

Advanced Power System Analysis

Lecture 9

Fault analysis

Lecturer: Teshome Goa (Assist. Prof.)

Lecture learning outcomes:

At the end of this lecture, you will be able to:

- i. Understand the importance of unbalanced fault analysis
- ii. Know the formulation of symmetrical matrix for unbalanced fault analysis.
- iii. Identify the three sequence networks and their relationship with Phase parameters.
- iv. Develop the sequence network for unbalanced fault analysis

Outlines

1. **Introduction: Unsymmetrical Fault**
2. **Sequence Networks**
3. **Symmetrical Analysis for Unbalanced Faults**
4. **Single Line-to-Ground (SL-G) Faults**
5. **The Line-to-Line (L-L) Unbalanced Faults**
6. **Double Line-to-Ground(LL-G) Faults**

Summary

References

1. Introduction: Unsymmetrical Fault

- Except for the balanced three-phase fault, faults result in an unbalanced system[1].
- The most common types of faults are single line-ground (SLG) and line-line (LL).
- Other types are double line-ground (DLG), open conductor, and balanced three phase.
- System is **only unbalanced** at point of fault!
- The easiest method to analyze unbalanced system operation due to faults is through the use of **symmetrical** components

2. Sequence Networks

- The key idea of symmetrical component analysis is to decompose the system into three sequence networks[2]. The networks are then coupled only at the point of the unbalance (i.e., the fault)
- The three sequence networks are known as the
 - positive sequence (this is the one we've been using)
 - negative sequence
 - zero sequence

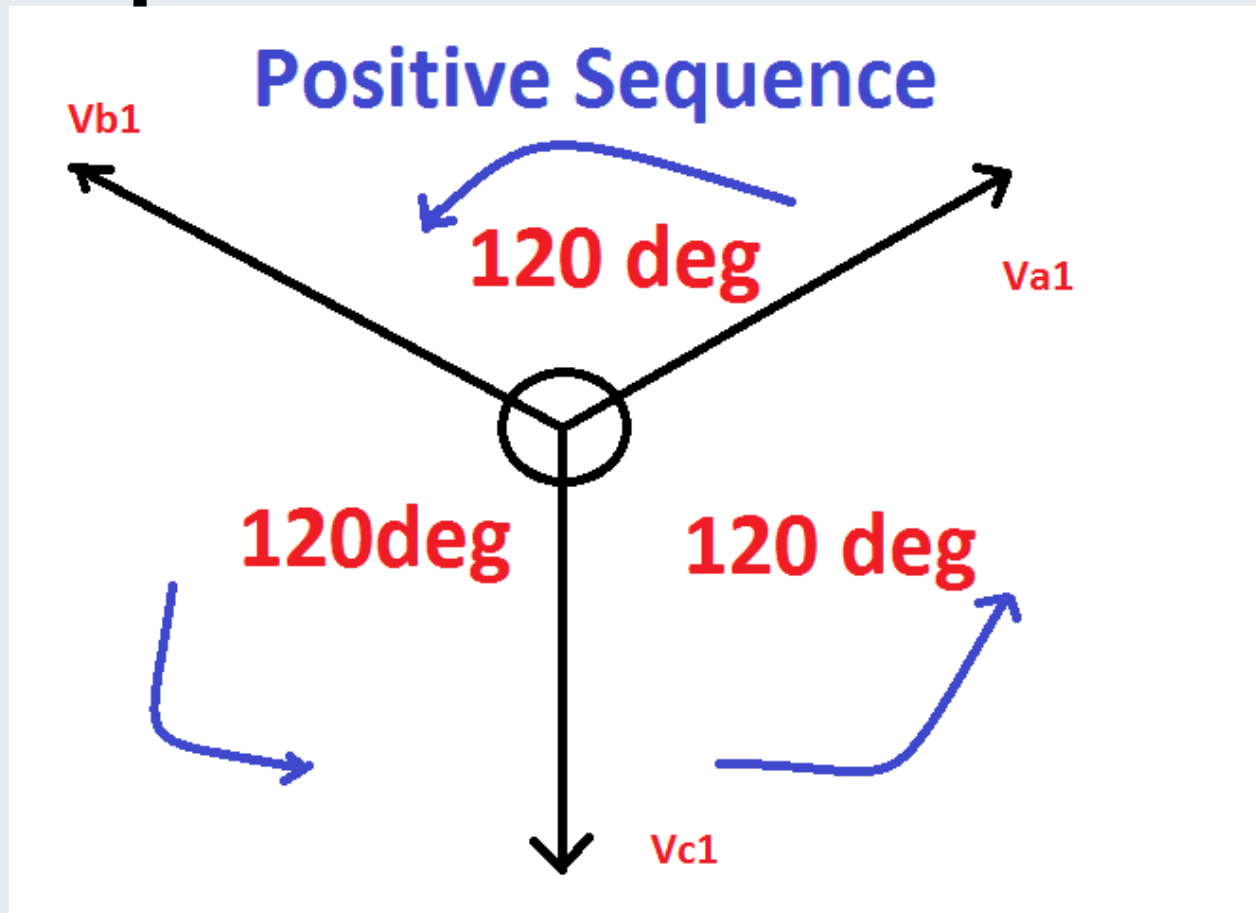
Sequence Networks

Cont.....

- **Positive Sequence Sets:** A regular three-phase electrical system that is balanced is represented by a positive sequence network.
- In symmetrical component analysis, which aids in the study of **unbalanced faults in electrical** power systems, it is one of three sequence networks.
- Only positive sequence voltages, currents, and impedances are present in the positive sequence network.
- The positive sequence sets have three phase currents/voltages with equal magnitude, with phase b lagging phase a by 120° , and phase c lagging phase b by 120° .
- We've been studying positive sequence sets

Sequence Networks

Cont.....



- Positive sequence sets have **zero** neutral current

Figure 1. Positive sequence network sequence diagram.

Url: <https://www.electrical4u.net/wp-content/uploads/2018/06/Symmetrical-Components-positive-Sequence.png>

Sequence Networks

Cont.....

- **Negative Sequence Sets:** The negative sequence impedance is **created due to negative sequence** current flow.
- Negative sequence current **in a three-phase system refers to a current component that** rotates in the opposite direction to the positive sequence current, indicating an unbalanced condition in the system.
- It's a **result of asymmetrical currents** or voltages, which can be caused by factors like unbalanced loads or faults.
- The negative sequence sets have three phase currents/voltages with equal magnitude, with phase b **leading** phase a by 120° , and phase c **leading** phase b by 120° .
- Negative sequence sets are similar to positive sequence, except the phase order is reversed as presented in Fig.2

Sequence Networks

Cont.....

