Course: Automatic Control System Technology

Lecture 8: Develop a control system block diagram model

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Develop a control system block diagram model

Session objectives:

By the end of this session, students will be able to :

- ❖Define a block diagram
- ❖Explain advantages and disadvantages of block diagram
- ❖Describe elements of a block diagram
- ❖Describe blocks connections within a block diagram
- ❖Apply block diagram reduction techniques
- ❖Draw a block diagram of linear systems

Define a Block diagram

- A control system may consist of a number of components.
- To show the functions performed by each component, in control engineering, we commonly use a diagram called the **block diagram**.
- A **block diagram** of a system is **a pictorial representation of the functions performed by each component and of the flow of signals.** Katsuhiko Ogata(2009), Modern Control Engineering,5th Edition, Prentice Hall, page 17.

Define a Block diagram

- The block diagram represents the cause and effect relationship or simply the functional relationship between the input signal and output signal.
- Within a block diagram of the system **variables are linked to each other through functional blocks**.
- The **functional block** or simply **block** is a symbol for the mathematical operation on the input signal to the block that produces the output.
- The mathematical operation is the transfer function of the block.

Define a Block diagram

- The block represents the system components and the function performed by each component is represented by the transfer function of the block.
- The transfer functions of the components are usually entered in the corresponding blocks and blocks are connected by arrows to indicate the direction of the flow of signals.
- The signal can only pass in the direction of the arrows.

Explain advantages and disadvantages of Block diagram

❖ **Advantages of Block diagram representation:**

1. It is easy to form the overall block diagram for the entire system by just connecting the blocks of the components according to the signal flow.
2. It is easy to evaluate the contribution of each component to the overall performance of the system
3. In the block diagram, the functional operation of the system can be visualized more readily by examining the individual blocks rather than examining the physical system itself.

Explain advantages and disadvantages of Block diagram

❖ Disadvantages of Block diagram representation:

1. In a block diagram, the main source of energy is not explicitly shown
2. The block diagram of a given system is not unique
3. It is difficult to identify the nature of physical elements connected in the system.

Palani S. (2022), Automatic Control Systems: With MATLAB, 2nd Edition,Springer, page 86-87.

Describe elements of Block diagram

❖ Any linear control system may be represented by a block diagram consisting of the following elements: ***blocks, signals (indicated by arrows), summing points, and takeoff points*** as shown in figure 1.

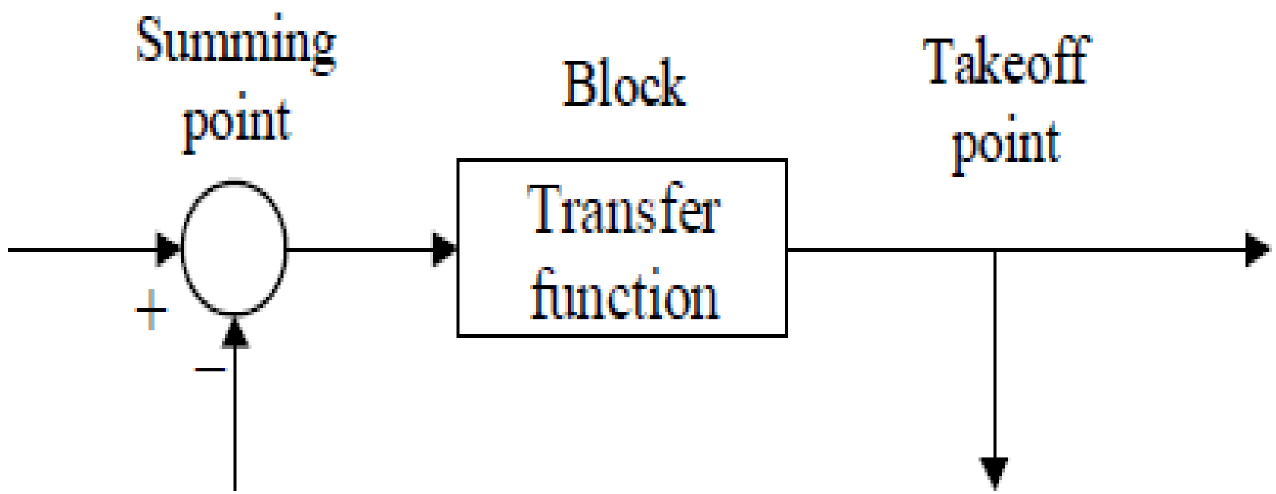


Figure 1. Elements of a block diagram

Describe components of Block diagram

- **Blocks:** Blocks represent the system components or subsystems. They are typically modeled by and labeled with **a transfer function**
- **Signals:** Signals are inputs and outputs of blocks. The direction of a signal is indicated by **arrow**. The arrowhead pointing toward the block indicates the input of the block, and the arrowhead leading way the block represents the output of the block

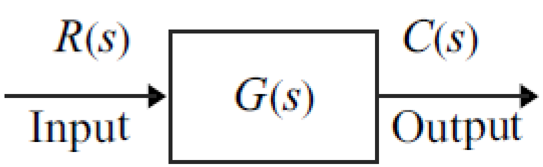
Describe components of Block diagram

- **Summing point/Summing junction:** It is where signals are algebraically added together. The plus or minus sign at each arrowhead indicates whether the signal is to be added or subtracted. It is important that the quantities being added or subtracted have the same dimensions and the same units.
- **Branch point/takeoff point**. It is a point from which the signal, from a block, goes **concurrently to other blocks** or **summing points.**

Describe blocks connections within a Block diagram

❖ In a Block diagram, blocks can be connected in three basic forms/topologies known as familiar forms: ***Cascade/series connection, parallel connection, and feedback connection.***

