Advanced Power System Analysis
Lecture 12

Power System Stability

Lecturer: Teshome Goa (Assist. Prof.)

Lecture learning outcomes:

At the end of this lecture, you will be able to:

i. Knows the importance of graphical representation of swing curve in Power stability

ii. Understand the necessities of Inertia constant and kinetic energy for stability.

iii. Understand equal area criteria importance in stability analysis

iv. Knows the basics of computerized methods for stability analysis

Outlines

1. Introduction
2. Constants Used in Stability Analysis
3. Change in Rotor Angle for Loaded Machine
4. Equal Area Criterion
5. Sudden three Phase fault in the Transmission Line stability
6. Computerized Method of Swing Equation solutions

Summary

References

1. Introduction

- Plotting the swing curve in a system is required to ascertain whether the power system is stable followin g a disruption.
- This curve indicates instability in the system if it indicates that the angle between any two machines ten ds to increase indefinitely[1].
- It is likely, but not guaranteed that the system is stable following all disruptions, the angle between the t wo machines achieves its maximum value and then starts to decrease, oscillating with a constant amplit ude.
- The machines can be made to come to rest in relation to one another using a straightforward graphical method, which is called the equal area criteria for stability.

Introduction Cont…..

- It gives information on stability, demonstrate δ's tendency to shift and/or rise above the point of return.
- The system is unstable if δ keeps rising over time.
- It is assumed that the system will stay stable if δ begins to decline after reaching a maximum value as presented in Fig.1.

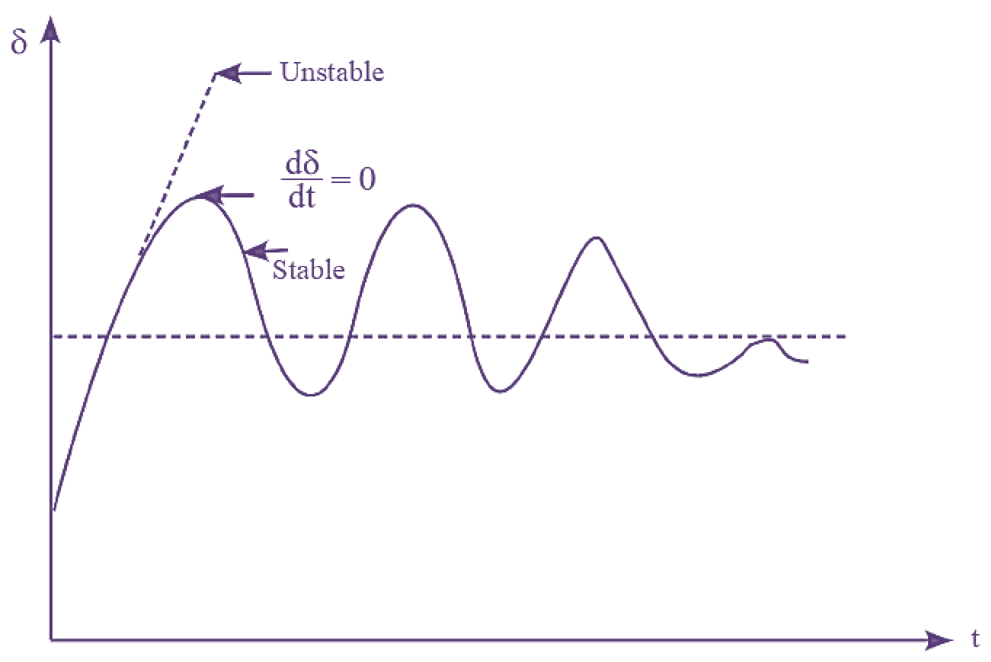


Figure 1. Swing Curve.

Url:https://electricalworkbook.com/wp-content/uploads/2022/12/Swing-Curve.png

2. Constants Used in Stability Analysis

- The three constant terms, Inertia constant (M), Kinetic energy (N) and inertia constant (N), which are the three fundamental components to address the swing equation and equal area criteria as given by[2]:
- From swing equation:

$$P_a = P_m - P_e = M \frac{\partial^2 \delta}{\partial^2 t} \qquad \text{eqn.(1)}$$

- Which means, if power is given by (W or Pu), δ is electrical angle and t is second , then, the constant M may be defined as the power in MW required to produce an angular acceleration per electrical angle, s.
- Then, the kinetic energy, N of the rotor at synchronous speed denoted by N is given by:

$$N = \frac{1}{2} M \omega_s \qquad \text{eqn.(2)}$$

- Where, ω_s =2πn_s(mechanical rad/s) or 2πf(360f) in electrical rad/s and n_s is

  the speed in r.p.s

Constants Used in Stability Cont…..

- Practically, generators of the same MVA ratings may not have the same kinetic energy and momentum.
- To express them in a common way, the constant H called inertia constant is used, which defined as the ratio of stored kinetic **energy to volt ampere rating** of machine as given by:

$$H = \frac{\text{kinetic energy}}{MVArating} = \frac{MJ}{MVA}$$

$$= \frac{N}{S}$$

$$\Leftrightarrow N = SH$$

eqn.(3)

- Then, finding out equivalent H constant is very important in which 'n' number of generators connected in parallel to the same bus bar in reality
- Assume, (S1, S2,….Sn), (H1, H2,..Hn) and (N1, N2…Nn) are the base MVA, inertia

Constant and Kinetic energy of individual machines, respectively .

Constants Used in Stability Cont.….