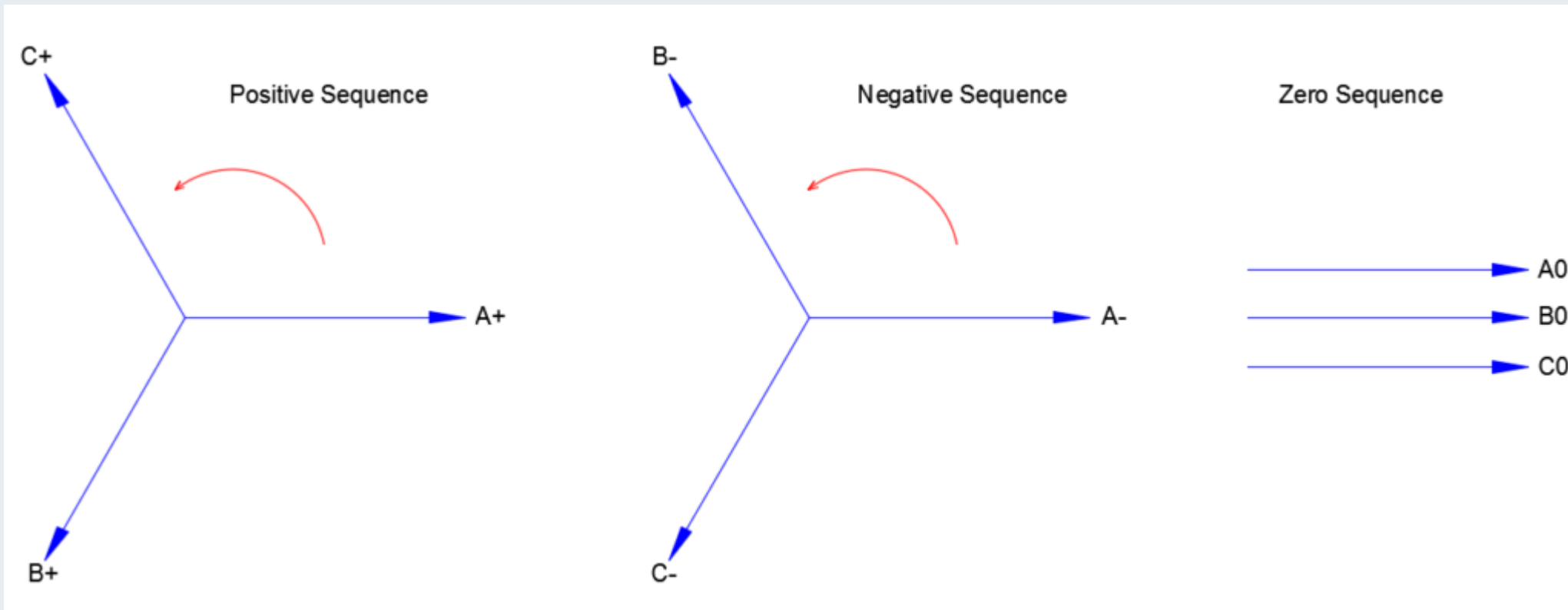


Figure 2. The three sequence currents/voltages of unsymmetrical fault.

[Url:https://voltage-disturbance.com/wp-content/uploads/2018/06/Positive-Negative-Zero-Sequence-Components-1024x396.png](https://voltage-disturbance.com/wp-content/uploads/2018/06/Positive-Negative-Zero-Sequence-Components-1024x396.png)

- Negative sequence sets have **zero** neutral current

Sequence Networks

Cont.....

- **Zero Sequence Sets:** When the phase currents in a three-phase system are out of balance and do not add up to zero, the current that occurs is referred to as zero sequence current.
- In a three-phase system, it is basically the "common" or "residual" current that flows when the phase currents are not balanced.
- Unbalanced loads, defects, or non-linear loads are some of the causes of this unbalanced situation.
- Three values with equal magnitude and angle make up zero sequence sets.
- Neutral current is present in zero sequence sets.

Sequence Networks

Cont.....

- **Sequence Set Representation:** Any arbitrary set of three phases, say I_a, I_b, I_c or V_a, V_b, V_c can be represented as a sum of the three sequence sets:

$$I_a = I_a^0 + I_a^+ + I_a^-$$

$$I_b = I_b^0 + I_b^+ + I_b^-$$

$$I_c = I_c^0 + I_c^+ + I_c^-$$

eqn.(1)

$$V_a = V_a^0 + V_a^+ + V_a^-$$

$$V_b = V_b^0 + V_b^+ + V_b^-$$

$$V_c = V_c^0 + V_c^+ + V_c^-$$

eqn.(1)

- Where, I_a^0, I_a^+, I_a^- and V_a^0, V_a^+, V_a^- are the zero, positive and negative sequence currents and voltages, respectively

Sequence Networks

Cont.....

- Now, the unsymmetrical components can be converted into symmetrical sequences while referring the three sequences network and the phase-sequence current relationship as given by:
- Consider, $\alpha=120^\circ$, which is the phase-shift of a balanced three phase power system and assume :

$$\alpha = 1\angle 120^\circ,$$

$$\alpha + \alpha^2 + \alpha^3 = 0$$

$$\alpha^3 = 1$$

eqn.(3)

- Which means

$$\alpha = 1\angle 120^\circ = \cos 120^\circ + j \sin 120^\circ$$

$$\alpha^2 = \cos 240^\circ + j \sin 240^\circ$$

$$\alpha^3 = \cos 360^\circ + j \sin 360^\circ$$

Sequence Networks

Cont.....

- Consider phase A as a reference to develop the symmetrical components.
- Considering the analogy of zero, positive and negative sequences network of Phase b, and c with

respect to phase A are:

$$\begin{aligned} I_b^0 &= I_a^0 & I_c^0 &= I_a^0 \\ I_b^+ &= \alpha^2 I_a^+ & I_c^+ &= \alpha I_a^+ \\ I_b^- &= \alpha I_a^- & I_c^- &= \alpha^2 I_a^- \\ I_b &= I_a^0 + \alpha^2 I_a^+ + \alpha I_a^- & I_c &= I_a^0 + \alpha I_a^+ + \alpha^2 I_a^- \end{aligned} \quad \text{eqn.(4)}$$

- Then, the symmetrical matrix is given as:

$$\begin{aligned} I_a &= I_a^0 + I_a^+ + I_a^- \\ I_b &= I_a^0 + \alpha^2 I_a^+ + \alpha I_a^- \\ I_c &= I_a^0 + \alpha I_a^+ + \alpha^2 I_a^- \end{aligned} \quad \text{eqn.(5)}$$

Sequence Networks

Cont.....