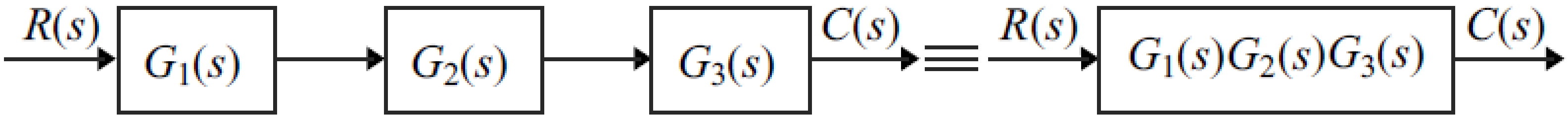
✓***A single block***



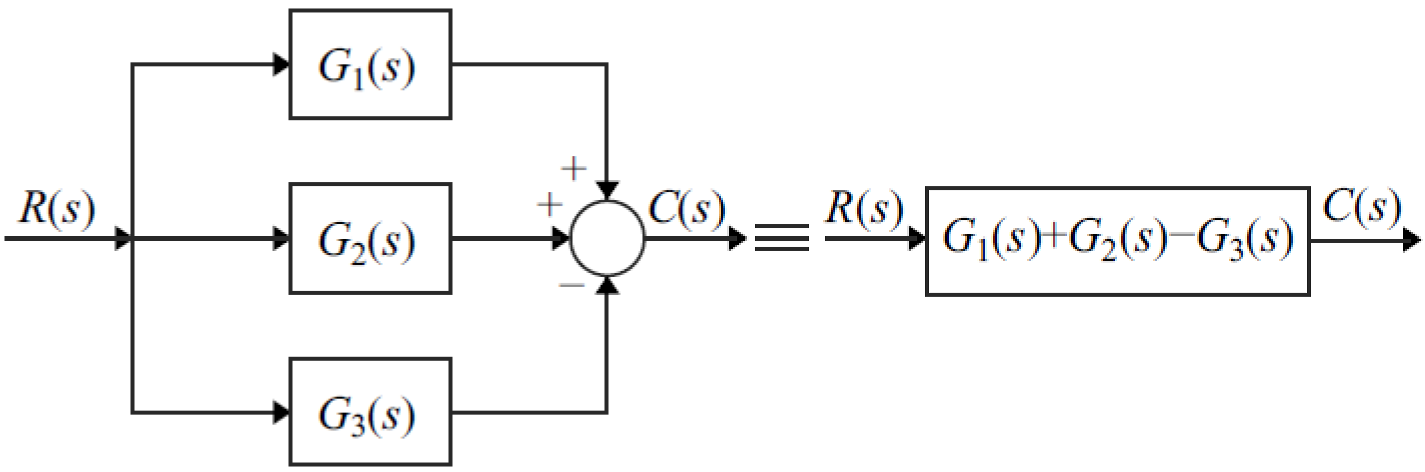
$$C(s)=G(s)R(s)$$

Describe blocks connections within a Block diagram

✓ ***Cascade/series connection***

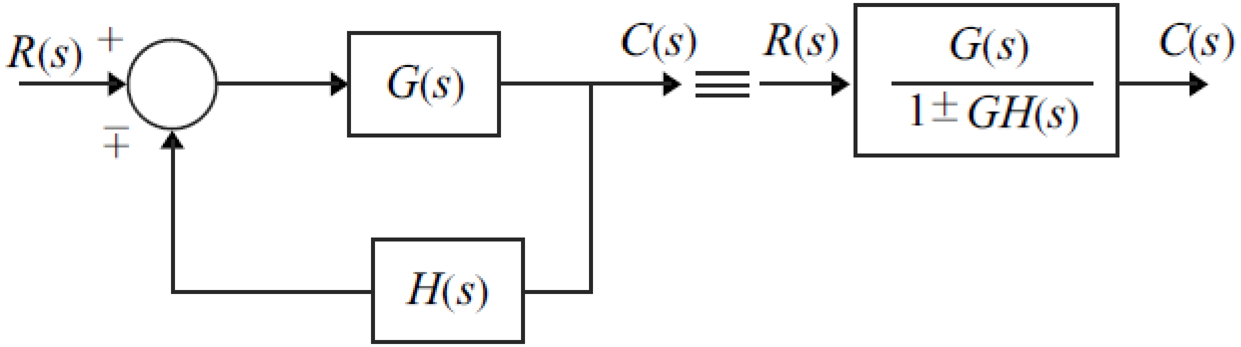


✓ ***Parallel connection***



Describe blocks connections within a Block diagram

✓***Feedback connection***



In the formula, we consider + sign for negative feedback and – sign for positive feedback.

Apply block diagram reduction techniques

- The reduction of block diagram to a single block representation can be derived by simple algebraic manipulation of the equations representing the blocks.
- These diagram transformations are known as block diagram reduction techniques/ rules or block diagram algebra
- Before we begin to reduce block diagrams, you should be aware that the familiar forms (cascade, parallel, and feedback) are not always apparent in a block diagram.