- And (Se, He , Ne and Sb) are the equivalent MVA, inertia constant, Kinetic energy of machines and com mon base MVA, respectively

- Then, the energy stored in equivalent machine is given by:

$$N_e = N_1 + N_2 + N_3 ..... + N_n \quad \text{eqn.(4)}$$

$$S_e H_e = S_1 H_1 + S_2 H_2 + S_3 H_3 .... S_n H_n$$

- If Se=Sb is the summation of each generator base and, then

$$H_e = \frac{S_1}{S_b} H_1 + \frac{S_2}{S_b} H_2 + \frac{S_3}{S_b} H_3 .... \frac{S_n}{S_b} H_n \quad \text{eqn.(5)}$$

- For identical machines, S1=S2=Sn=S, and H1=H2=H3=Hn, means Sb= n*S.

- Then, by substitution: $H_e = \frac{S}{nS} H + \frac{S}{nS} H + \frac{S}{nS} H .... \frac{S}{nS} H = \frac{nS}{nS} H$ eqn.(6)

$$\Leftrightarrow H_e = H$$

Constants Used in Stability Cont.....

- As a result, the equivalent H constant of multiple identical machines running in parallel is equal to the H constant of any single machine.
- **Equivalent M constant of two machines**: a similar machine connected via a reactance to an infinite bus can be used in place of two synchronous machines connected by a reactance.
- Two machines swing equation is provided by:

$$P_{m1} - P_{e1} = M_1 \frac{\partial^2 \delta_1}{\partial^2 t} \qquad \text{eqn.(7)}$$

$$\Leftrightarrow \frac{\partial^2 \delta_1}{\partial^2 t} = \frac{P_{m1} - P_{e1}}{M_1}$$

- And $$P_{m2} - P_{e2} = M_2 \frac{\partial^2 \delta_2}{\partial^2 t} \qquad \text{eqn.(8)}$$

$$\Leftrightarrow \frac{\partial^2 \delta_2}{\partial^2 t} = \frac{P_{m2} - P_{e2}}{M_2}$$

Constants Used in Stability Cont.….

- By subtracting, eqn.(8) from eqn.(7)

$$\Leftrightarrow \frac{\partial^2\delta_1}{\partial^2 t} - \frac{\partial^2\delta_2}{\partial^2 t} = \frac{P_{m1} - P_{e1}}{M_1} - \frac{P_{m2} - P_{e2}}{M_2}$$

$$\frac{\partial^2\delta_1}{\partial^2 t} - \frac{\partial^2\delta_2}{\partial^2 t} = \frac{M_2(P_{m1} - P_{e1}) - M_1(P_{m2} - P_{e2})}{M_1 M_2}$$

eqn.(9)

- If δ is the relative angle between the rotors of the two machines, then $\delta = \delta_1 - \delta_2$ gives:

$$\frac{\partial^2(\delta_1 - \delta_2)}{\partial^2 t} = \frac{M_2(P_{m1} - P_{e1}) - M_1(P_{m2} - P_{e2})}{M_1 M_2}$$

eqn.(10)

$$\frac{\partial^2(\delta)}{\partial^2 t} = \frac{(M_2 P_{m1} - M_1 P_{m2})}{M_1 M_2} - \frac{(M_2 P_{e1} - M_1 P_{e2})}{M_1 M_2}$$

Constants Used in Stability Cont…..

- Multiplying both sides of eqn.(10) by M1M2 /(M1 + M2), gives

$$\frac{\partial^2(\delta)}{\partial^2 t} = \frac{(M_2 P_{m1} - M_1 P_{m2})}{M_1 M_2} - \frac{(M_2 P_{e1} - M_1 P_{e2})}{M_1 M_2}$$

$$\frac{\mathrm{M1M2}}{(\mathrm{M1}+\mathrm{M2})}\left(\frac{\partial^2(\delta)}{\partial^2 t}\right) = \frac{\mathrm{M1M2}}{(\mathrm{M1}+\mathrm{M2})}\left(\frac{(M_2 P_{m1} - M_1 P_{m2})}{M_1 M_2} - \frac{(M_2 P_{e1} - M_1 P_{e2})}{M_1 M_2}\right)$$

$$\frac{\mathrm{M1M2}}{(\mathrm{M1}+\mathrm{M2})}\left(\frac{\partial^2(\delta)}{\partial^2 t}\right) = \frac{(M_2 P_{m1} - M_1 P_{m2})}{(\mathrm{M1}+\mathrm{M2})} - \frac{(M_2 P_{e1} - M_1 P_{e2})}{(\mathrm{M1}+\mathrm{M2})}$$

eqn.(11)

- The swing equation of an equivalent machine is given by:

$$M'\left(\frac{\partial^2(\delta)}{\partial^2 t}\right) = P_m' - P_e'$$

eqn.(12)

- Finally, the relationship between inertia constant M and inertia constant H is:

$$M = \frac{2N}{\omega_s} = \frac{2N}{360 f} = \frac{N}{180 f}, \mathrm{MJ \cdot s/electrical\ degree}$$

eqn.(13)

$$M = \frac{SH}{180 f}, \mathrm{MJ \cdot s/electrical\ radian}$$

3. Change in Rotor Angle for Loaded Machine

- As the machine is loaded, its angle is changing and determined as:
- From swing equation:

$$M\frac{\partial^2\delta}{\partial^2 t} = P_m - P_e = P_a$$

$$\Rightarrow \frac{\partial^2\delta}{\partial^2 t} = \frac{P_a}{M} = \frac{\pi f_0}{2H}P_a = \frac{\pi f_0}{2H}(P_m - P_e) \qquad \text{eqn.(14)}$$

where, Pa /M is a constant and M= $\frac{2H}{\pi f_o}$

- Multiplying both sides by $2\frac{d\delta}{dt}$, we get

$$2\frac{d\delta}{dt}\frac{d^2\delta}{dt^2} = \frac{2\pi f_o}{2H}(P_m - P_e)\frac{d\delta}{dt} \quad \text{or} \quad \frac{\partial}{\partial t}[\frac{\partial\delta}{\partial t}]^2 = \frac{2P_a}{M}\frac{\partial\delta}{\partial t} \qquad \text{eqn.(15)}$$