- Thus,

$$\begin{vmatrix} I_a \\ I_b \\ I_c \end{vmatrix} = I_a^0 \begin{vmatrix} 1 \\ 1 \\ 1 \end{vmatrix} + I_a^+ \begin{vmatrix} 1 \\ \alpha^2 \\ \alpha \end{vmatrix} + I_a^- \begin{vmatrix} 1 \\ \alpha \\ \alpha^2 \end{vmatrix}$$

eqn.(6)

- Which is:

$$\begin{vmatrix} I_a \\ I_b \\ I_c \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 \\ 1 & \alpha^2 & \alpha \\ 1 & \alpha & \alpha^2 \end{vmatrix} \begin{vmatrix} I_a^0 \\ I_a^+ \\ I_a^- \end{vmatrix}$$

eqn.(7)

Sequence Networks

Cont.....

- While defining the symmetrical component Transformation matrix as 'A' , which is:

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha^2 & \alpha \\ 1 & \alpha & \alpha^2 \end{bmatrix} \quad \text{eqn.(8)}$$

- Then, the phase voltage/current and sequence components are related as:

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = A \begin{bmatrix} I_a^0 \\ I_a^+ \\ I_a^- \end{bmatrix}, \quad \begin{bmatrix} V \\ V_b \\ V_c \end{bmatrix} = A \begin{bmatrix} V_a^0 \\ V_a^+ \\ V_a^- \end{bmatrix} \quad \text{eqn.(9)}$$

$$I = AI_s$$

$$V = AV_s$$

Sequence Networks

Cont.....

- Considering the Inverse matrix, the sequence parameters are determined if phase values are given as :
- From the above formula:

$$I_s = \frac{I}{A} = A^{-1}I$$
$$V_s = \frac{V}{A} = A^{-1}V$$
$$A^{-1} = 1/3 \begin{vmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{vmatrix}$$

eqn.(10)

$$I_s = \frac{I}{A} = 1/3 \begin{vmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{vmatrix} I$$

eqn.(11)

$$V_s = \frac{V}{A} = 1/3 \begin{vmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{vmatrix} V$$

Sequence Networks

Cont.....

- Example: For three phase power systems phase current with equal magnitude and 120° as given below. Determine the three sequence currents using symmetrical Matrix.

$$I = \begin{bmatrix} 20\angle 0 \\ 20\angle -120 \\ 20\angle 120 \end{bmatrix} = \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}$$

- Solution :
- Determine sequence Matrix A as:

$$A = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha^2 & \alpha \\ 1 & \alpha & \alpha^2 \end{bmatrix} = \begin{bmatrix} 1 & & 1 \\ 1 & (\cos 240 + j\sin 240) & (\cos 120 + j\sin 120) \\ 1 & (\cos 120 + j\sin 120) & (\cos 240 + j\sin 240) \end{bmatrix}$$

Sequence Networks

Cont.....

- Then, the sequence currents are;

$$I_s = A^{-1}I$$
$$= 1/3 \begin{vmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{vmatrix} * \begin{vmatrix} I_a \\ I_b \\ I_c \end{vmatrix}$$

Which means

$$I_s = A^{-1}I$$
$$I_s^0 = (I_a + I_b + I_c)/3$$
$$= (20\angle 0 + 20\angle -120 + 20\angle 120)/3$$
$$= (20(\cos 0 + \sin 0) + 20(\cos(-120) + \sin(-120)) + 20(\cos 120 + \sin 120))/3$$
$$= (20 + 20(-0.5 - 0.87) + 20(-0.5 + 0.87))/3$$
$$= 0$$
$$I_s = A^{-1}I$$
$$I_s^+ = \frac{I_a + \alpha * I_b + \alpha^2 * I_c}{3}$$
$$= \frac{20\angle 0 + (1\angle 120 * 20\angle -120) + 1\angle 120 * 20\angle 240}{3}$$
$$= \frac{20(\cos 0 + \sin 0) + 20(\cos(0) + \sin(0)) + 20(\cos 360 + \sin 360)}{3}$$
$$= \frac{20 + 20 + 20(1 + 0)}{3}$$
$$= 20\angle 0$$

Sequence Networks

Cont.....

$$I_s = A^{-1}I$$

$$I_s^- = \frac{I_a + \alpha^2 * I_b + \alpha * I_c}{3}$$

$$= \frac{20\angle 0 + (1\angle 240 * 20\angle -120) + 1\angle 120 * 20\angle 120}{3}$$

$$= \frac{20(\cos 0 + \sin 0) + 20(\cos(120) + \sin(120)) + 20(\cos 240 + \sin 240)}{3}$$

$$= 0$$

- Then, the three sequence currents are:

$$\begin{vmatrix} I_s^0 \\ I_s^+ \\ I_s^- \end{vmatrix} = \begin{vmatrix} 0 \\ 20\angle 0 \\ 0 \end{vmatrix}$$

Sequence Networks

Cont.....

- It's observed that for balanced three phase currents we have only positive sequence currents.
- But, in fault condition the system will not be the same as presented in example one.

Example 2: Assume the three phase voltage are give below and determine the sequence voltages

$$\begin{matrix} |V_a| \\ |V_b| \\ |V_c| \end{matrix} = \begin{matrix} |10\angle 0 \\ 10\angle -120 \\ 8\angle 100 \end{matrix}$$

Solution:

$$\begin{aligned} V_s^0 &= \frac{1}{3}[10\angle 0 + 10\angle -120 + 8\angle 60] \\ &= \frac{1}{3}[10 + 10(\cos(-120) + j\sin(-120)) + 8(\cos 60 + j\sin 60)] \\ &= \frac{1}{3}[10 - 5 - j8.7 + 4 + j8.7] = 3\angle 0 \end{aligned}$$

Symmetric Components

Cont.....

- The positive and negative sequence voltages are given by:

$$V_s^+ = \frac{1}{3}[10\angle 0 + (1\angle 120 * 10\angle -120) + (1\angle 240 * 8\angle 60)]$$

$$= \frac{1}{3}[10 + 10(\cos(0) + \sin(0)) + 8(\cos 300 + \sin 300)]$$

$$= \frac{1}{3}[10 + 10 + 4 - j8.7]$$

$$= 8 - j2.32$$

$$= 8.32\angle -16.17$$

$$V_s^- = \frac{1}{3}[10\angle 0 + (1\angle 240 * 10\angle -120) + (1\angle 120 * 8\angle 60)]$$

$$= \frac{1}{3}[10 + 10(\cos(120) + \sin(120)) + 8(\cos 180 + \sin 180)]$$

$$= \frac{1}{3}[10 - 5 - j8.7 - 8 + 0]$$

$$= -1 - j2.9$$

$$= 3.06\angle 70.97$$

$$\begin{matrix} V_s^0 \\ V_s^+ \\ V_s^- \end{matrix} = \begin{matrix} 3\angle 0 \\ 8.32\angle -16.17 \\ 3.06\angle 70.97 \end{matrix}$$

- Then, the sequence voltage are

3. Symmetrical Analysis for Unbalanced Faults

- By splitting the unbalanced system into three balanced systems: positive, negative, and zero sequence components:
- **Symmetrical components make it easier** to analyze unbalanced faults in three-phase systems[3].
- This approach is essential for comprehending and evaluating a variety of fault situations, especially those with varying phase magnitudes or sequences.
- Accordingly, the symmetrical components analysis aims at developing the relationship between symmetrical currents and voltage due to the three sequence reactance's
- Then, the phase voltage and currents will be determined using symmetrical matrix relations

Symmetrical Analysis

Cont....

