- Because, the available Q always determine the voltage at that bus.

G-S Cont….

Advantages and Disadvantages of Gauss–Seidel Method

A. *Advantages*

- The calculations are simple and so there is less programming task to perform.
- The memory requirement is small.
- Useful for the small systems.
- Faster computation per iteration compared to some other methods

G-S Cont.…

- *Disadvantages*
  - Requires a large number of iterations to converge.
  - Not suitable for large systems.
  - Convergence time increases with the size of the system.
- Highly depends on the initial guess

3. Newton-Raphson Power Flow (NRPF)

- The Newton–Raphson method is regarded as the most crucial of the many load flow analysis solution techniques available for big interconnected power systems.
- There are numerous benefits associated with the Newton-Raphson method.
- When compared to the other processes, its convergence properties are comparatively strong.
- When sparse network equations are solved using the scarcity programmed ordered elimination technique, significantly short computation times are obtained.

NRPF Cont.....

- The reliability of the Newton–Raphson method is comparatively good, since it can solve cases that lead to divergence with the other popular processes, but the method is by no means reliable
- Failure does not occur on some ill-conditioned problems.
- The number of iterations required to obtain a solution is independent of the system size, but more functional evaluations are required at each iteration.
- Since in the load flow problem real power and magnitude of bus voltage are specified for the PV buses , the load flow equation is formulated in the polar form

NRPF Cont.….

- N-R iterative method approximate set of non-linear simultaneous eqns. to set of linear simultane ous eqns. By Taylor's expansion[4]:
- Let's consider:

$$f_1(x_1,x_2)=K_1$$
$$f_2(x_1,x_2)=K_2$$

eqn.(9)

Where, K1 and K2 are constants.

- $x_{1(0)}$ and $x_{2(0)}$ are estimate solution and We designate
- $\Delta x_{1(0)}$ & $\Delta x_{2(0)}$ to correct the solution, so we can write:

$$K_1=f_1(x_1,x_2)=f_1\left(x_1^{(O)}+\Delta x_1,x_2^{(O)}+\Delta x_2\right)$$
$$K_2=f_2(x_1,x_2)=f_2\left(x_1^{(O)}+\Delta x_1,x_2^{(O)}+\Delta x_2\right)$$

eqn.(10)

NRPF Cont.….

- Now we can solve for $\Delta x_{1(0)}$ & $\Delta x_{2(0)}$ by expanding eqn.10. in Taylor's series to give:

$$K_1 = f_1(x_1, x_2) = f_1\left(x_1^{(0)}, x_2^{(0)}\right) + \Delta x_1^{(0)} \left.\frac{\partial f_1}{\partial x_1}\right|^{(0)} + \Delta x_2^{(0)} \left.\frac{\partial f_1}{\partial x_2}\right|^{(0)} + \ldots\ldots.$$

$$K_2 = f_2(x_1, x_2) = f_2\left(x_1^{(0)}, x_2^{(0)}\right) + \Delta x_1^{(0)} \left.\frac{\partial f_2}{\partial x_1}\right|^{(0)} + \Delta x_2^{(0)} \left.\frac{\partial f_2}{\partial x_2}\right|^{(0)} + \ldots\ldots$$

eqn.(11)

- If partial derivatives of first order only taken, the above equations in matrix form:

$$\begin{vmatrix} K_1 - f_1\left(x_1^{(0)}, x_2^{(0)}\right) \\ K_2 - f_2\left(x_1^{(0)}, x_2^{(0)}\right) \end{vmatrix} = \begin{vmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} \end{vmatrix}^{(0)} \begin{vmatrix} \Delta x_1^{(0)} \\ \Delta x_2^{(0)} \end{vmatrix}$$

eqn.(12)

NRPF Cont.….