Apply block diagram reduction techniques

- The basic approach to simplify a block diagram can be summarized as follows:

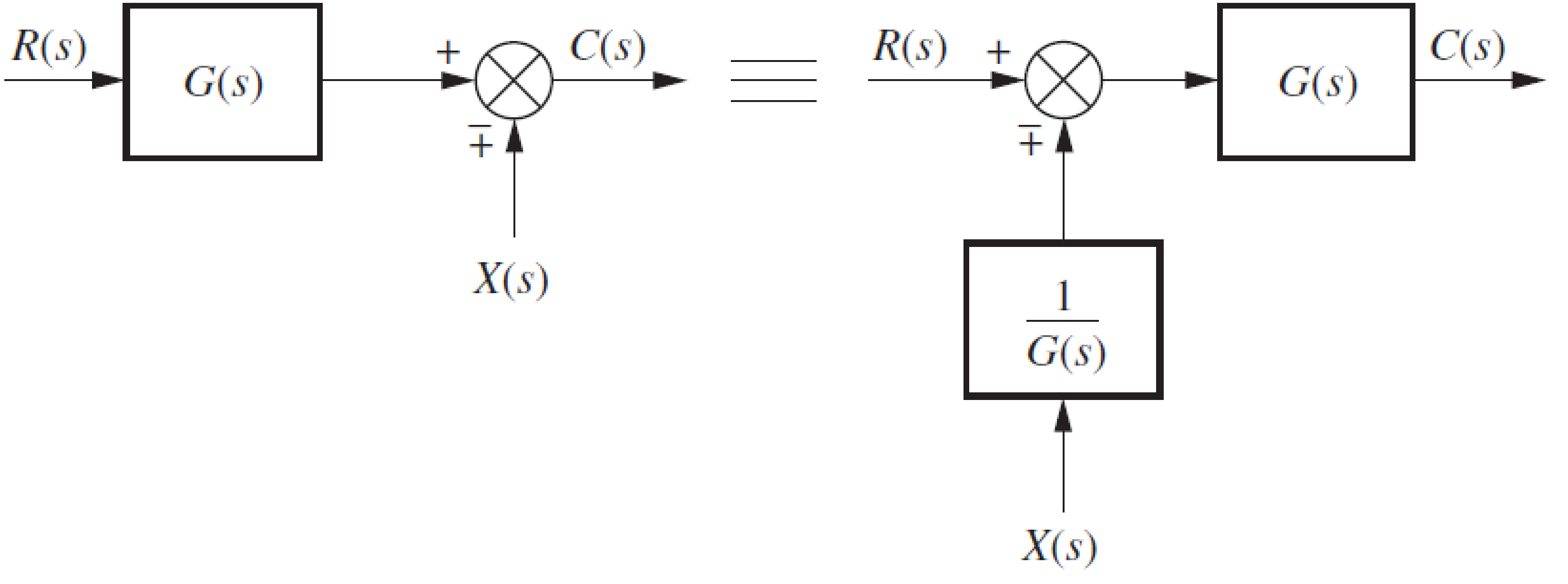
1. Combine all cascade blocks,
2. Combine all parallel blocks
3. Eliminate all minor (interior) feedback loops,
4. **Shift summing points to the left/ahead of the block**
5. **Shift takeoff points to the right/behind of the block**
6. Repeat steps 1 to 5 until the canonical form is obtained

- It is also possible to move summing point to the right/behind of the block or takeoff to the left/ahead of the block.

Apply block diagram reduction techniques

❖ Note that takeoff points may be interchanged among them and summing points may also be interchanged.

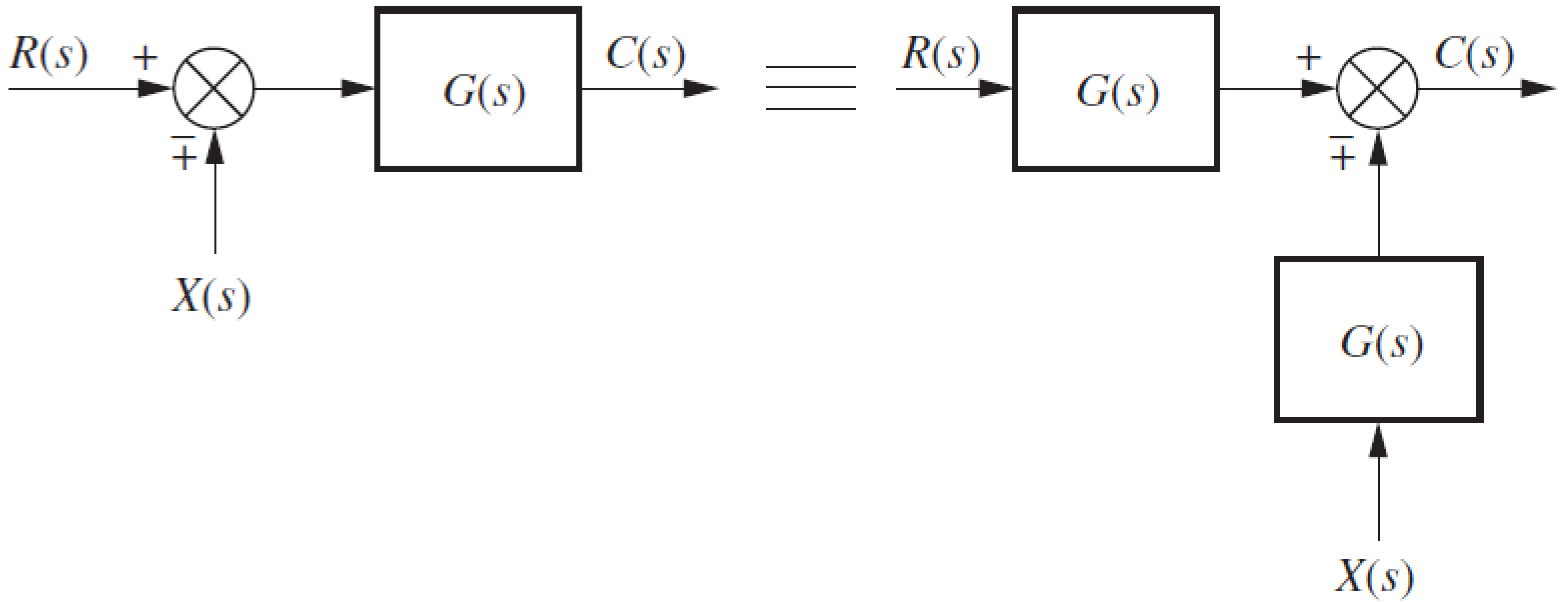
❖ **Shift or move summing points to the left/ahead of the block**