- Consider the star connected load as presented in Fig.3
- In a three-phase star-connected balanced load, the line voltage is $\sqrt{3}$ times the phase voltage, and the line current is equal to the phase current.
- Each phase in a balanced star connection has the same magnitude of voltage and current, and the phase difference between them is 120

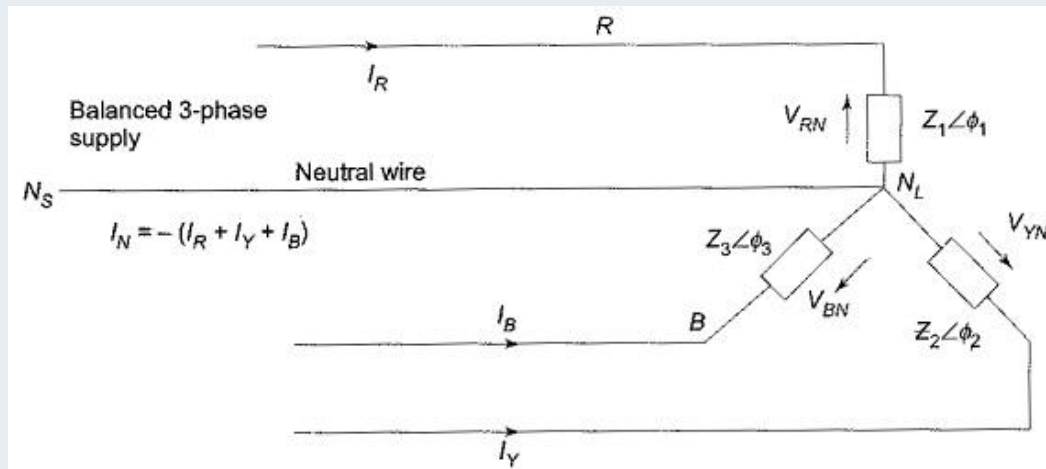


Figure 3. Three phase balanced load.

Url: <https://www.eeeguide.com/wp-content/uploads/2019/12/Unbalanced-Three-Phase-Circuit-Analysis-4.jpg>

Symmetrical Analysis

Cont....

- Consider the following wye-connected load:
- If $R=a$, $B=b$ and $Y=c$, then

$$I_n = I_a + I_b + I_c$$

$$V_{ag} = I_a Z_y + I_n Z_n$$

$$V_{bg} = I_b Z_y + I_n Z_n$$

$$V_{cg} = I_c Z_y + I_n Z_n$$

eqn.(12)

- By substitution :

$$V_{ag} = (Z_Y + Z_n)I_a + I_b Z_n + I_c Z_n$$

$$V_{bg} = I_a Z_n + (Z_Y + Z_n)I_b + I_c Z_n$$

$$V_{cg} = I_a Z_n + I_b Z_n + (Z_Y + Z_n)I_c$$

eqn.(13)

Symmetrical Analysis

Cont....

- The matrix form is

$$\begin{bmatrix} V_{ag} \\ V_{bg} \\ V_{cg} \end{bmatrix} = \begin{bmatrix} Z_Y + Z_n & Z_n & Z_n \\ Z_n & Z_Y + Z_n & Z_n \\ Z_n & Z_n & Z_Y + Z_n \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}$$

Thus,

$$A^{-1}ZA = \begin{bmatrix} Z_Y + Z_n & 0 & 0 \\ 0 & Z_Y & 0 \\ 0 & 0 & Z_Y \end{bmatrix}$$

- which is similar with:

$$V = ZI$$

$$AV_s = ZAI_s$$

$$\Rightarrow V_s = A^{-1}ZAI_s$$

eqn.(14)

eqn.(15)

Symmetrical Analysis

Cont....

- Accordingly, the sequence voltage and currents are related as:

$$\begin{bmatrix} V_s^0 \\ V_s^+ \\ V_s^- \end{bmatrix} = \begin{bmatrix} Z_Y + Z_n & 0 & 0 \\ 0 & Z_Y & 0 \\ 0 & 0 & Z_Y \end{bmatrix} \begin{bmatrix} I_s^0 \\ I_s^+ \\ I_s^- \end{bmatrix}$$

eqn.(16)

- This is can be decoupled as:

$$V_s^0 = (Z_Y + Z_n) * I_s^0$$

$$V_s^+ = Z_Y * I_s^+$$

$$V_s^- = Z_Y * I_s^-$$

eqn.(17)

Symmetrical Analysis

Cont....

- Their sequence diagram of generator is presented in Fig.4.