Change in Rotor Angle for Loaded Machine Cont.…

- Upon integration of the above eqn. 15 gives

$$\int \partial[\frac{\partial\delta}{\partial t}]^2 = \frac{2P_a}{M}\int \partial\delta$$

$$[\frac{\partial\delta}{\partial t}]^2 = \frac{2P_a}{M}\delta$$

$$\frac{\partial\delta}{\partial t} = \sqrt{\frac{2P_a}{M}} * \delta^{1/2}$$

eqn.(16)

$$\Leftrightarrow \frac{\partial\delta}{\delta^{1/2}} = \sqrt{\frac{2P_a}{M}} * \partial t$$

- Integrating again gives :

$$\int \delta^{-1/2}\partial\delta = \int \sqrt{\frac{2P_a}{M}} * \partial t$$

eqn.(17)

$$\frac{\delta^{(-1/2+1)}}{-1/2+1} = \sqrt{\frac{2P_a}{M}} * t$$

$$\Leftrightarrow \delta^{(1/2)} = \frac{1}{2} * \sqrt{\frac{2P_a}{M}} * t$$

Change in Rotor Angle for Loaded Machine Cont.…

- Then by substitution, we get:

$$\frac{\partial \delta}{\partial t} = \sqrt{\frac{2P_a}{M} * \delta^{1/2}}$$

$$\frac{\partial \delta}{\partial t} = \sqrt{\frac{2P_a}{M}} * \frac{1}{2} * \sqrt{\frac{2P_a}{M}} * t \qquad \text{eqn.(18)}$$

$$\frac{\partial \delta}{\partial t} = \frac{P_a}{M} * t = \omega$$

- Which is the change in rotor angle at anytime

4. Equal area criteria

- In a system to determine whether a power system is stable after a disturbance, it is necessary to plot the swing curve[3].
- If this curve shows that the angle between any two machines tends to increase without limit, system is

unstable.

- If after all disturbances, the angle between the two machines reaches the maximum value and thereafter decreasing, thus oscillating with constant amplitude, is probability or ot certain the system is stable.
- There is a simple graphical method of determining whether the machines come to rest with respect to ea ch other, method used to determine this is known as the equal area criterion for stability.

Equal area criteria Cont.…

- Consider a machine connected to an infinite bus presented in Fig.2

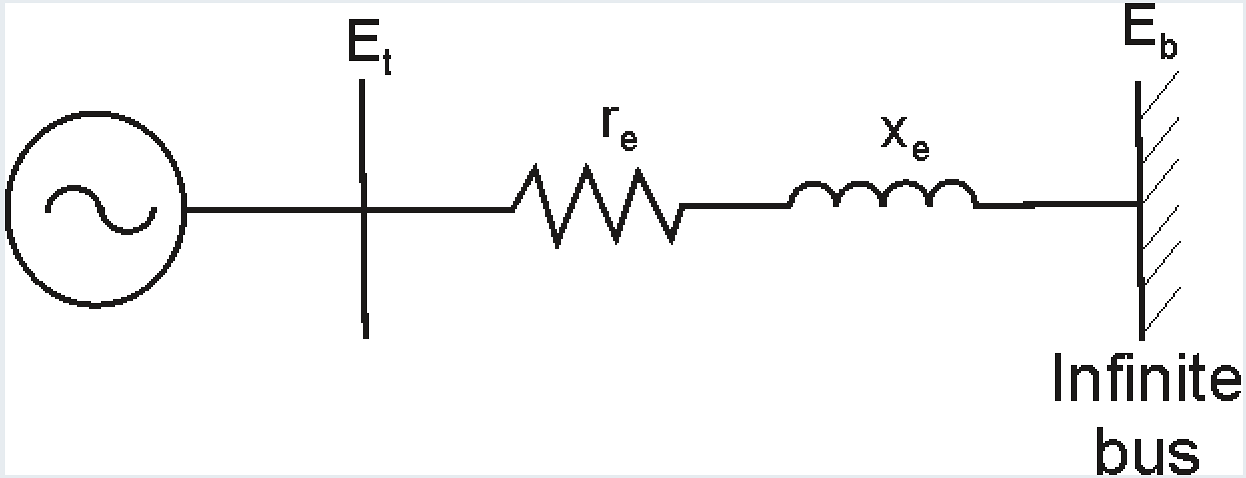


Figure 2. Single Machine to Infinite bus(SMIB).

