Advanced Power System Analysis
Lecture 10

Fault analysis

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Lecture learning outcomes:

At the end of this lecture, you will be able to:

i. Knows how to solve the symmetrical components current

ii. Understand the basic steps in formulating sequence diagrams.

iii. Performs the fault analysis for different unbalanced faults.

iv. Know the basic procedures regarding large power network fault analysis

Outlines

1. Introduction
2. Solved problems
3. Sequence Diagram Formulation
4. Examples on Unsymmetrical Fault
5. Fault Analysis in Large Network

Summary

References

1. Introduction

- The analysis of unbalanced fault with some solved problems is very important to understand the behavior of power network during different types of creditable contingency like (L-G , L-L and LL-G) fault conditions.
- It helps to know the characteristic of fault currents during unbalanced fault conditions.
- In addition, the importance of symmetrical analysis for unbalanced system is checked with some solved problem[1]
- Then, the circuit protective capacity can be selected based on the magnitude and types of faults

Introduction Cont.….

- In large power networks, the analysis both symmetrical and unsymmetrical faults are developed using the same scenarios as presented before with slight modification.
- Because, large power system network comprises a lot of generation units, transformers , transmission lines and loads are interconnected together and needs special attentions , and knowledge of power system analysis.
- Thus , the symmetrical faults in large system involve all three phases of either the bus or lines experiencing a short circuit to each other or to ground
- while unsymmetrical faults affect only one or two phases of buses or lines
- The large system analysis needs formulation of bus admittance matrix and supper position theorems[2].

2. Solved Problems

Examples on determining the symmetrical Components of Unbalanced system.

Example 1: If a Three-phase of four 4-wire system having the current at each phase is given below. Calculate the positive, negative and zero sequence currents in phase-a and the return current.

$$I_a = 90\angle 60A, I_b = 40\angle 260A, I_c = 20\angle 180,$$

Solution :

$$I_s = A^{-1} I$$

$$= 1/3 \begin{vmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{vmatrix} * \begin{vmatrix} I_a \\ I_b \\ I_c \end{vmatrix}$$

$$I_s^0 = \frac{90\angle 60A + 40\angle 260A + 20\angle 180}{3}$$

$$= \frac{90(\cos 60 + j\sin 60) + 40(\cos 260 + j\sin 260) + 20(\cos 180 + j\sin 180)}{3}$$

$$= \frac{90(0.5 + j0.87) + 40(-0.17 - 0.98) + 20(-1 + 0)}{3} = 6 + j13$$

$$= 14.32\angle 65.22$$

Solved Problems Cont.….

$$I_s^+ = \frac{1\angle 0 * 90\angle 60A + 1\angle 120 * 40\angle 260A + 1\angle 240 * 20\angle 180}{3}$$

$$= \frac{90(\cos 60 + j\sin 60) + 40(\cos 380 + j\sin 380) + 20(\cos 420 + j\sin 420)}{3}$$

$$= \frac{90(0.5 + j0.87) + 40(0.94 + j0.34) + 20(0.5 + j0.87)}{3} = 31 + j36$$

$$= 47.5\angle 49.3$$

$$I_s^- = \frac{1\angle 0 * 90\angle 60A + 1\angle 240 * 40\angle 260A + 1\angle 120 * 20\angle 180}{3}$$

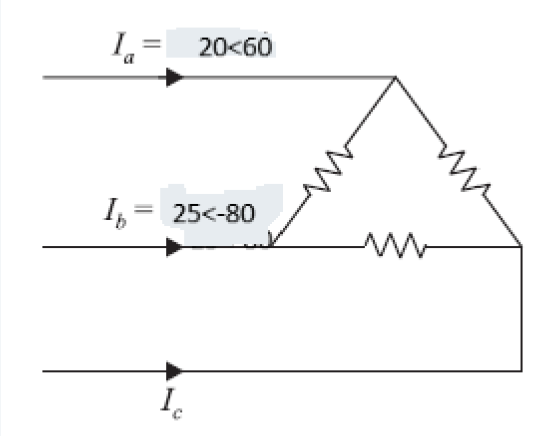
$$= \frac{90(\cos 60 + j\sin 60) + 40(\cos 500 + j\sin 500) + 20(\cos 300 + j\sin 300)}{3}$$

$$= \frac{90(0.5 + j0.87) + 40(-0.76 + j0.64) + 20(0.5 - j0.87)}{3} = 8.33 + j29$$

$$= 30.17\angle 73.97$$

Solved Problems Cont.….