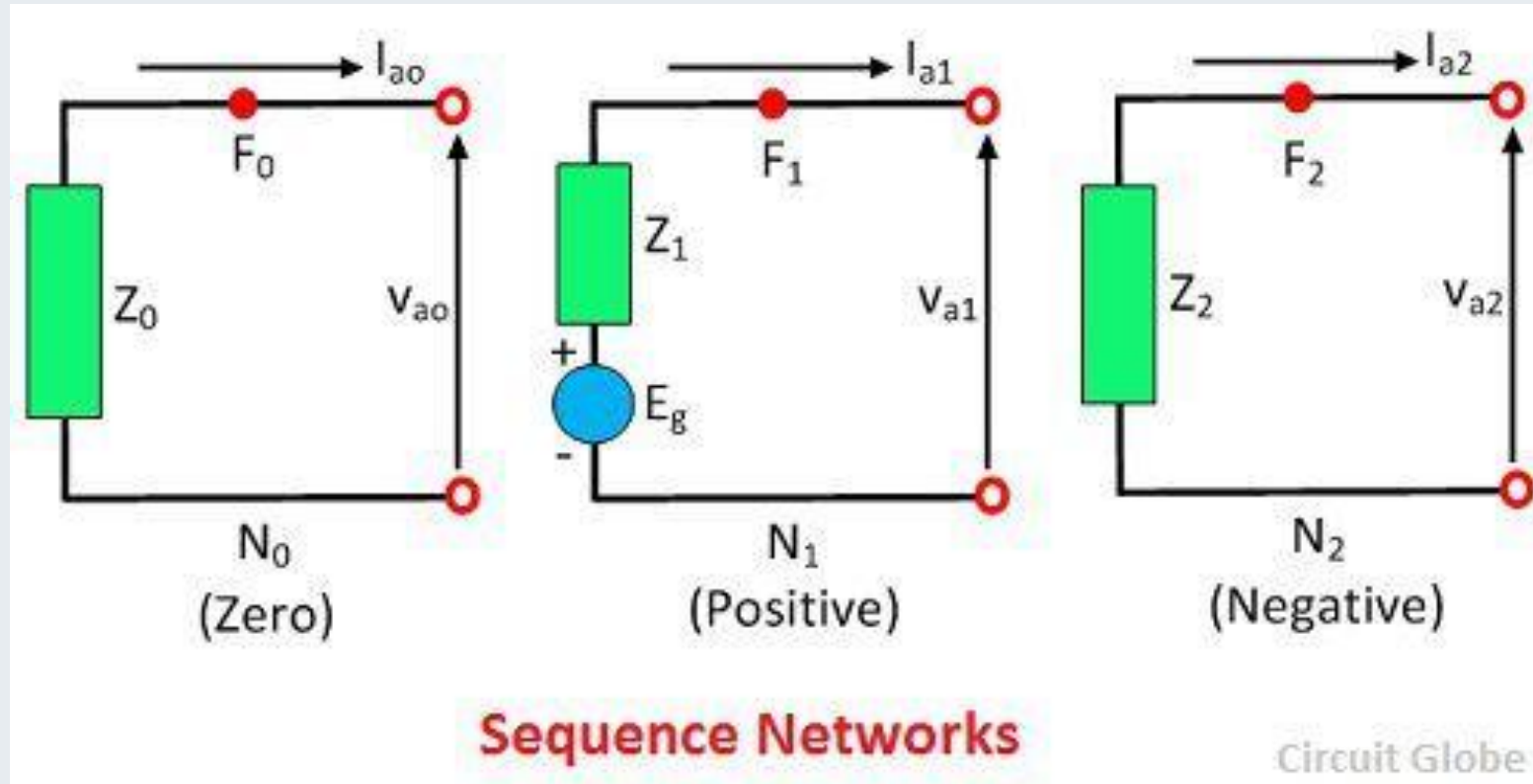


Figure 4. The three sequence Network.

Url: <https://circuitglobe.com/wp-content/uploads/2017/04/sequence-network-image-2.jpg>

Symmetrical Analysis

Cont....

- The key points from Fig.4 is that the generators only produce positive sequence voltages
- Thus, only the positive sequence network has a voltage source and the others are shorted while performing the unbalanced fault analysis
- In addition, during a fault condition the positive and negative sequence impedance of generators are assumed to be the same approximately equal to sub-transient reactance ($Z^+ \approx Z^- \approx X_d''$)
- The zero sequence **impedance** is usually substantially **smaller**.
- The value of Z_n depends on whether the generator is grounded

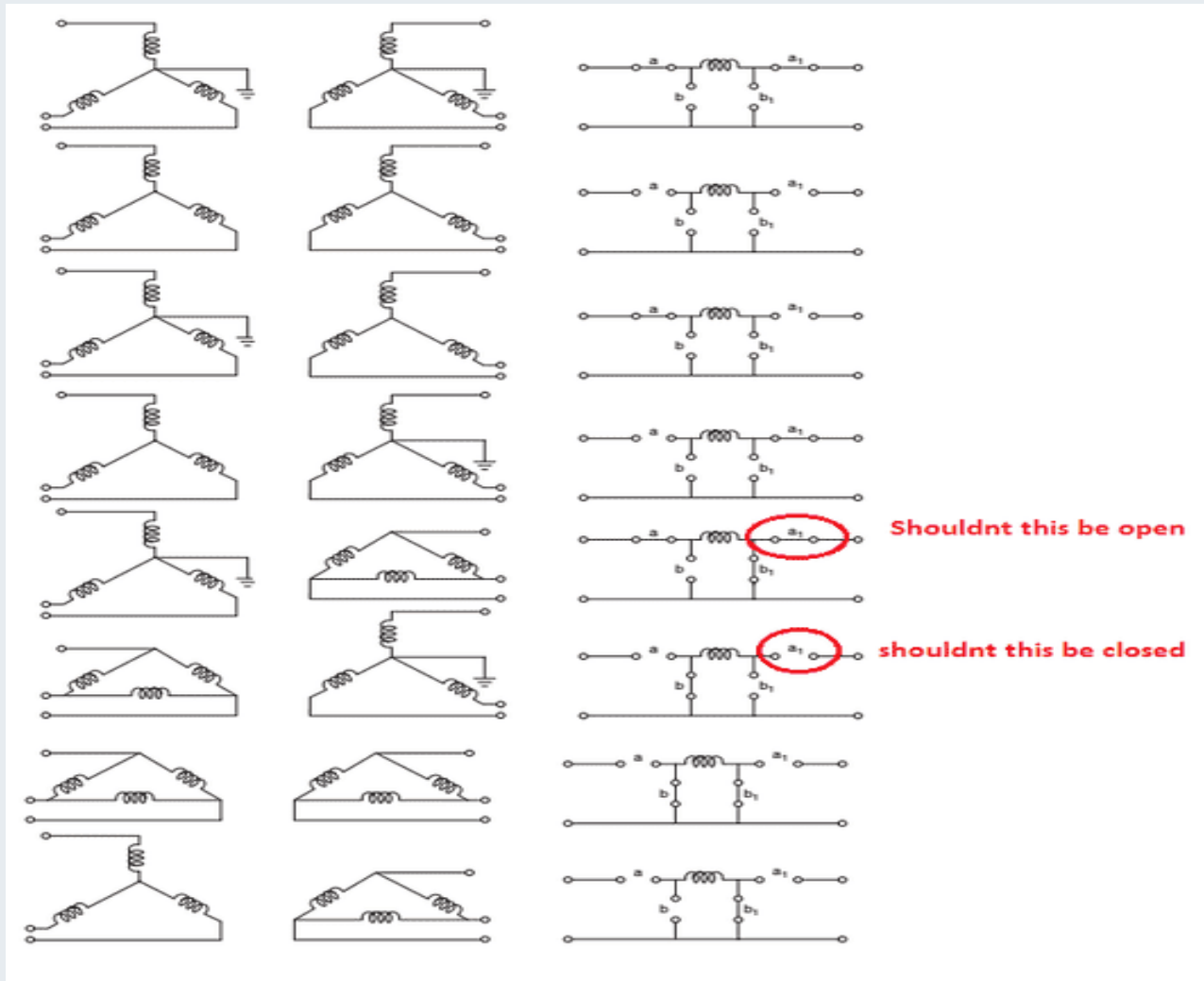
Symmetrical Analysis

Cont....