- Equation 11, can be written as

$$\begin{vmatrix} \Delta K_1^{(0)} \\ \Delta K_2^{(0)} \end{vmatrix} = J^{(0)} \begin{vmatrix} \Delta x_1^{(0)} \\ \Delta x_2^{(0)} \end{vmatrix} \qquad \text{eqn.(12)}$$

- Where, **$J^{(0)}$** is called Jacobean of initial estimate and by finding it we can determine $\Delta x_{1(0)}$ & $\Delta x_{2(0)}$.
- By correcting the initial estimate we have;

$$x_1^{(1)} = x_1^{(0)} + \Delta x_1^{(0)}$$
$$x_2^{(1)} = x_2^{(0)} + \Delta x_2^{(0)} \qquad \text{eqn.(13)}$$

- Then, the process is repeated until the correction is small
- To use the above discussion for power flow problem, we choose polar coordinate and designate:

$$V_i = |V_i| < \delta_i$$
$$V_j = |V_j| < \delta_j$$
$$Y_{ij} = |Y_{ij}| < \theta_{ij} \qquad \text{eqn.(14)}$$

- We have

$$P_i - jQ_i = \sum_{j=1}^{N} |V_i V_j Y_{ij}| \angle -\theta_{ij} - \delta_j + \delta_i$$

NRPF Cont…..

- From this;

$$P_i = \sum_{i=1}^{N} \left|V_i V_j Y_{ij}\right| \cos\left(\delta_i - \delta_j - \theta_{ij}\right)$$
$$Q_i = -\sum_{i=1}^{N} \left|V_i V_j Y_{ij}\right| \sin\left(\delta_i - \delta_j - \theta_{ij}\right)$$

eqn.(15)

- $J^{(i)}$ consists of the partial derivatives of P & Q with respect to each variables(voltage and its angle)

- For 3 bus system if bus 1 is swing, matrix eqns. for each iteration is :

$$\begin{bmatrix} \Delta P_2 \\ \Delta P_3 \\ \Delta Q_2 \\ \Delta Q_3 \end{bmatrix} = \left[\begin{array}{cc|cc} \frac{\partial P_2}{\partial \delta_2} & \frac{\partial P_2}{\partial \delta_3} & \frac{\partial P_2}{\partial |V_2|} & \frac{\partial P_2}{\partial |V_3|} \\ \frac{\partial P_3}{\partial \delta_2} & \frac{\partial P_3}{\partial \delta_3} & \frac{\partial P_3}{\partial |V_2|} & \frac{\partial P_3}{\partial |V_3|} \\ \hline \frac{\partial Q_2}{\partial \delta_2} & \frac{\partial Q_2}{\partial \delta_3} & \frac{\partial Q_2}{\partial |V_2|} & \frac{\partial Q_2}{\partial |V_3|} \\ \frac{\partial Q_3}{\partial \delta_2} & \frac{\partial Q_3}{\partial \delta_3} & \frac{\partial Q_3}{\partial |V_2|} & \frac{\partial Q_3}{\partial |V_3|} \end{array}\right] \begin{bmatrix} \Delta \delta_2 \\ \Delta \delta_3 \\ \Delta V_2 \\ \Delta V_3 \end{bmatrix}$$

J1 J2 J3 J4

eqn.(16)

NRPF Cont.….

Elements of Jacobean matrix J1:

- The diagonal elements are:

$$\frac{\partial P_i}{\partial \delta_i} = \sum_{j \neq i} |V_i||V_j||Y_{i,j}|\sin(\phi_{i,j} - \delta_i + \delta_j) \qquad \text{eqn.(17)}$$

- The off-diagonal elements are:

$$\frac{\partial P_i}{\partial \delta_j} = -|V_i||V_j||Y_{i,j}|\sin(\phi_{i,j} - \delta_i + \delta_j),\ j \neq i \qquad \text{eqn.(18)}$$

NRPF Cont.….

Elements of Jacobean matrix J2 :

- The diagonal elements are:

$$\frac{\partial P_i}{\partial V_i} = 2|V_i||Y_{i,j}|\cos\phi_{i,i} + \sum_{j\neq i}|V_j||Y_{i,j}|\cos(\phi_{i,j} - \delta_i + \delta_j)$$ eqn.(19)

- The off-diagonal elements are:

eqn.(20)

$$\frac{\partial P_i}{\partial V_j} = |V_i||Y_{i,j}|\cos(\phi_{i,j} - \delta_i + \delta_j),\ j \neq i$$

NRPF Cont.….