❖ *When we move summing point to the left/ahead of the block, the transfer function of the block divides in the corresponding branch.*

Apply block diagram reduction techniques

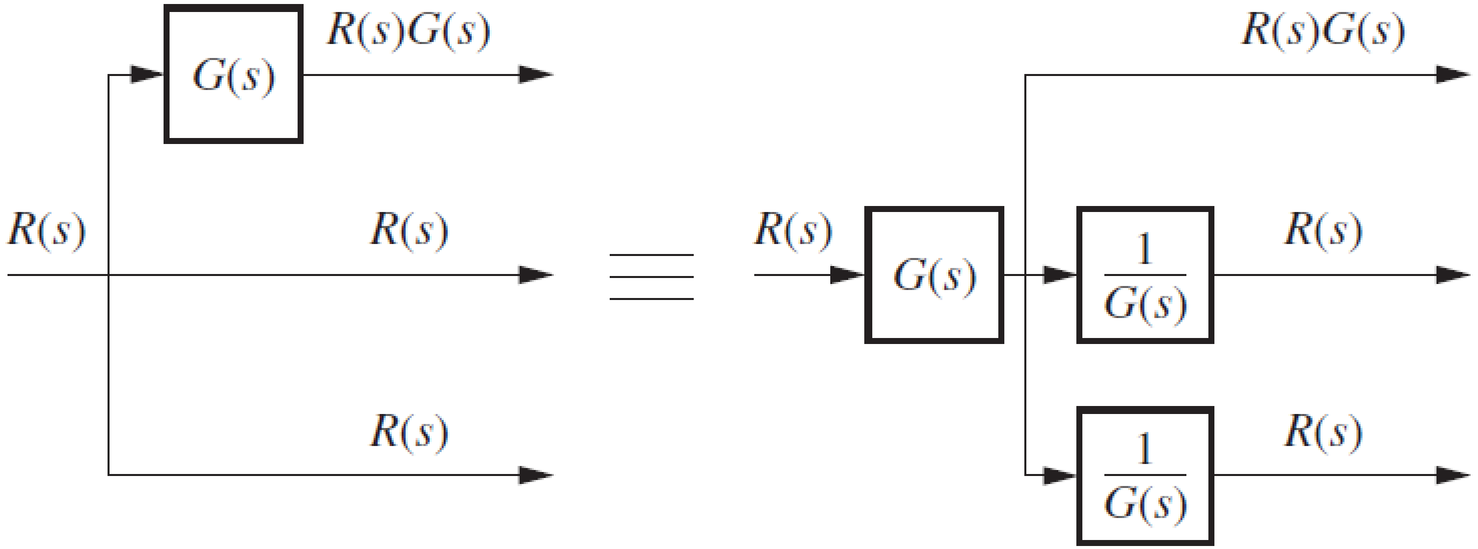
❖ **Shift or move summing points to the right/behind of the block**



❖ *When we move summing point to the right/behind of the block, the transfer function of the block multiplies in the corresponding branch.* Nise, Norman S. (2015), Control Systems Engineering, 7th Edition, John Wiley & Sons, page 241.

Apply block diagram reduction techniques

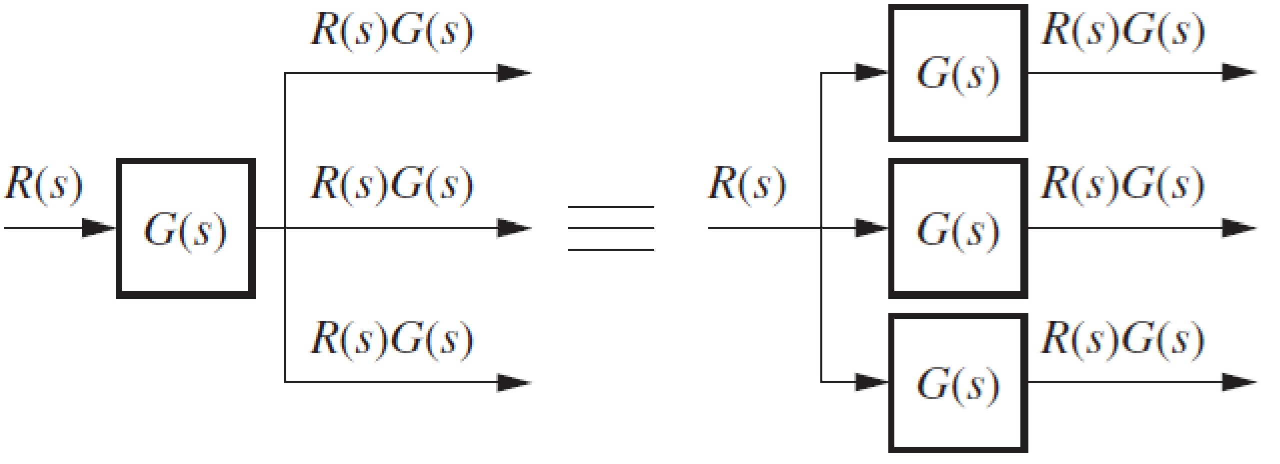
❖ **Shift or move takeoff point to the right/behind of the block**



❖ *When we move takeoff point to the right/behind of the block, the transfer function of the block divides in the corresponding branch.*

Apply block diagram reduction techniques

❖ **Shift or move takeoff point to the left/ahead of the block**



❖ *When we move takeoff point to the left/ahead of the block, the transfer function of the block multiplies in the corresponding branch.*

Apply block diagram reduction techniques

❖ **Example 1.** Reduce the block diagram shown in the figure 2 and obtain its closed loop transfer function $\frac{\boldsymbol{C(s)}}{\boldsymbol{R(s)}}$

(a)