Url:https://www.researchgate.net/publication/236009431/figure/fig1/AS:363465500839938@1463668336782/Single-Machine-connected-to-infinite-bus-system.png

Equal area criteria Cont.…

- The swing equation with damping neglected is represented as below:

$$\frac{2H}{\pi f_o}\frac{d^2\delta}{dt^2} = P_m - P_e = P_a \qquad \text{eqn.(19)}$$

- where P_a is the acceleration power. From the above equation, we have

$$\frac{d^2\delta}{dt^2} = \frac{\pi f_o}{2H}(P_m - P_e) \qquad \text{eqn.(20)}$$

- Multiplying both sides by $2\frac{d\delta}{dt}$, we get

$$2\frac{d\delta}{dt}\frac{d^2\delta}{dt^2} = \frac{2\pi f_o}{2H}(P_m - P_e)\frac{d\delta}{dt} \qquad \text{eqn.(21)}$$

Equal area criteria Cont….

Or

$$\left[\left(\frac{d\delta}{dt}\right)^2\right] = \frac{2\pi f_0}{H}(P_m - P_e)_{d\delta}$$ eqn.(22)

- Integrating both sides” $\left(\frac{d\delta}{dt}\right)^2 = \frac{2\pi f_0}{H}\int_{\delta_0}^{\delta}(P_m - P_e)d\delta$

Or $$\frac{d\delta}{dt} = \sqrt{\frac{2\pi f_0}{H}\int_{\delta_0}^{\delta}(P_m - P_e)d\delta}$$ eqn.(23)

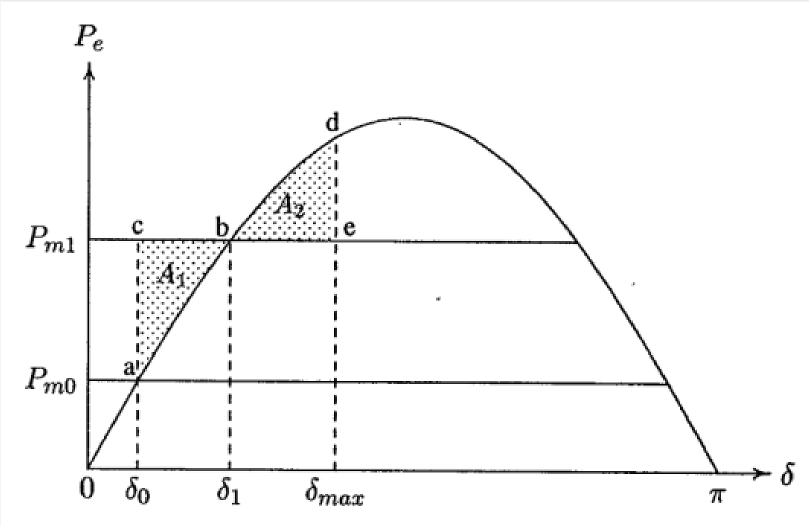
- $\frac{d\delta}{dt}$ - is relative speed of the machine with respect to the synchronously revolving reference frame.
- For stability, this speed must become zero after disturbance, therefore:

$$\int_{\delta_0}^{\delta}(P_m - P_e)d\delta = 0$$ eqn.(24)

Equal area criteria Cont….

- Let us consider a machine operating at the equilibrium point δ_0, corresponding to the mechanical power input $P_{m0} = P_{e0}$ as shown in the Fig.3..

- Consider a sudden step increase in the input power represented by the horizontal line P_{m1}.



- Since $P_{m1} > P_{e0}$, the accelerating power on the rotor is posi tive and the power angle δ increases.
- The excessive energy stored in the rotor during the initial a cceleration is

$$\int_{\delta_0}^{\delta_1}(P_m - P_e)d\delta = area\ abc = area\ A_1$$

eqn.(25)

Figure 3. Equal area criteria for sudden change in load.
Url:https://www.researchgate.net/publication/365064451/figure/fig1/AS:11431281094570778@1667510256364/Equal-area-criteria-for-sudden-change-in-load.ppm

Equal area criteria Cont.…

- With increase in δ, the electrical power increases, and when $\delta = \delta_1$, the electrical power matches the new input power P_{m1}.
- Though the accelerating power is zero at this point, the rotor is running above synchronous speed, hence, δ and electrical power P_e will continue to increase.
- Now $P_m < P_e$, causing the rotor to decelerate towards synchronous speed until $\delta = \delta_{max}$.
- Accordingly, the rotor must swing past point ‘b’ until an equal amount of energy is given up by the rotating masses.
- The energy given up by the rotor as it decelerates back to synchronous speed is

$$\int_{\delta_1}^{\delta_{max}} (P_{m1} - P_e) d\delta = area\ bde = area\ A_2 \quad \text{eqn.(26)}$$

Equal area criteria Cont….

- The resultant is that the rotor swings to point b and the angle δmax , at which point:

  |area A_1| = |area A_2|

- This is known as the equal-area criterion.
- The rotor angle would oscillate back and forth between δ_0 and δ_{max} and its natural frequency.
- If there is damping in the machine, it would make these oscillations to subside and a new steady-state operation established at point **b.**
- The equal area criterion can be used to determine the maximum additional power P_m which can be applied to system but stability still maintained.