- The positive and negative sequence **diagrams for transformers are** similar to the transmission lines.
- However, the zero sequence network impedance depends on both how the **transformer is grounded and the type of connection(Star or Delta)**.
- For instance, if the transformer primary and secondary is connected as Y-Y grounded both side, the transformer zero sequence reactance single line diagram similar with transmission line.
- All types of transformers connection and the zero sequence reactance representation is presented in Fig.5

Symmetrical Analysis

Cont....



- Once the positive, negative and zero sequence reactance is known using thevenin equivalent form, then the fault current and voltage can be calculated based on the types of fault as given by:

Figure 5. Representation of zero sequence diagram for transformer.

[Url:https://www.physicsforums.com/attachments/upload_2018-1-28_19-33-30-png.219267/](https://www.physicsforums.com/attachments/upload_2018-1-28_19-33-30-png.219267/)

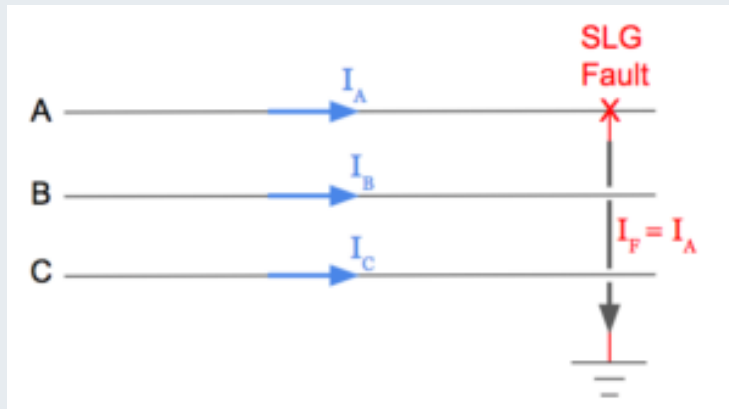
4. Single Line-to-Ground (SL-G) Faults

- The faults , unbalance the network, **but only at the fault location if it is an unbalanced fault.**
- **This causes a coupling of the sequence networks, flow of the sequence current and hence sequence voltage.**
- But, the way how the sequence networks are coupled is highly depends on the fault type.
- Therefore, it's important develop a mathematical relationship for several common faults.
- Assume, a SLG fault **only one phase has non-zero fault current** is takes place at phase A, which is presented in Fig.6.

SL-G Faults

Cont.....

- Thus, the fault currents are:



$$\begin{vmatrix} I_a^f \\ I_b^f \\ I_c^f \end{vmatrix} = \begin{vmatrix} I_a^f \neq 0 \\ 0 \\ 0 \end{vmatrix} \quad \text{eqn.(18)}$$

Figure 6. Single line to ground Fault.

Url: <https://www.electricalpereview.com/wp-content/uploads/2018/02/Single-Line-to-ground-fault-transmission-three-phase-diagram.png>

- From sequence network:

$$\begin{vmatrix} I_f^0 \\ I_f^+ \\ I_f^- \end{vmatrix} = \frac{1}{3} \begin{vmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{vmatrix} \begin{vmatrix} I_a^f \\ 0 \\ 0 \end{vmatrix} \quad \text{eqn.(19)}$$

$$\Leftrightarrow I_f^0 = I_f^+ = I_f^- = \frac{1}{3} I_a^f$$

SL-G Faults

Cont.....

- Then, the voltage at faulted location is determined as

$$V_a^f = Z_f I_a^f \quad \text{eqn.(20)}$$

- The phase voltage and sequence voltages are related as:

$$\begin{bmatrix} V_a^f \\ V_b^f \\ V_c^f \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha^2 & \alpha \\ 1 & \alpha & \alpha^2 \end{bmatrix} \begin{bmatrix} V_f^0 \\ V_f^+ \\ V_f^- \end{bmatrix} \quad \text{eqn.(21)}$$

$$\Leftrightarrow V_a^f = V_f^0 + V_f^+ + V_f^-$$

- Finally, the only way that three sequence currents are equal and the fault voltage is the summation of sequence voltage is when the network is connected in radial form as represented in Fig.7

SL-G Faults

Cont.....

- From the presented connection the fault currents can be easily calculated: if $I_{a1} = I_f^+, I_{a2} = I_f^-, I_{a0} = I_f^0$

and $Z_1 = Z^+, Z_2 = Z^-, Z_0 = Z^0$ the fault current is :

$$I_f^+ = \frac{E_a}{Z^+ + Z^- + Z^0} = I_f^- = I_f^0$$

$$I_a^f = 3I_f^+$$

eqn.(22)

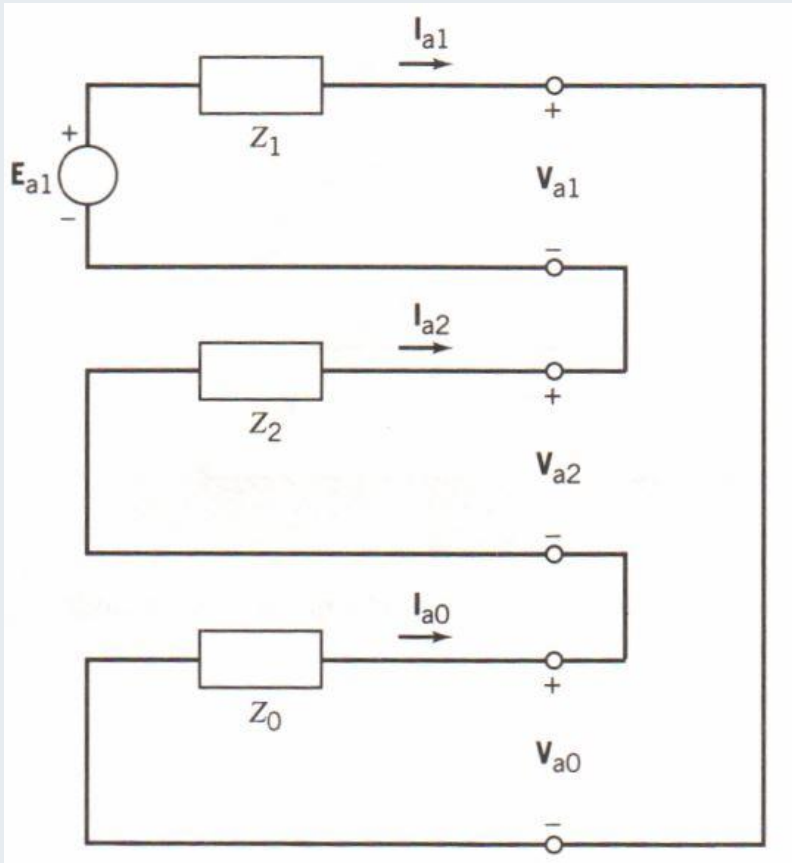


Figure7. The circuit diagram of SL-G fault.