Sequence Networks :Example 2: A delta connected balance resistive load is connected across an unbalanced three-phase supply shown in Fig. below with currents in line a and b given. Determine the symmetrical components of the currents.



- Solution:

For delta connected load, the summation of three phase current is equal to zero as given by:

$$
\begin{aligned}
&I_a + I_b + I_c = 0 \\
&I_c = -(I_a + I_b) \\
&= -(20\angle 60 + 25\angle -80) \\
&= -(20(\cos 60 + j\sin 60) + 25(\cos -80 + \sin -80)) \\
&= -(20(0.5 + j0.87) + 25(0.17 - j0.985) \\
&= -5.75 - j\text{-}42.025 = 42.42\angle 82.2
\end{aligned}
$$

Solved Problems Cont.….

- Then, the symmetrical component current is determined from the relation

$$I_s = A^{-1} I$$

$$= 1/3 \begin{vmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{vmatrix} * \begin{vmatrix} I_a \\ I_b \\ I_c \end{vmatrix}$$

Which means

$$I_s^0 = \frac{20\angle 60A + 25\angle -80A + 42.42\angle 82.2}{3}$$

$$= \frac{20(\cos 60 + j\sin 60) + 25(\cos(-80) + j\sin(-80)) + 42.42(\cos 82.2 + j\sin 82.2}{3}$$

$$= \frac{90(0.5 + j0.87) + 25(0.17 - 0.98) + 42.42(0.135 + j0.99)}{3} = 18.32 + j31.93$$

$$= 36.81\angle 60.15$$

Solved Problems Cont…..

- The positive and negative sequence currents are:

$$I_s^+ = \frac{20\angle 60A + 1\angle 120 * 25\angle -80A + 1\angle 240 * 42.42\angle 82.2}{3}$$

$$= \frac{20(\cos 60 + j\sin 60) + 25(\cos(40) + j\sin(40)) + 42.42(\cos 322.2 + j\sin 322.2}{3}$$

$$= \frac{90(0.5 + j0.87) + 25(0.77 + j0.64) + 42.42(0.79 - j0.61)}{3} = 32.59 + j22.8$$

$$= 39.77\angle 34.97$$

$$I_s^- = \frac{20\angle 60A + 1\angle 240 * 25\angle -80A + 1\angle 120 * 42.42\angle 82.2}{3}$$

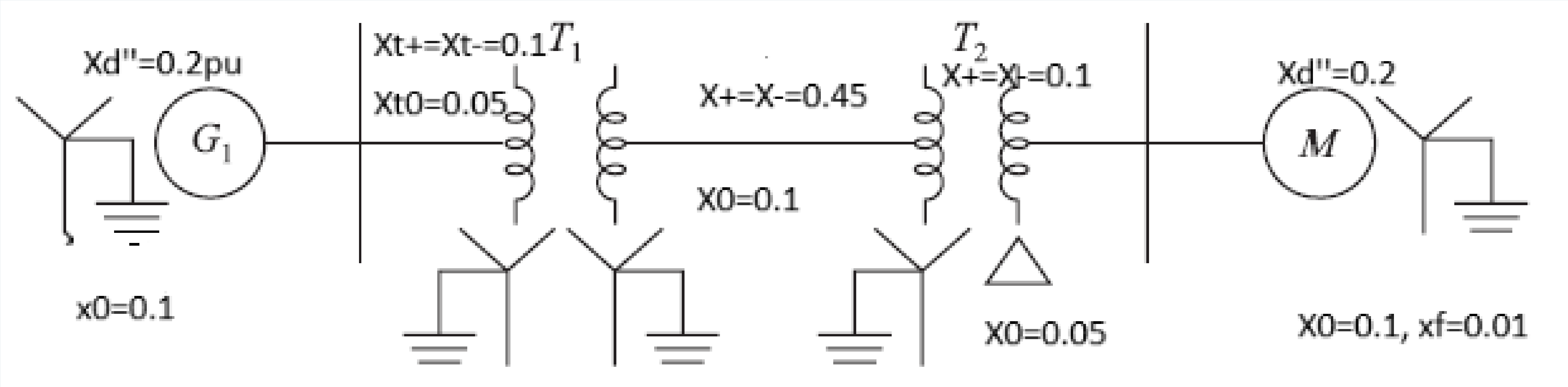
$$= \frac{20(\cos 60 + j\sin 60) + 25(\cos(160) + j\sin(160)) + 42.42(\cos 202.2 + j\sin 202.2}{3}$$