Equal area criteria Cont.…

- Stability can only be maintained if area A_2 at least equal to area A_1 can still be located above P_m.
- The limit of stability occurs when δ_{max} is at the intersection of line P_m and the power angle curve for $90^0 < \delta < 180^0$ as shown in the Fig.4

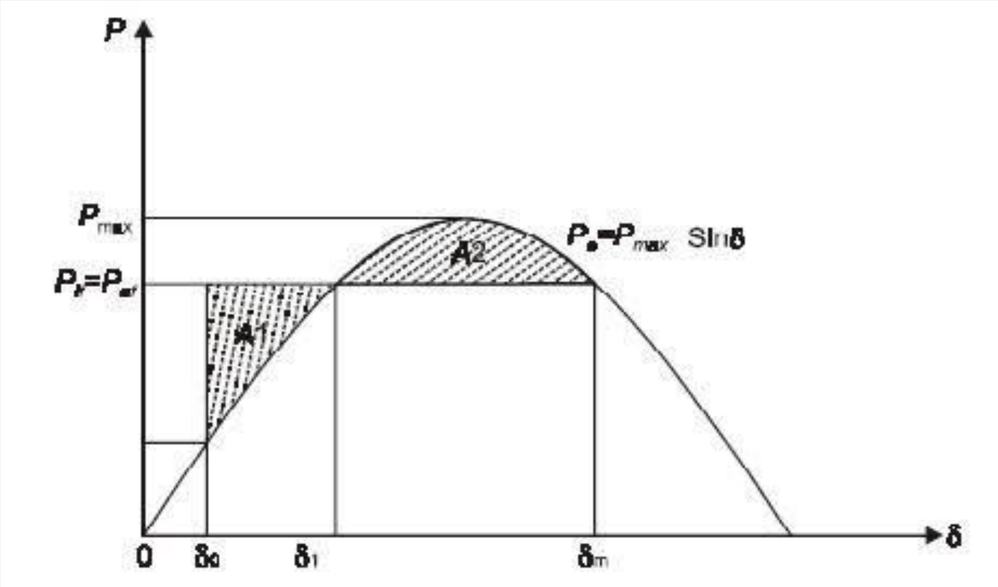


Figure 4. The transient stability limit.

Url: https://encrypted-tbn0.gstatic.com/images?q=tbn:ANd9GcSGO3IYm2yk4KopQLW6qdaAjVT3fZ2PzkiiQORLdAjPcfxXSPGLiP4BqxsSdHjieyLyZrg&usqp=CAU

Equal area criteria Cont.…

- Applying the equal criterion to the Fig.4, it gives:

$\int_{\delta_0}^{\delta_1}(P\mathrm{m} - Pe)d\delta = \int_{\delta_1}^{\delta_{max}}(P\mathrm{e} - Pm)d\delta$ eqn.(27)

Where, $Pe = P_{max}\sin\delta$ and by substitution:

$$P_m(\delta_1 - \delta_0) - \int_{\delta_0}^{\delta_1} P_{max}\sin\delta\, d\delta = \int_{\delta_1}^{\delta_{max}} P_{max}\sin\delta\, d\delta - P_m(\delta_{max} - \delta_1)$$ eqn.,(28)

Integrating the above equation, we get

$(\delta_{max} - \delta_0)P_m = P_{max}(\cos\delta_0 - \cos\delta_{max})$ eqn.(29)

Subst., $Pe = P_{max}\sin\delta$

$(\delta_{max} - \delta_0)\sin\delta_{max} + \cos\delta_{max} = \cos\delta_0$ eqn.(30)

Equal area criteria Cont.…

- The above non-linear algebraic equation can be solved by an iterative technique for δ_max.
- Once δ_max is obtained, the maximum permissible power or the transient stability limit is found from:

$$P_m = P_{max} \sin \delta_1 \quad \text{eqn.(31)}$$

- Where,

$$\delta_1 = \pi - \delta_{max} \quad \text{eqn.(32)}$$

5. Sudden three Phase fault in the Transmission Line

- To determine the rotor angle, fault clearing time of circuit breaker and its speed in transient fault analysis , first network classified into pre-fault, during fault and post fault conditions[4].

- Consider the three-phase short-circuit fault which occurs at the sending of the transmission line 2 of sing le machine connected to an infinite system with double circuit transmission line as presented in Fig.5

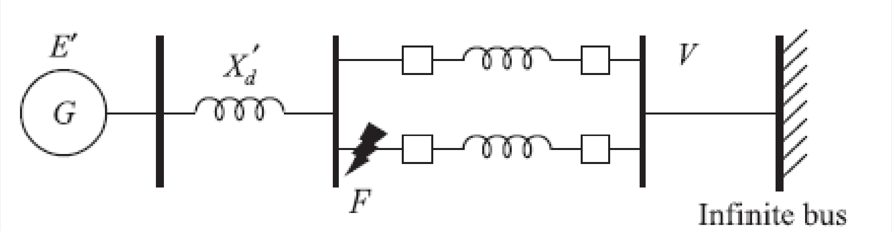


Figure 5. Double Transmission line

- **Pre-fault condition:** prior to a fault, both transmission lines are connected as shown in Fig.5.
- The power angle curve is given by

$$P_{e1} = \frac{EV}{X_T}\sin\delta = P_{\max 1}\sin\delta \qquad \text{eqn.(33)}$$

Sudden three Phase Cont..

- Where,

$$X_T = X_d^{'} + (\frac{X_{T1} * X_{T2})}{X_{T1} + X_{T2}} \quad \text{eqn.(34)}$$

- **During fault condition**, upon the occurrence of three phase fault at the sending of the transmission line 2, the generator gets isolated from the power system for the purpose of power flow looks the shortest pat h, and the fault lasts. Which means, $P_{e2} = 0$.
- The rotor therefore accelerates and δ angle increases, synchronism will be lost unless the fault is cleared in time.
- **Post-fault condition:** The circuit breakers at the two ends of the faulted line open

at time t_c (corresponding to angle δc), the clearing time, disconnecting the faulted line

- The power angle curve is given by: $P_{e3} = \frac{EV}{X_{(III)}} \sin\delta = P_{\max 3} \sin\delta$ eqn.(35)

Sudden three Phase Cont..