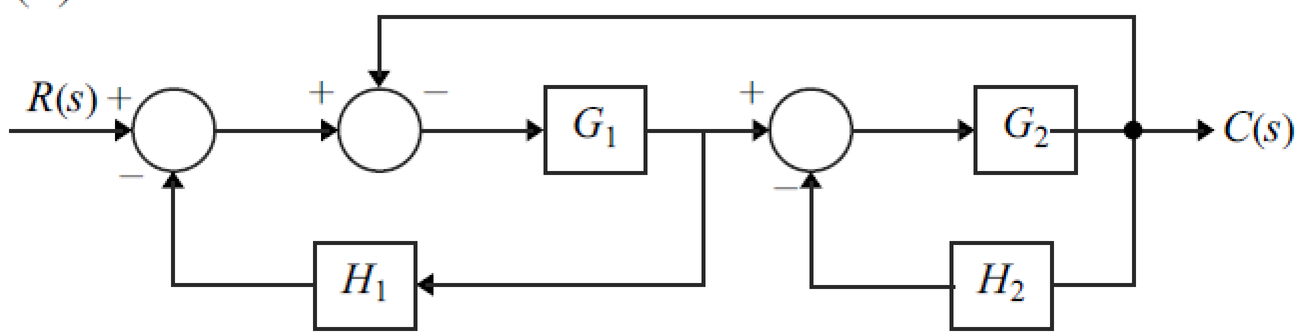


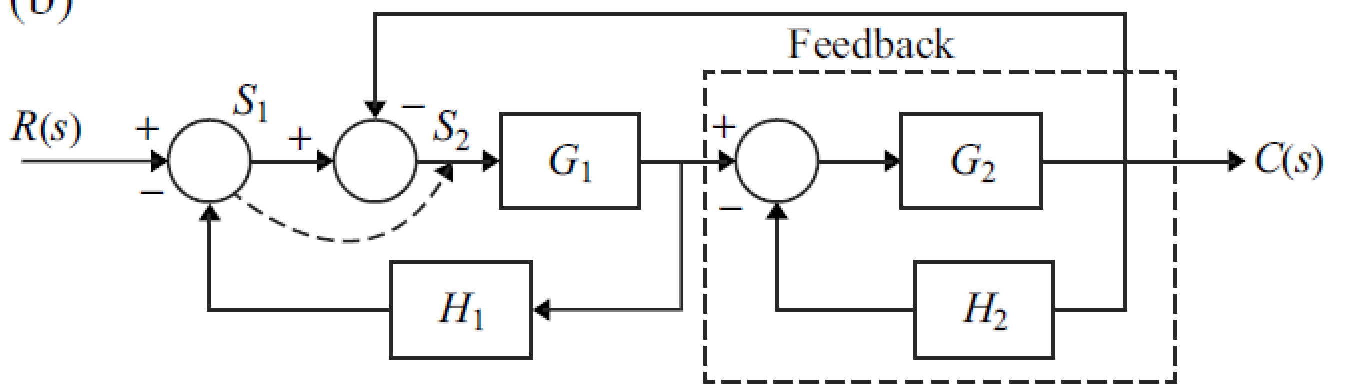
Figure 2. (a) Example of Block diagram to be reduced

Palani S. (2022), Automatic Control Systems: With MATLAB, 2^{nd} Edition, Springer, page 93.

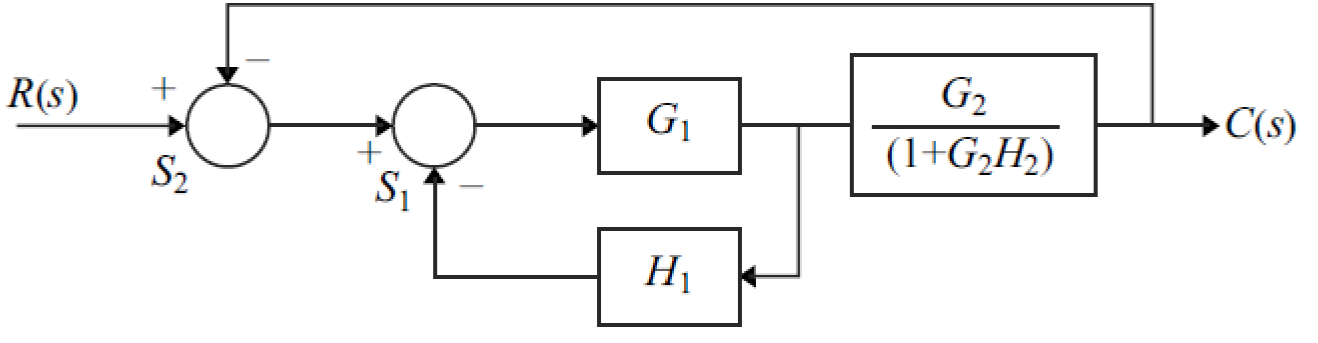
Apply block diagram reduction techniques

❖ **Example 1.***(cont.)*

(b)



(c)

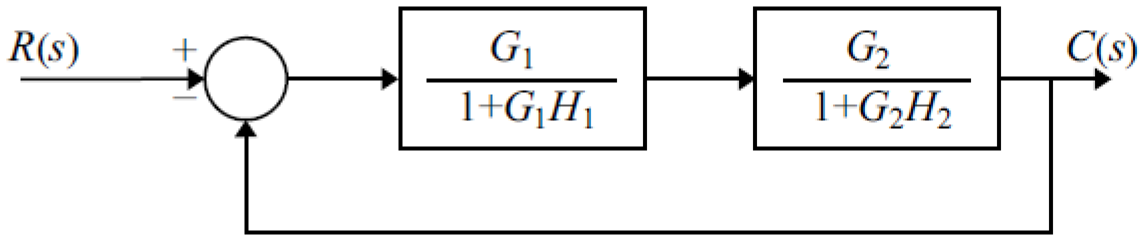


❖ G_2 and H_2 blocks , in figure (b), are connected in negative feedback, and summers S_1 and S_2 are interchanged to get fig. (c).