Elements of Jacobean matrix J3:

- The diagonal elements are:

$$\frac{\partial Q_i}{\partial \delta_i} = \sum_{j \neq i} |V_i||Y_{i,j}| \cos(\phi_{i,j} - \delta_i + \delta_j) \qquad \text{eqn.(21)}$$

- The off-diagonal elements are:

$$\frac{\partial Q_i}{\partial \delta_j} = -|V_i||V_j||Y_{i,j}| \cos(\phi_{i,j} - \delta_i + \delta_j), j \neq i \qquad \text{eqn.(22)}$$

NRPF Cont.….

Elements of Jacobean matrix J4 :

- The diagonal elements are:

$$\frac{\partial Q_i}{\partial V_i} = -|V_i||Y_{i,j}|\sin\phi_{i,j} - \sum_{j\neq i}|V_j||Y_{i,j}|\sin(\phi_{i,j} - \delta_i + \delta_j) \quad \text{eqn.(23)}$$

- The off-diagonal elements are:

$$\frac{\partial Q_i}{\partial V_j} = -|V_i||Y_{i,j}|\sin(\phi_{i,j} - \delta_i + \delta_j),\ j \neq i \quad \text{eqn.(24)}$$

- Difference in scheduled to calculated power (power residuals) is given by :

$$\Delta P_i^k = P_{i,sched} - P_i^k$$

$$\Delta Q_i^k = Q_{i,sched} - Q_i^k \quad \text{eqn.(25)}$$

- Then, the new estimate for voltage and its angle are:

$$\delta_i^{k+1} = \delta_i^k + \Delta\delta_i^k$$

$$V_i^{k+1} = V_i^k + \Delta V_i^k \quad \text{eqn.(26)}$$

NRPF Cont.….

- $\Delta\delta_k$ and $\Delta|V_k|$ found from the eqn.26 is added in pervious value to calculate new value of P & Q, and

  process is repeated until desired precision index achieved.

- To achieve quick convergence, N-R is sensitive for initial estimate.

- As initial estimate, the nominal voltages values can be used.

- For PV bus **J** with respect to constant V will be omitted & row with respect to Q also omitted as presente

  d in eqn.26, and both parameters are calculated after convergence.

NRPF Cont.….

- The steps applied for N-R power flow problem are as follow:
- Step-1: Determine: $\begin{vmatrix} \Delta P^{(i)} \\ \Delta Q^{(i)} \end{vmatrix} = \begin{vmatrix} P\text{-}P(x^{(i)}) \\ Q\text{-}Q(x^{(i)}) \end{vmatrix}$ eqn.(27)
- Where; $x^{(i)} = \begin{vmatrix} \delta^{(i)} \\ V^{(i)} \end{vmatrix}$

Step-2: Calculate Jacobean matrix:

$$\begin{vmatrix} \Delta P^{(i)} \\ \Delta Q^{(i)} \end{vmatrix} = \begin{vmatrix} J_1^{(i)} & J_2^{(i)} \\ J_3^{(i)} & J_4^{(i)} \end{vmatrix} \begin{vmatrix} \Delta \delta^{(i)} \\ \Delta V^{(i)} \end{vmatrix} \qquad \text{eqn.(28)}$$

- Step-3: Solve the following equation to find $\Delta\delta_{(i)}$ and $\Delta V_{(i)}$

$$x^{(i+1)} = \begin{vmatrix} \delta^{(i+1)} \\ V^{(i+1)} \end{vmatrix} = \begin{vmatrix} \delta^{(i)} \\ V^{(i)} \end{vmatrix} + \begin{vmatrix} \Delta\delta^{(i)} \\ \Delta V^{(i)} \end{vmatrix} \qquad \text{eqn.(29)}$$

NRPF Cont.….

- Step 4: Power flows and losses on the network branches are computed as follows once bus voltages a nd angles have been solved for:
- Network branches include transformers and transmission cables.
- A branch element's (shown on a medium length line) direction of positive current flow is defined.
- Each branch end has a set power flow.
- Step 5: Lastly, determine the slack bus's real and reactive power.