- Where,

$$X_{III} = X_d^{'} + X_{T1} \qquad \text{eqn.(36)}$$

- Obviously, (P_{max2}, and P_{max3}) < P_{max1}, means the rotor now starts to decelerate as shown in Fig.6.
- The system will be stable if a decelerating area $A2$ can be found equal to accelerating area $A1$ before δ reaches the maximum allowable value δmax.
- As area $A1$ depends upon the clearing time t_c, the clearing time must be less than a certain value (critic al clearing time) for the system to be stable.
- It is to be observed that the equal area criterion helps to determine the critical clearing angle and not the critical clearing time.
- Critical clearing time can be obtained by numerical solution of the swing equation.

Sudden three Phase Cont..

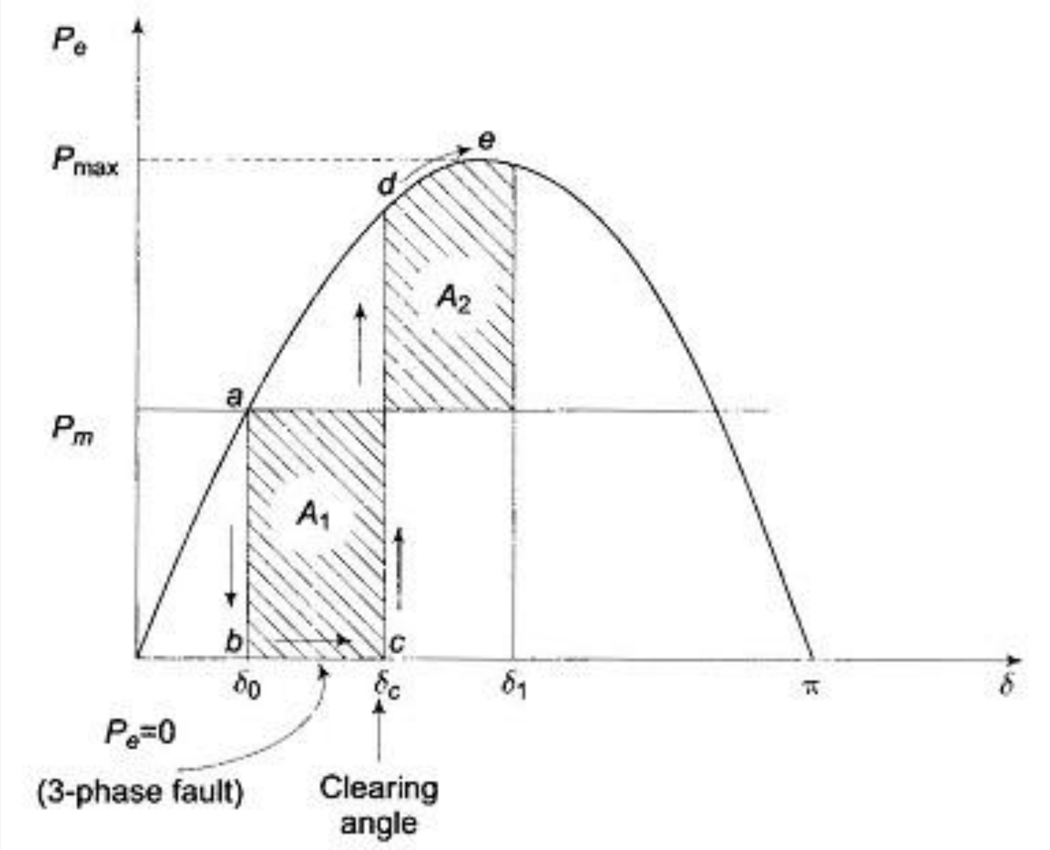


Figure 6. Power curve of 3-phase fault at the beginning of line 2.

Url: https://www.eeeguide.com/wp-content/uploads/2016/12/Equal-Area-Criterion-in-Power-System-042.jpg

Sudden three Phase Cont..

Example 1: The moment of inertia of a 4 pole, 100 MVA, 15 kV, 3-Φ, 0.85 power factor, 50 Hz turbo alternator is 20,000 kg·m^2. Calculate H and M.

Solution:

$$J = 20000 kg.m^2$$

$$N_s = \frac{120f}{p} = \frac{120*50}{2} = 3000rpm$$

$$n_s = \frac{N_s}{60} = 50rps$$

$$\omega_s = 2\pi n_s = 100\pi$$

$$N = \frac{1}{2}J\omega_s^2 = \frac{1}{2}*20000*(100\pi)^2 = 985.96\text{MJ}$$

$$\text{H} = \frac{\text{N}}{\text{S}} = \frac{985.96}{100} = 9.86 MJ/MVA$$

$$M = \frac{SH}{180f} = \frac{100*9.86}{180*50} = 0.109\text{MJ.s/electrical degree}$$

Sudden three Phase Cont..

- Example 2: A 50 Hz, 4 pole, turbo alternator rated 100 MVA, 15 kV has an inertia constant of 10 MJ/MV A. Determine:

a. the energy stored in the rotor at synchronous speed

b. the rotor acceleration if the mechanical input is suddenly raised to 100MW for an electric load of 45M W. (Neglect mechanical and electrical losses).