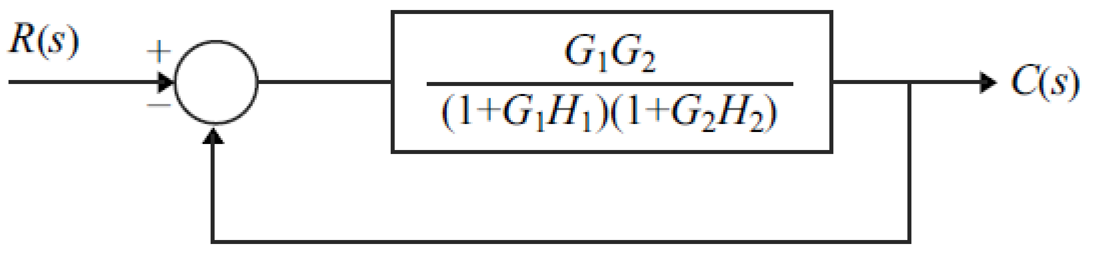
Apply block diagram reduction techniques

❖ **Example 1.** *(cont.)*

(d)



(e)



❖ G_1 and H_1 blocks , in figure (c), are connected in negative feedback, and blocks in figure (d) are combined to get result in figure (e).

Apply block diagram reduction techniques

- **Example 1.** *(cont.)*, finally the transfer function of the system is:

$$\frac{C}{R}(s) = \frac{G_1 G_2}{(1 + G_1 H_1)(1 + G_2 H_2) + G_1 G_2}$$

Apply block diagram reduction techniques

- **Example 2.** For the block diagram shown in figure 3. (a) Using block diagram reduction technique, determine $\frac{C(s)}{R(s)}$

(a)

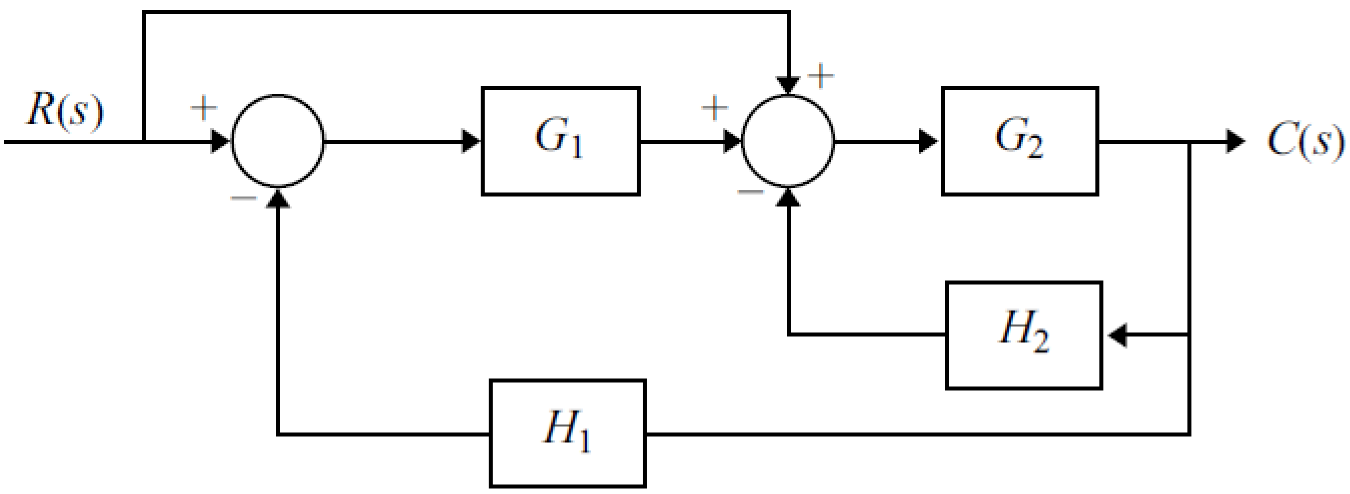


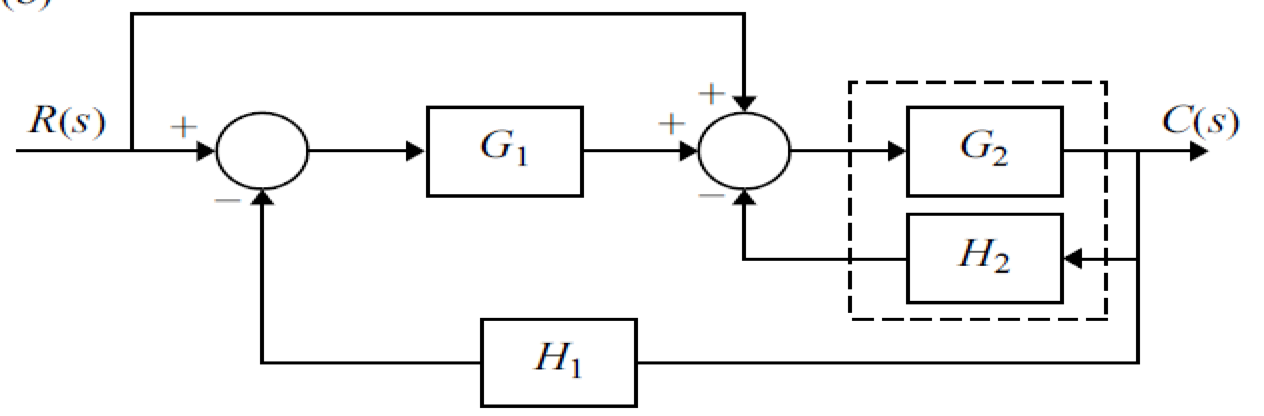
Figure 3. (a) Example of Block diagram to be reduced

Palani S. (2022), Automatic Control Systems: With MATLAB, 2nd Edition, Springer, page 97.