NRPF Cont…..

Advantages and disadvantages of NR method

i. Advantages

- Fast convergence as long as initial guess is close to solution
- large region of convergence
- Used in the most actual power network analysis

ii. Disadvantages:

- it requires the derivative of the function and may struggle with convergence if the derivative is zero or the initial guess is poor.
- Each iteration takes much longer than a Gauss Seidel iteration
- more complicated to code, particularly when implementing sparse matrix algorithms

4. Fast Decoupled Load Flow (FDLF)

- One of the enhanced techniques, the Fast Decoupled Load Flow (FDLF) method, was developed by simplifying the Newton–Raphson method.
- This approach became the most popular in load flow analysis because of its quick convergence, dependable results, and ease of computation.
- FDLF, however, does not always converge well in situations with high R/X ratios or excessive loading (low voltage) at some buses.
- In these situations, numerous attempts and advancements have been made to get over these convergence barriers.

FDLF Cont……

- Some of them focused on systems with low voltage buses, while others aimed to converge systems with high R/X ratios.
- But one of the most recent innovations is a Robust Fast Decoupled Load Flow, which solves the low bus voltage and high R/X ratio problems at the same time.
- It is based on generic voltage normalization techniques and heuristic justification.
- This technique takes advantage of the power system's characteristic whereby the true power flow-voltag e a

$$P = \frac{V_1 V_2}{X_1} \sin\delta$$

eqn.(30)

$$Q = \frac{V_1 V_2}{X_1} \cos\delta - \frac{V_2^2}{X}$$

FDLF Cont……

- The transmission lines have high X/R ratio
- For such system the real power changes ΔP are less sensitive to changes in the voltage magnitude and are most sensitive to change in phase angle $\Delta\delta$.
- Similarly the reactive power changes ΔQ is less sensitive to change in angles $\Delta\delta$ and is mainly dependent on changes in voltage magnitude:
- Which means , the $J2 = J3=0$

FDLF Cont.…

- It is given as :

$$\begin{bmatrix} \Delta P_2 \\ \Delta P_3 \\ \Delta Q_2 \\ \Delta Q_3 \\ \end{bmatrix} = \begin{bmatrix} \frac{\partial P_2}{\partial \delta_2} & \frac{\partial P_2}{\partial \delta_3} & \frac{\partial P_2}{\partial |V_2|} & \frac{\partial P_2}{\partial |V_3|} \\ \frac{\partial P_3}{\partial \delta_2} & \frac{\partial P_3}{\partial \delta_3} & \frac{\partial P_3}{\partial |V_2|} & \frac{\partial P_3}{\partial |V_3|} \\ \frac{\partial Q_2}{\partial \delta_2} & \frac{\partial Q_2}{\partial \delta_3} & \frac{\partial Q_2}{\partial |V_2|} & \frac{\partial Q_2}{\partial |V_3|} \\ \frac{\partial Q_3}{\partial \delta_2} & \frac{\partial Q_3}{\partial \delta_3} & \frac{\partial Q_3}{\partial |V_2|} & \frac{\partial Q_3}{\partial |V_3|} \end{bmatrix} \begin{bmatrix} \Delta \delta_2 \\ \Delta \delta_3 \\ \Delta V_2 \\ \Delta V_3 \\ \end{bmatrix} \Rightarrow \begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} J_1 & 0 \\ 0 & J_2 \end{bmatrix} [\ \ ] \qquad \text{eqn.(31)}$$

- FDLF technique is very useful in contingency analysis where numerous
- outages are to be simulated or a load flow solution is required for online control.

Summary

- This lecture note is presents the importance of iterative power flow solutions to get the four power system parameters.
- Accordingly, the load flow equations for different types of iterative power flow solution are developed
- Besides this, the importance of initial guess, the size of problem and techniques used are the main points while using iterative power flow solutions. Thus, the power flow result either converge, diverge or oscillate based on these parameter effect.
- The advantage of Fast-decoupled load flow solution is also discussed.
- Finally, the advantage and disadvantage of each power flow solution techniques are also discussed.

References

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Thank you !