$$= \frac{90(0.5 + j0.87) + 25(-0.94 + j0.34) + 42.42(-0.92 - j0.37)}{3} = -5.84 + j23.7$$

$$= 24.4\angle -76.15$$

3. Sequence Diagram Formulation

Example 3: Develop the three (positive , negative and zero) sequence diagrams for the single line diagram presented below. Assume the positive and negative sequence reactance of generator and motor is equal to their respective sub-transient reactance.



Sequence Diagram Formulation Cont.…

Solution:

- From the sequence diagram formulation, only the positive sequence network has a source voltage and the other two sequence network will not have source voltage
- Accordingly , the positive sequence diagram while only consider the positive sequence reactance of all the connected parameters are given by:

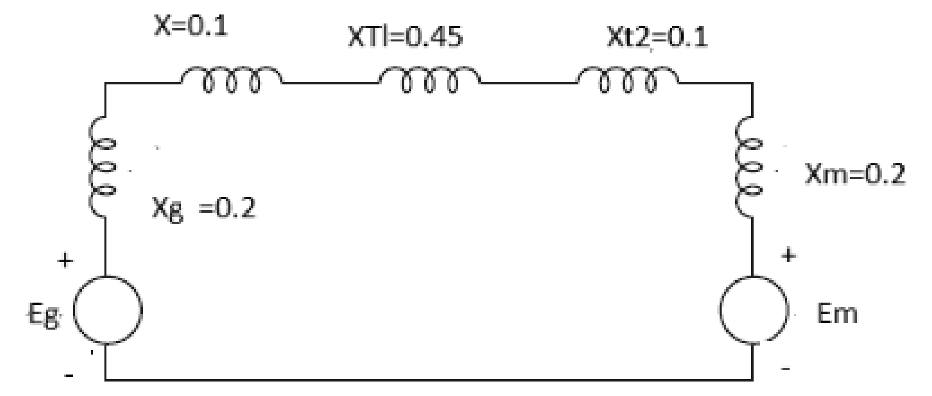


Figure 1. Positive sequence reactance diagram

Sequence Diagram Formulation Cont.…

- Then, the negative sequence diagram is developed and presented below.
- For negative sequence there is no source voltage, means both generator and motor voltages are shorted
- Finally, since the positive and negative sequence value for all components are same, the circuit is given as :

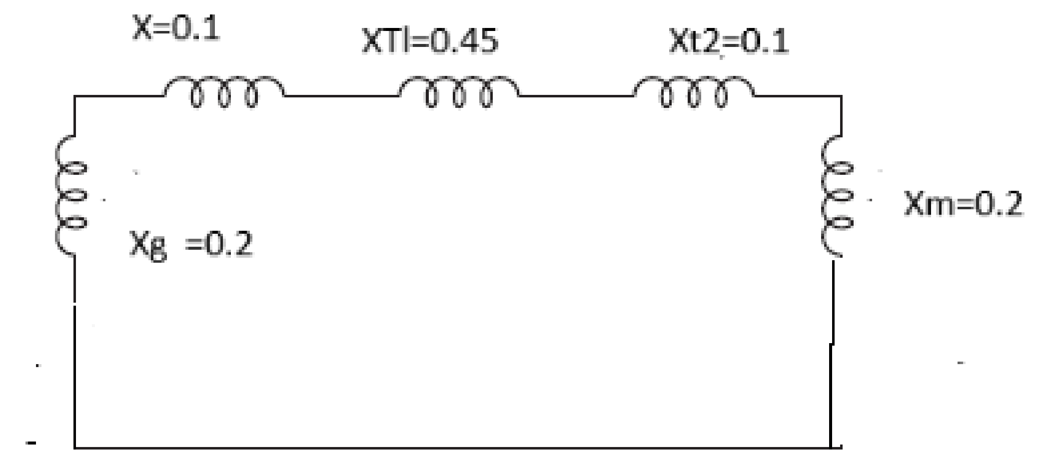


Figure 2. Negative sequence Diagram

Sequence Diagram Formulation Cont….