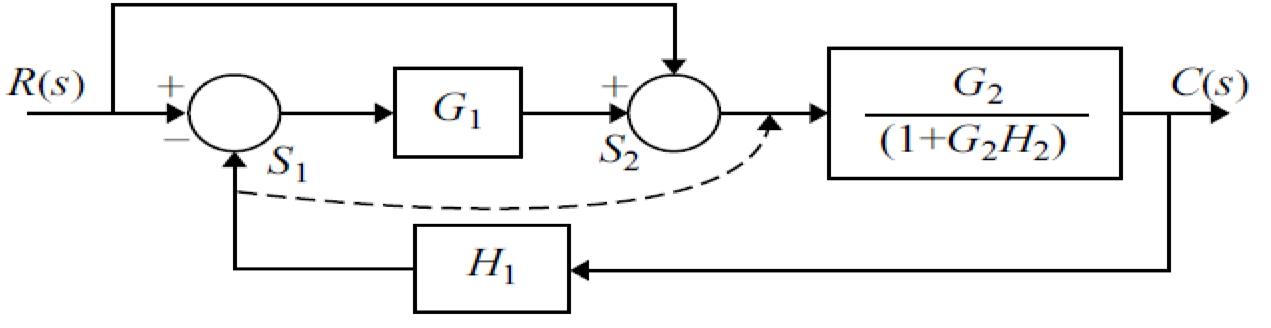
Apply block diagram reduction techniques

❖ **Example 2.** *(cont.)*

(b)



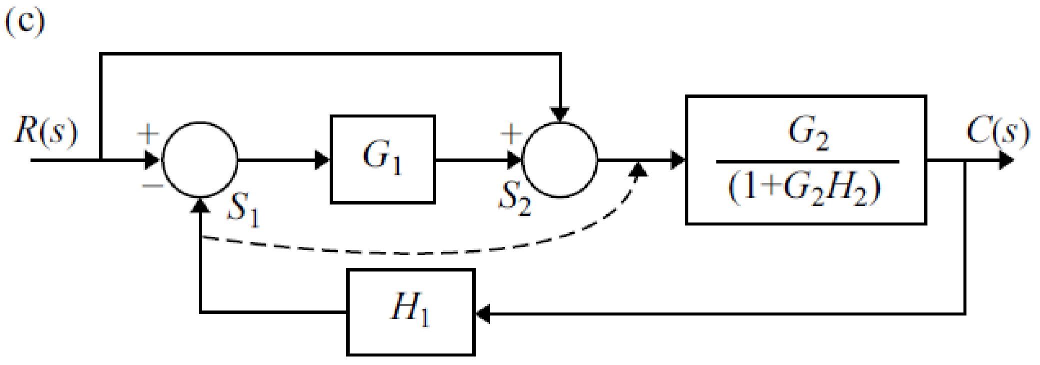
(c)

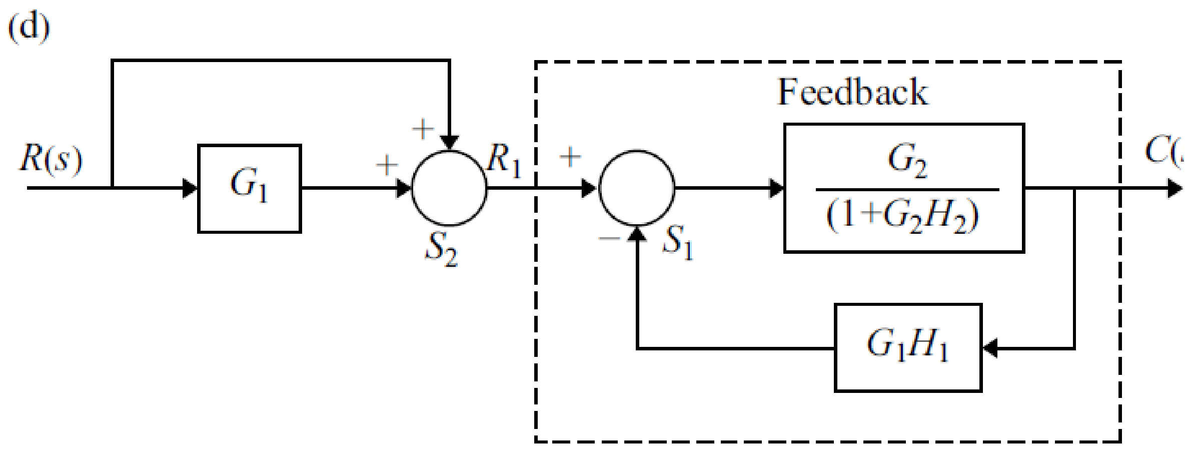


❖ The blocks G_2 and H_2 are connected in negative feedback

Apply block diagram reduction techniques

❖ **Example 2.** *(cont.)*



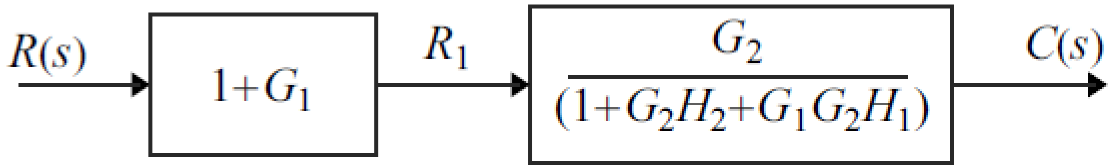


❖ From figure (c) , move s_1 to right of block G_1 and interchange summers s_1 and s_2 to get figure (d). Here G_1 multiplies H_1

Apply block diagram reduction techniques

❖ **Example 2.** *(cont.)*

(e)



❖ Finally, from figure (e) the transfer function of the system is obtained as:

$$\frac{C(s)}{R(s)} = \frac{G_2(1 + G_1)}{(1 + G_2H_2 + G_1G_2H_1)}$$

Draw a block diagram of linear systems

❖ To draw a block diagram for a system, follow these steps:

1. First, write equations that describe the dynamic behavior of each component.
2. Then, take the Laplace transforms of these equations, assuming zero initial conditions.
3. And, represent each Laplace-transformed equation individually in block form.
4. Finally, assemble those individual blocks into a complete block diagram.