Solution:

$$a.\quad H = 10MJ/MVA; S = 100MVA$$

$$N = HS = 1000MJ$$

$$b.\ swing\quad equation:$$

$$M\frac{\partial^2\delta}{\partial^2 t} = P_m - P_e$$

$$= (100 - 45)$$

$$M\frac{\partial^2\delta}{\partial^2 t} = 55MW = Pa$$

$$M = \frac{SH}{180f} = \frac{1000}{180*50} = 0.111\text{MJ.s}$$

$$\text{Then, accelartion, } \frac{\partial^2\delta}{\partial^2 t} = \frac{Pa}{\text{M}} = \frac{55}{0.111} = 495.49 \text{ electrical degree/s}$$

Sudden three Phase Cont..

Example 3: A generator *A* is rated at 50 Hz, 80 MW, 90 MVA, 3000 rpm and has an inertia constant *H* = 6 MJ/MVA. The corresponding data for another generator *B* is 50 Hz, 110 MW, 150 MVA, 1500 rpm, and 5 M J/MVA.

a. If two generators operate in parallel, calculate *H* for the equivalent generator on a base of 100 MVA.

b. If the power station is connected to another power station which has two of each type of genera or, cal culate *H* for the equivalent generator connected to an infinite bus bar.

Solution : a. we know that :

$$S_e H_e = H_1 S_1 + H_2 S_2$$

$$H_e = \frac{(6*90)+(5*150)}{S_e} = 12.9\text{MJ/MVA}$$

b. For another power station of the same generators :

$$S_e H_e = H_1 S_1 + H_2 S_2$$

$$H_{e2} = 2*H_{e1} = 2*12.9\text{MJ/MVA}$$

$$= 25.8\text{ MJ/MVA and when they are connected parellel}$$

$$\text{H}_\text{e} = \frac{\text{H}_{\text{e1}}*\text{H}_{\text{e2}}}{\text{H}_{\text{e1}}+\text{H}_{\text{e2}}} = \frac{12.9*25.8}{12.9+25.8} = 8.6 MJ/MVA$$

Sudden three Phase Cont..

Example 4. A 150 MVA, 15 kV, 50 Hz, 4 pole turbo alternator has an inertia constant of 8MJ/MVA.

a. Determine the stored energy in the rotor at synchronous speed.

b. The machine is operating at a load of 130 MW when the load suddenly increases to 180 MW. Determine the rotor delay. Neglect losses.

c. The retardation/delay calculated above is maintained for 4 cycles. Determine the change in power angle and the rotor speed in rpm at the end of this period. Solution :

$$H = 8MJ/MVA, S = 150MVA$$

$$a.\ N = HS = 8*150 = 1200MJ$$

$$b.\ \ P_a = 180 - 130 = 50MW(delay/accelartion)$$

$$M = \frac{N}{180f} = \frac{1200}{180*50} = 0.1333MJ.s/electrical\,\text{deg}\,ree$$

$$\frac{\partial\delta^2}{\partial t^2} = \frac{P_a}{M} = \frac{50}{0.1333} = 375\,\text{electrical degree/s}^2$$

Sudden three Phase Cont..

- It's know that change in rotor angle. where t is the time period for acceleration or retardation.

$$\frac{\partial\delta}{\partial t} = \frac{P_a}{M} * t$$

$$c.\ for\ 4cycle, \quad t = \frac{4*1}{50} = 0.08\text{s}$$

$$\frac{\partial\delta}{\partial \text{t}} = 375 * 0.08s = 30 \quad electrical\ \text{deg}\, ree / s$$

$$= \frac{30*\pi}{180} 0.5236 \quad electrical\ rad / s$$

$$\frac{\partial\delta}{\partial \text{t}} = \frac{0.5236}{p} = 0.618\, mechnical \quad rad / s = 2\pi n_s$$

$$\Rightarrow n_s = \frac{0.2618}{2\pi} = 0.0417$$

$$N_S = n_s * 60 = 2.5\, rpm$$

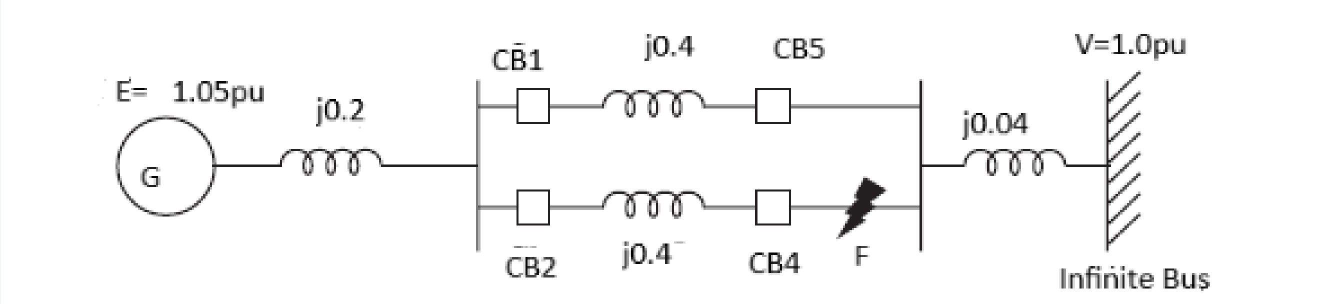
$$\text{Since the accelartion is for 4 cycles,}$$

$$\text{New speed} = \text{synchronous speed} - \text{Ns}$$

$$= 1500 rpm - 2.5 rpm = 1497.5 rpm$$

Sudden three Phase Cont..