- Lastly, the zero sequence diagram is formulated
- It’s known that zero sequence reactance's depends on the types of transformer connection and how the transformer is grounded or not
- Accordingly, the generator side transformer is connected as Y-Y grounded both side means its reactance diagram is similar with line reactance
- Whereas, the load side transformer is connected as Delta-Y grounded , its connection is open from delta side and connected the other network from star-side , and then grounded.

Sequence Diagram Formulation Cont.…

- Then, the circuit diagram of zero sequence network while considering the zero sequence reactance of each components, the ground impedance of motor and the transformer connection types, and its grounding is presented as:

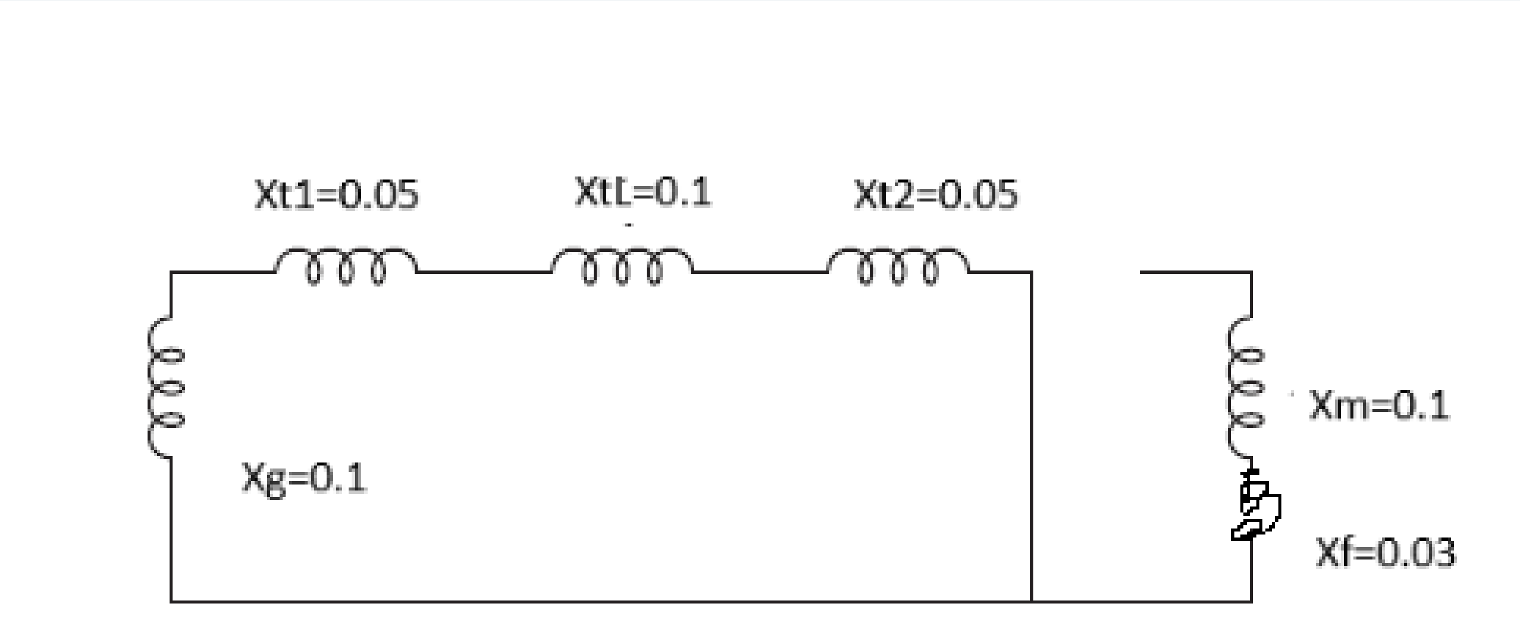


Figure 3. Zero sequence reactance diagram

4.Examples on Unsymmetrical Fault

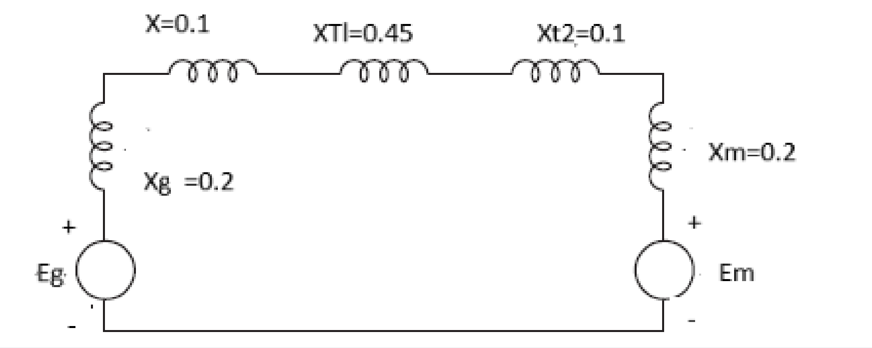
- **Example Four**: Consider the single line given at example three and assume the SL-G fault is takes place at the terminal of the motor. Assume the generator and motor are at 100 MVA and 15 Kv each and all the reactance's are at common bases. Determine the fault current at the terminal of motor.