5. The Line-to-Line (L-L) Unbalanced Faults

- The other, second most common types of fault is line-to-line, which occurs when two of the conductors come in contact with each other.
- Assume the fault is between phase b and c on Fig.7
- The current and voltage relations are:

$$I_a^f = 0$$

$$I_b^f = -I_c^f, I_f^0 = 0 \quad \text{eqn.(23)}$$

$$V_{bg} = V_{cg}$$

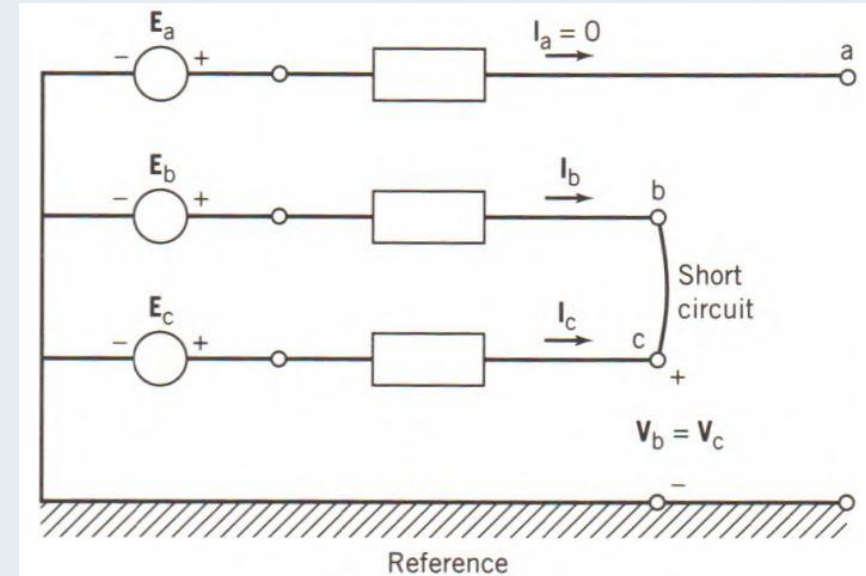


Figure 7. L-L fault(b-c). [https://blogger.googleusercontent.com/img/b/R29vZ2xl/AVvXsEiYI9FdpG1VjKFqI3edf6IStlleuVjFDNx1wxCW3iZ7MnAs0vbQdfAAwbd-OqbCXBjIo_Y_XleuloKc5xuxlyxSkiaDdrYQFLZgPmia1I3TEFimBhHwZyRm7hFPEo4gvcx](https://blogger.googleusercontent.com/img/b/R29vZ2xl/AVvXsEiYI9FdpG1VjKFqI3edf6IStlleuVjFDNx1wxCW3iZ7MnAs0vbQdfAAwbd-OqbCXBjIo_Y_XleuloKc5xuxlyxSkiaDdrYQFLZgPmia1I3TEFimBhHwZyRm7hFPEo4gvcxLweLhfrwscaO8/s1600/A-line-to-line-fault..jpg)

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L-L Unbalanced Faults

Cont....

- Using the current and voltage symmetry :

$$\begin{bmatrix} I_f^0 \\ I_f^+ \\ I_f^- \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{bmatrix} \begin{bmatrix} 0 \\ I_b^f \\ -I_b^f \end{bmatrix}$$

$$\Leftrightarrow I_f^0 = 0$$

$$I_f^+ = \frac{1}{3} I_b^f (\alpha - \alpha^2), I_f^- = \frac{1}{3} I_b^f (\alpha^2 - \alpha)$$

$$\Rightarrow I_f^+ = -I_f^-$$

$$\begin{bmatrix} V_f^0 \\ V_f^+ \\ V_f^{-0} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{bmatrix} \begin{bmatrix} V_{ag}^f \\ V_{bg}^f \\ V_{cg}^f \end{bmatrix}$$

$$V_f^+ = \frac{1}{3} [V_{ag}^f + (\alpha + \alpha^2)V_{bg}^f]$$

$$V_f^- = \frac{1}{3} [V_{ag}^f + (\alpha + \alpha^2)V_{bg}^f]$$

$$\Rightarrow V_f^+ = V_f^-$$

eqn.(24)

L-L Unbalanced Faults

Cont....

- The above two conditions are satisfied when the positive and negative sequence reactance's are connected parallel as presented in Fig.8

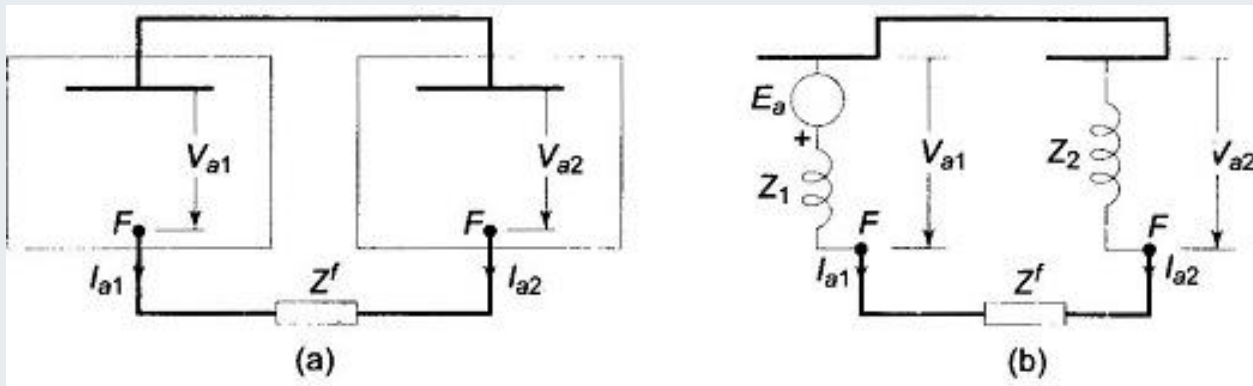


Figure 8. Line to Line fault sequence network. [Url:https://www.eeeguide.com/wp-content/uploads/2016/12/Line-to-Line-Fault-012.jpg](https://www.eeeguide.com/wp-content/uploads/2016/12/Line-to-Line-Fault-012.jpg)

Then, the currents are determined as:

$$I_f^+ = \frac{E_a}{Z^+ + Z^-} = -I_f^- \quad \text{eqn.(25)}$$

6. Double Line-to-Ground(LL-G) Faults

- A double line-to-ground (DLG) problem occurs when two line conductors make contact with ground and one another.
- Assume Fig.9, the fault between Phase b and c with ground impedances .