Example 5. Given the circuit as shown in the figure below where three-phase fault is applied on one end of a line near circuit breaker CB4. Find the critical fault clearing angle for clearing the fault with simultaneous opening of breaker CB2 and CB4. The generator is delivering 1.0 p.u. MW at the instant preceding the fault . All the p.u. quantities are on the common MVA.



Solution. During , pre-fault condition the thevining equivalent reactance is :

$$X_{T1} = j0.2 + \frac{j0.4 * j0.4}{j0.4 + j0.4} + j0.04 = j0.44$$

- Then, the electrical power transferred to infinite bus is given by:

$$P_{e1} = \frac{EV}{X_{T1}} Sin\delta = \frac{1.05*1}{0.44} \sin\delta = 2.386 \sin\delta = P_{e1\max} \sin\delta$$

Sudden three Phase Cont..

- **During fault condition:** The fault occurs at the end of the line 2 or near bus 2, no power is transferred fr om the circuit, Pe2=0
- **Post-fault condition:** With the opening of the faulted line, say by simultaneous opening of the circuit bre akers CB2 and CB4, the post-fault transfer reactance while removing line 2 is given by:

$$X_{T3} = j0.2 + j0.4 + j0.04 = j0.64$$

- The power transferred to infinite bus is:

$$P_{e3} = \frac{EV}{X3} Sin\delta = \frac{1.05*1}{0.64} \sin\delta = 1.640 \sin\delta = P_{e3\max} \sin\delta$$

- The initial angle:

$$P_{m0} = P_{me} = 1 = P_{\max 1} \sin\delta_0$$

$$\delta_0 = Sin^{-1}(\frac{1}{2.386)} = 24.78^0$$

Sudden three Phase Cont..

And,

$$\delta_{\max} = 180^0 - \sin^{-1}(\frac{P_{m0}}{P_{e3}}) = 180 - \sin^{-1}(\frac{1}{1.64})$$

$$= 180 - 37.57 = 142.43^0$$

Then,

$$\delta_{\max} = 180^0 - \sin^{-1}(\frac{P_{m0}}{P_{e3}}) = 180 - \sin^{-1}(\frac{1}{1.64})$$

$$= 180 - 37.57 = 142.43^0$$

$$Cos\delta_{Cr} = \frac{P_m(\delta_{\max} - \delta_0)\frac{\pi}{180} - P_{e\max 2}\cos\delta_0 + P_{e\max 3}\cos\delta_{\max}}{P_{e\max 3} - P_{e\max 2}}$$

$$Cos\delta_{Cr} = \frac{1.0(142.43^0 - 24.78)\frac{\pi}{180} - 0 + 1.64\cos 142.43^0}{1.64 - 0} = \frac{2.05 + (-1.3)}{1.64}$$

$$\delta_{cr} = \cos^1(0.7667) = 39.9$$

5. Computerized Method of Swing Equation solutions

- The dynamic behavior of a synchronous generator's rotor angle during disturbances is described by the swing equation, a second-order differential equation.
- Since there is no analytical solution for this non-linear differential equation, approximate solutions are obtained by computerized techniques. Some of them are:

a. Numerical Integration

b. Step by step method.

c. Euler’s method.

d. Modified Euler’s method.

e. Runge–Kutta method.

Computerized Method Cont.…

- **Numerical Integration**: Most computerized methods employ numerical integration techniques to approximate the solution over time.
- Point-by-Point Method: A simple but less accurate method where the solution is advanced one time step at a time using a basic approximation of the derivative.
- Modified Euler Method: A more accurate method that involves predicting values at each time step and then correcting them using average derivatives.
- Runge-Kutta Methods: A more accurate class of methods that use intermediate variable values at each time step to calculate weighted average changes, resulting in higher accuracy
- In order to apply these methods you can either use simulation software's or codes

Summary

- In this lecture, the importance of plotting the swing curve to ascertain whether the power system is stable following a disruption is discussed
- If this curve shows that the angle between any two machines tends to increase without limit, the system i s unstable
- In line with this, the importance of constants like kinetic energy, inertia constant (H) and M in transient st ability analysis is presented.
- In addition, the effect of sudden load change in power angle stability is also presented.
- Then, the simple graphical method of determining whether the machines come to rest with respect to eac h other called equal area criteria is discussed.
- Finally, the compterized method to solve stability analysis is also included

References

[1]. Y., Su; S., Cheng; J., Wen and J., Zhang. "Power System Dynamic Stability Analysis and Stability Type Discrimination“. Proceedings of the 41st International Universities Power Engineering Conference, Newcastle upon Tyne, UK, 2006, pp. 516-520. **DOI:** 10.1109/UPEC.2006.367531

[2]. Z., Liu; C., Hong and C., Guo. "Equivalent Inertia Estimation of Power System Based on Kinetic Energy Theorem." IEEE International Conference on Power Science and Technology (ICPST), Kunming, China, 2023, pp. 905-910. DOI: 10.1109/ICPST56889.2023.10165186.

[3]. S., Yong; M., Jinpeng; K., Jürgen and Z., Meng. ‘’’Equal-area criterion in power systems revisited’’. Proc. R. Soc. http://doi.org/10.1098/rspa.2017.0733.

[4]. X., Chierthaichingpangxao; S., Phommixay and K., Somsai. "Improvement of Power Oscillation Damping by Using a Unified Power Flow Controller: Case Study of NamTheun 2 Transmission Line“. International Electrical Engineering Congress (iEECON), Krabi, Thailand, 2023, pp. 295-299. doi: 10.1109/iEECON56657.2023.10126868.

Thank you !