Solution:

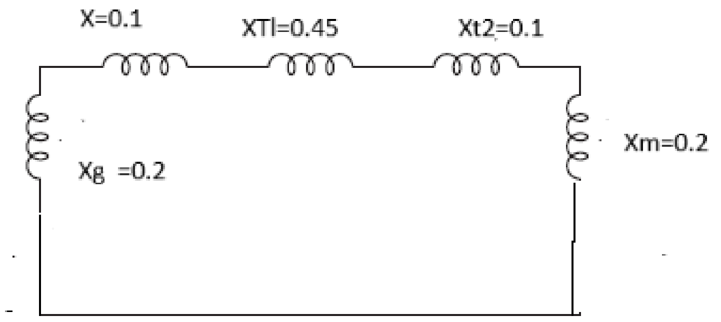
1. Develop the three sequence diagram
2. Find out the thevinen equivalent of each sequence
3. Develop the SL-G fault sequence diagram
4. Find the sequence currents
5. Determine the fault current in p.u and amp
6. Determine the momentary current of CB

Examples on Unsymmetrical Fault Cont.…

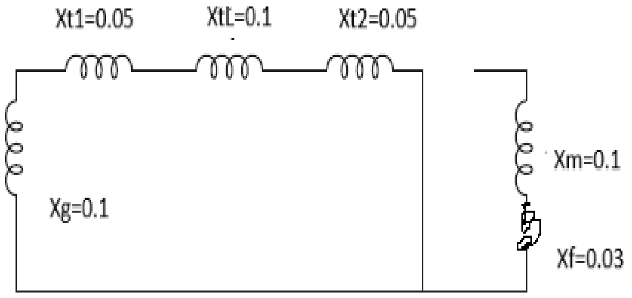
- The three sequence diagram is already developed earlier as:



Positive sequence



Negative sequence



zero sequence

- If the fault is at the terminal of motor , their Thenenin equivalent reactance's are developed as :

Examples on Unsymmetrical Fault Cont.…

- Which are:

$$X_{th}^{+} = \frac{(0.2+0.1+0.45+0.1*0.2)}{0.2+0.1+0.45+0.1+0.2}$$

$$= \frac{0.85*0.2}{0.85+0.2} = j0.162\text{p.u}$$

$$X_{th}^{0} = j0.2 + 3*X_{f}$$

$$= j0.2 + j0.03$$

$$= j0.23p.u$$

$$X_{th}^{+} = X_{th}^{-} = j0.162\,p.u$$

- Then, the three sequence impedance are connected in series and the sequence currents and fault current is determined as:

Examples on Unsymmetrical Fault Cont.…

- For single line-ground fault all the sequence currents are equal as:

$$I_f^+ = \frac{E_a}{Z^+ + Z^- + (Z^0 + 3X_f)} = I_f^- = I_f^0$$

$$= \frac{1.pu}{0.162 + 0.16 + 0.23} = 1.8\text{p.u}$$

- Then, the fault current at a phase in p.u and amp is:

$$I_a^f = 3 * I_f^+$$

$$= 3 * 1.8 = 5.4\, p.u$$

$$I_c^f = 0$$

$$I_b^f = 0$$

$$I_{Base} \frac{MVA_b}{\sqrt{3}V_b} * 10^3 = \frac{100}{\sqrt{3} * 15} * 10^3 = 993.84A$$

$$I_a^f = 5.4 * 993.84A$$

$$= 5{,}366.736\text{A}$$

Examples on Unsymmetrical Fault Cont.…

- Then, the voltage across faulted line and ground is:

$$V_{ag} = I_a^f z_f$$

$$= V_f^+ + V_f^- + V_f^0$$

$$= I_f^+ X_{th}^+ + I_f^- X_{th}^- + I_f^0 X_{th}^0$$

$$= 1.8 * 0.162 + 1.8 * 0.162 + 1.8 * 0.23$$

$$= 0.9972\text{p.u}$$

$$= 0.9972 * 15\text{kv}$$

$$= 14.958\text{kV}$$

Examples on Unsymmetrical Fault Cont.…

- Example Five : Consider the single line given at example three and assume the L-L fault is takes place at the terminal of the motor. Assume the generator and motor are at 100 MVA and 15 Kv each and all the reactance's are at common bases. Determine the fault current at the terminal of motor if the fault is between line b and c.

Solution

1. Develop the three sequence diagram
2. Find out the thevinen equivalent of each sequence
3. Develop the L-L fault sequence diagram
4. Find the sequence currents
5. Determine the fault current in p.u and amp
6. Determine the momentary current of CB

Examples on Unsymmetrical Fault Cont.…

- The sequence diagram is similar with that of S-L ground fault if the fault is at the same place
- The thevenin equivalent impedance of each sequence is the same with the SL-G fault
- The only difference is the sequence network of L-L, which connecting the negative and positive sequence parallel
- The fault current in phase b is equal with phase b but opposite in sign, whereas the fault current at phase a is zero.
- The voltage at b and c are equal
- The zero sequence current is zero.
- If they are connected in parallel, the sequence currents are determined as:

Examples on Unsymmetrical Fault Cont.…

'

$$I_f^+ = \frac{E_a}{Z^+ + Z^-} = -I_f^-$$

$$I_f^+ = \frac{1p.u}{0.162 + 0.162} = -I_f^-$$

$$= 3.08\text{pu}$$

- Then, from symmetrical network the fault current are:

$$I_b^f = (-j\sqrt{3}I_f^+) = -I_c^f$$

$$= -j\sqrt{3} * j3.08$$

$$= 5.33\text{p.u}$$

$$= 993.84A * 5.33$$

$$= 5{,}297.1672\text{A}$$

Examples on Unsymmetrical Fault Cont.…

- Example Six: Consider the single line given at example three and assume the LL-G fault is takes place at the terminal of the motor. Assume the generator and motor are at 100 MVA and 15 Kv each and all the reactance's are at common bases. Determine the fault current at the terminal of motor if the fault is between line b and c with ground .

Solution

1. Develop the three sequence diagram
2. Find out the thevinin equivalent of each sequence
3. Develop the LL-G fault sequence diagram
4. Find the sequence currents
5. Determine the fault current in p.u and amp
6. Determine the momentary current of CB

Examples on Unsymmetrical Fault Cont.…

- All the steps are the same except how the L-L-G fault sequence network is connected.
- It's discussed that in this types of fault, the equivalent circuit is when the negative and zero sequence networks are connected parallel and then connected radial to positive sequence
- In addition, the zero sequence current not zero line L-L fault
- The fault voltage at b and c are equal
- Generally, it's determined as

$$I_f^+ = \frac{E_a}{Z^+ + Z^- // (Z^0 + 3X_f)}$$

$$= \frac{1pu}{0.162 + (\frac{0.166 * 0.23}{0.162 + 0.23})} = 3.89\text{pu}$$

Examples on Unsymmetrical Fault Cont….