Katsuhiko Ogata(2009), Modern Control Engineering,5th Edition, Prentice Hall, page 27.

Draw a block diagram of linear systems

- **Example:** Consider the RC circuit shown in figure.

1. And develop its Block diagram model.
2. Using block diagram reduction rules, obtain its transfer function

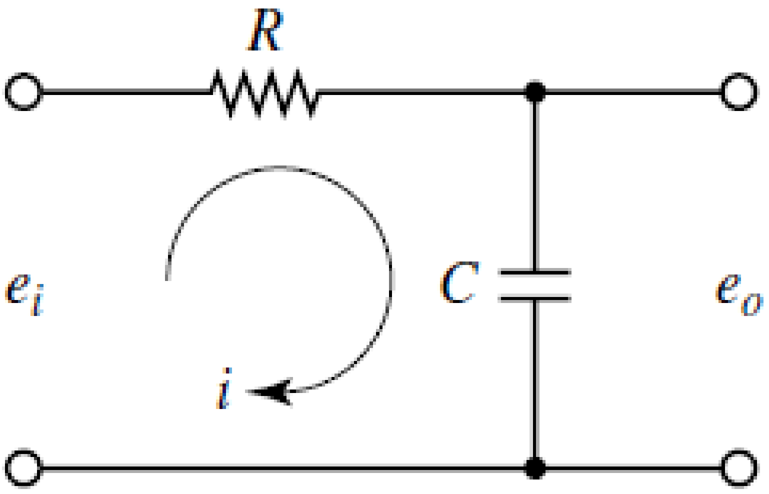


Figure 4. RC circuit

Katsuhiko Ogata(2009), Modern Control Engineering,5th Edition, Prentice Hall, page 27.

Draw a block diagram of linear systems

❖ **Example** *(cont.)*: **1.Block diagram model development:**

- Write equations that describe the dynamic behavior of each component.

$$e_i(t) = Ri(t) + e_o(t)$$

$$e_o(t) = \frac{1}{C}\int_0^t i(\tau)\,d\tau$$

- Take the Laplace transforms of these equations, assuming zero initial conditions.

$$E_i(s) = RI(s) + E_o(s) \rightarrow I(s) = \frac{1}{R}(E_i(s) - E_o(s))$$

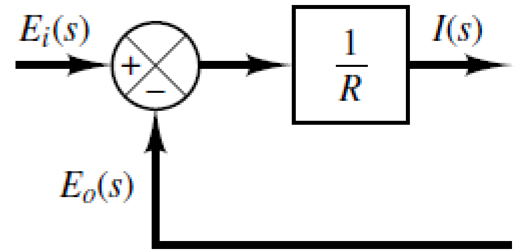
$$E_o(s) = \frac{1}{Cs}I(s)$$

Draw a block diagram of linear systems

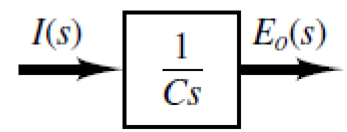
❖ **Example** *(cont.):* **1.Block diagram model development:**

- Represent each Laplace-transformed equation individually in block form.

$$I(s) = \frac{1}{R}(E_i(s) - E_o(s))$$



$$E_o(s) = \frac{1}{Cs} I(s)$$



- Assemble these blocks into a complete block diagram.

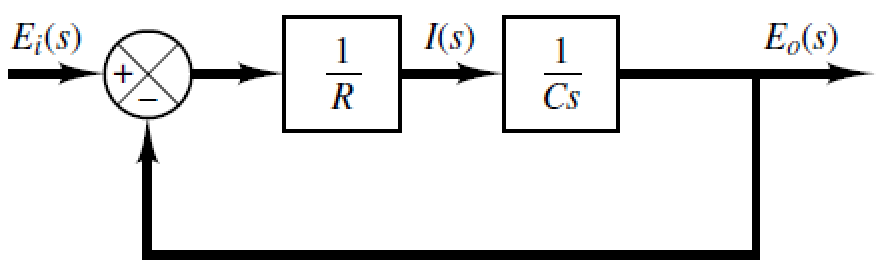


Figure 5.Block diagram of RC circuit

Draw a block diagram of linear systems

❖ **Example** *(cont.)*: **2. Transfer function of the circuit**

❖ From figure 4, it is clear that $\frac{1}{R}$ and $\frac{1}{Cs}$ are connected in cascade. Thus, the transfer function of a combined block is: $\frac{1}{RCs}$

❖ The resultant block diagram is of a negative feedback form. Hence, it can be reduced as:

$$\frac{E_0(s)}{E_i(s)} = \frac{\frac{1}{RCs}}{1 + \frac{1}{RCs}} = \frac{1}{RCs + 1}$$

❖ We find the same result as in Lecture 7 about transfer function model of RC circuit.

References

1. Palani S. (2022), Automatic Control Systems: With MATLAB, 2nd Edition, Springer.
2. Nise, Norman S. (2015), Control Systems Engineering,7th Edition,John Wiley & Sons.
3. Katsuhiko Ogata(2009), Modern Control Engineering,5th Edition, Prentice Hall.

THANK YOU