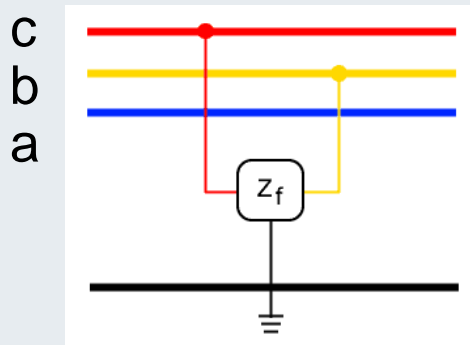


Figure 9. Double line to ground fault. Url: <https://www.electricaltechnology.org/wp-content/uploads/2022/07/Double-Line-to-Ground-L-L-G-Fault.png>

$$I_a^f = 0 \qquad V_{bg}^f = V_{cg}^f = Z_f (I_b^f + I_c^f) \qquad \text{eqn.(25)}$$

LL-G Faults

Cont.....

- Because of the ground impedance the zero sequence current is not zero and the current relation is:

$$\begin{bmatrix} I_a^f \\ I_b^f \\ I_c^f \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha^2 & \alpha \\ 1 & \alpha & \alpha^2 \end{bmatrix} \begin{bmatrix} I_f^0 \\ I_f^+ \\ I_f^- \end{bmatrix}$$

eqn.(26)

$$I_a^f = 0 \Rightarrow I_f^0 + I_f^+ + I_f^- = 0$$

- Then, the voltage relation is:

From the voltage relationships we get

$$\begin{bmatrix} V_f^0 \\ V_f^+ \\ V_f^- \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{bmatrix} \begin{bmatrix} V_{ag}^f \\ V_{bg}^f \\ V_{cg}^f \end{bmatrix} \rightarrow$$

eqn.(27)

$$\text{Since } V_{bg}^f = V_{cg}^f \rightarrow V_f^+ = V_f^-$$

$$\text{Then } V_{bg}^f = V_f^0 + (\alpha^2 + \alpha)V_f^+$$

$$\text{But since } 1 + \alpha + \alpha^2 = 0 \rightarrow \alpha^2 + \alpha = -1$$

$$V_{bg}^f = V_f^0 - V_f^+$$

LL-G Faults

Cont.....

$$\begin{aligned}V_{bg}^f &= V_f^0 - V_f^+ \\ &= Z_f (I_b^f + I_c^f)\end{aligned}$$

eqn.(28)

Also, since

$$\begin{aligned}I_b^f &= I_f^0 + \alpha^2 I_f^+ + \alpha I_f^- \\ I_c^f &= I_f^0 + \alpha I_f^+ + \alpha^2 I_f^-\end{aligned}$$

Adding these together (with $\alpha + \alpha^2 = -1$)

eqn.(29)

$$V_{bg}^f = Z_f (2I_f^0 - I_f^+ - I_f^-) \quad \text{with } I_f^0 = -I_f^+ - I_f^-$$

eqn.(30)

$$V_f^0 - V_f^+ = 3Z_f I_f^0$$

LL-G Faults

Cont.....

- Based on the above relations the three sequence networks are connected as given in Fig.10

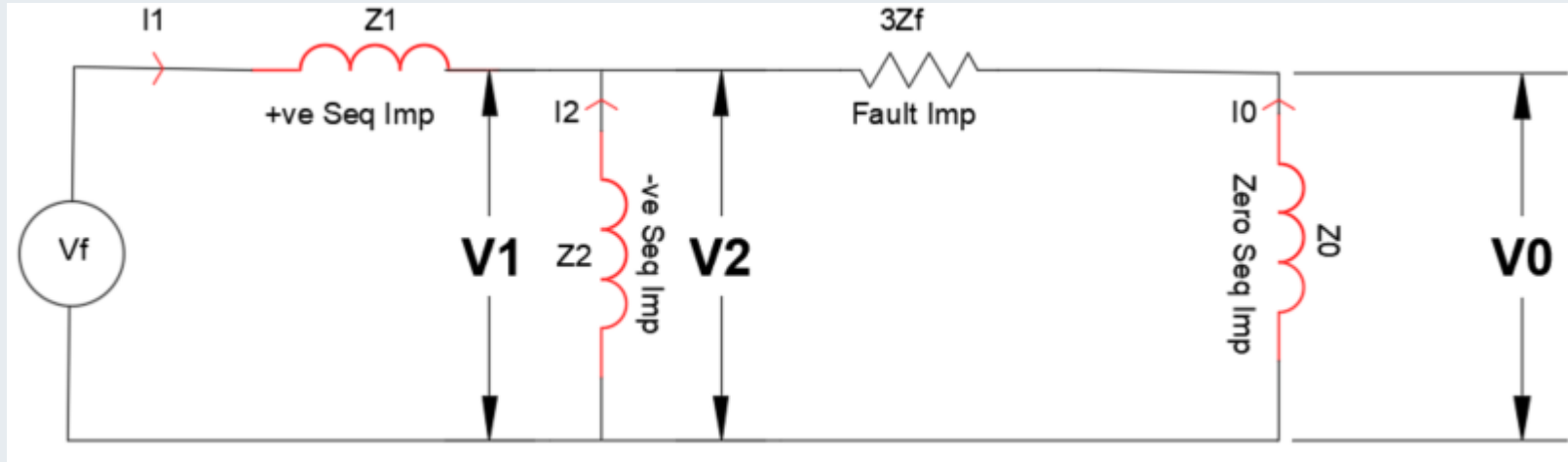


Figure10: DL-G fault sequence network. [Url:https://voltage-disturbance.com/wp-content/uploads/2019/11/Sequence-network-for-phase-ground-fault.png](https://voltage-disturbance.com/wp-content/uploads/2019/11/Sequence-network-for-phase-ground-fault.png)

$$I_f^+ = \frac{E_a}{Z^+ + Z^- \parallel (Z^0 + 3Z_f)} \quad \text{eqn.(31)}$$

Summary

- In this lecture, the unbalanced three phase fault analysis is presented.
- These type of fault the most common types of faults in power system, which occurs more frequently compared to balanced three phase fault.
- Around 85% of the unbalanced fault is L-G, the remaining 15 % is L-L and L-L-G faults
- To perform the unsymmetrical fault a symmetrical component method is used
- The symmetrical matrix, which relates sequence currents and voltage with phase currents and voltage better for unbalanced fault analysis
- Using the symmetrical components the sequence network for positive , negative and zero sequences are well developed

References

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Thank you !