- Since positive and negative sequence voltages are equal, we can determine the negative sequence current using

$$\mathrm{V_f^+ = I_f^+ * Z^+ = 3.89pu * 0.162 = V_f^-}$$
$$\mathrm{= 0.630pu}$$
$$\Rightarrow \mathrm{I_f^- = \frac{0.630}{0.162} = 3.89pu}$$

- By the same steps the fault currents are

determined as:

$$V_{bg}^f = V_f^0 - V_f^+$$
$$= Z_f (I_b^f + I_c^f)$$

Also, since

$$I_b^f = I_f^0 + \alpha^2 I_f^+ + \alpha I_f^+$$
$$I_c^f = I_f^0 + \alpha I_f^+ + \alpha^2 I_f^+$$

Adding these together (with $\alpha + \alpha^2 = -1$)

$$V_{bg}^f = Z_f (2I_f^0 - I_f^+ - I_f^-) \qquad \text{with } I_f^0 = -I_f^+ - I_f^-$$
$$V_f^0 - V_f^+ = 3Z_f I_f^0$$

5. Fault Analysis in Large Network

- Fault analysis in meshed and highly interconnected large power network needs to know how the network is integrated, loads are connected and hence their types[3].
- It requires the development of admittance matrix, then converting into per unit value, determining the fault currents either using analytical method simulating faults and analyzing their impact to ensure system stability and integrity.
- This is crucial for determining the appropriate circuit protective devices settings and switchgear capacities,
- Its intention is to improve the overall reliability of the power grid.

Fault Analysis in Large Network Cont.….

- Formulating bus admittance matrix /impedance matrix is the first step in large network fault analysis
- The best method in solving large network fault analysis is using supper-position method:
- This method starts in analyzing the network current and voltage prior to fault and during fault as:

$$I_i^{\,f} = I_i^1 + I_i^2$$

eqn.(1)

$$V_i^{\,f} = V_i^1 + V_i^2$$

- Where, the superscript 1 and 2 means pre-fault and faulted condition, respectively

Fault Analysis in Large Network Cont.….

- The superposition approach can be easily used as:
- The current is zero except for faulted point as given by:

$$YV = I$$

$$I = \begin{vmatrix} 0 \\ 0 \\ 0 \\ -I_f \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{vmatrix} \qquad \text{eqn.(2)}$$

- To use this approach the fault currents should be firs determined .

Fault Analysis in Large Network Cont.….

- Steps are:

Determine Y bus matrix and Z-bus matrix, which is the inverse of Y-bus

$$V = ZI,$$

$$Then,$$

$$\begin{bmatrix} Z_{11}.............Z_{1n} \\ . \\ . \\ . \\ Z_{n1}.............Z_{nn} \end{bmatrix} \begin{vmatrix} . \\ . \\ -I_f \\ . \\ . \end{vmatrix} = \begin{vmatrix} .V_1^2 \\ V_2^3 \\ . \\ . \\ V_{n-1}^2 \\ V_{n-1}^2 \end{vmatrix}$$

eqn.(3)

For fault at bus i we get $-I_f Z_{ii} = V_f = -V_i^2$

Fault Analysis in Large Network Cont.….

- Then, voltage during fault is also determined using the superposition method as

$$V_i = V_i^1 + V_i^2 \qquad \text{eqn.(4)}$$

- Then , the generalized step is: assume the pre-fault voltages are known
- Calculate Zbus for each sequence
- For a fault at bus i, the Zii values are the thevenin equivalent impedances; the pre-fault voltage is the positive sequence thevenin voltage
- Connect and solve the thevenin equivalent sequence networks to determine the fault current
- Sequence voltages throughout the system are

Fault Analysis in Large Network Cont.….

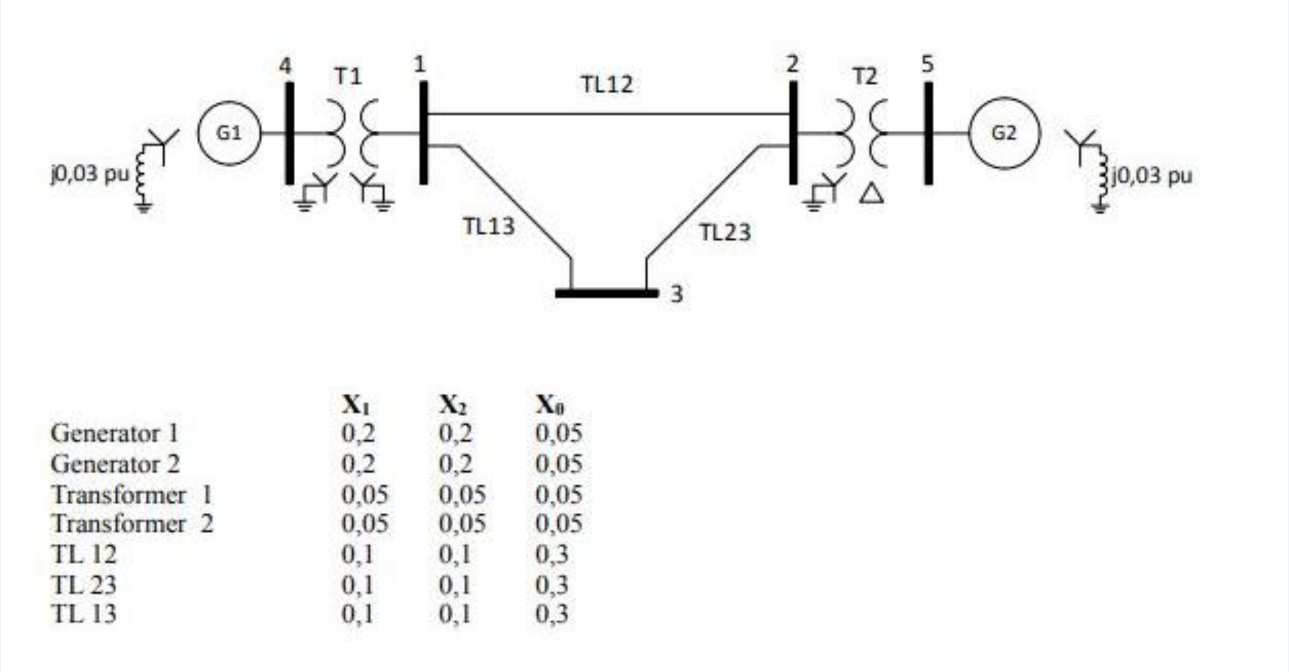
- Sequence voltages throughout the system are given by

$$\mathbf{V} = \mathbf{V}^{prefault} + Z\begin{bmatrix} 0 \\ \vdots \\ 0 \\ -I_f \\ 0 \\ \vdots \\ 0 \end{bmatrix} \qquad \text{eqn.(5)}$$

- Phase values are determined from the sequence values

Fault Analysis in Large Network Cont.….

Project assignment: For the following figure determine the fault current considering the fault analysis for large power network. Assume the Single line to ground fault is takes place at bus 3.



	X_1	X_2	X_0
Generator 1	0,2	0,2	0,05
Generator 2	0,2	0,2	0,05
Transformer 1	0,05	0,05	0,05
Transformer 2	0,05	0,05	0,05
TL 12	0,1	0,1	0,3
TL 23	0,1	0,1	0,3
TL 13	0,1	0,1	0,3

Figure 4. The single diagram of three bus system.

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Summary

- In this lecture, solved problems for unbalanced fault is discussed.
- The formulation sequence networks for positive, negative and zero sequences are presented.
- Specifically, the importance of transformer connection types and grounding on zero sequence network formulation is discussed.
- The fault analysis example for symmetrical components, L-G, L-L and LL-G is discussed
- In addition, the principle of superposition analysis for faults in large power network is also discussed
- Finally, the project works based on the large network analysis given

References

[1]. C., Chivon; D., Yahya; S., Sarot and L.,Young-II. ‘Symmetrical components-based robust stabilizing control of a grid-connected inverter under unbalanced voltage sag’’. Journal of engineering, 2024. https://doi.org/10.1049/tje2.70000.

[2]. C.,Hantao. "Bus Admittance Matrix Revisited: Performance Challenges on Modern Computers “. IEEE Open Access Journal of Power and Energy,. 2024. pp:.99. DOI:10.1109/OAJPE.2024.3366117

[3]. M., Zhen-Lei and L., Xiao-Jian. "Data-driven fault detection for large-scale network systems: A mixed optimization approach’’. Elsevier: Applied Mathematics and Computation. V.426(1), 2022. https://doi.org/10.1016/j.amc.2022.127134.

Thank you !