Course: Automatic Control System Technology

Lecture 11: Conduct the analysis of second order system time response

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Conduct the analysis of second order system time response

Session objectives:

By the end of this session, students will be able to :

- ❖ Represent the standard equation of second-order system
- ❖ Classify second-order system according to damping ratio
- ❖ Determine a unit step response of second-order system
- ❖ Represent poles and unit step responses of a second order system
- ❖ Determine time response specifications of a second order system

Represent the standard equation of second-order system

- ❖ The **standard equation of a second-order systems** is:

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2+2\xi\omega_n+\omega_n^2} \quad \text{--- eq1}$$

Where

- ✓ ω_n = Natural frequency of oscillations in rad/s
- ✓ ξ = damping ratio

Katsuhiko Ogata(2009), Modern Control Engineering,5th Edition, Prentice Hall, page 166.

Represent the standard equation of second-order system

- The equation eq1 is also called **prototype second order system** equation. Thus, $\boldsymbol{\omega_n}$ and $\boldsymbol{\xi}$ are two parameters of a second order system.
- The equation eq1 is represented in a closed loop form in figure 1.

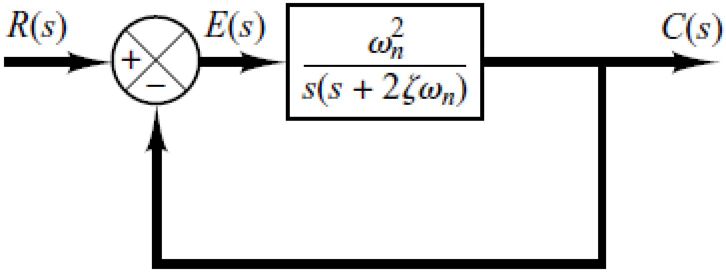

Figure 1. Closed loop block diagram of second-order system

Katsuhiko Ogata(2009), Modern Control Engineering,5th Edition, Prentice Hall, page 166.

Classify second-order systems according to damping ratio

- ❖ The dynamic behavior of a second-order system can then be described in terms of second-order system's parameters $\boldsymbol{\xi}$ and $\boldsymbol{\omega_n}$.
- ❖ By dynamic behavior, we mean the ways in which the **system responds to sudden input changes.**
- ❖ The characteristic equation of a second-order closed loop system is: $\boldsymbol{s^2 + 2\xi\omega_n\, s + \omega_n^2 = 0}$
- ❖ Roots (poles of transfer function) , $s_1$and s_2, of the characteristic equation are determined as:
  - ✓ $\Delta = \boldsymbol{4\xi^2\omega_n^2 - 4\omega_n^2}$

Classify second-order systems according to damping ratio

- ✓ $s_1 = \frac{-2\xi\omega_n + \sqrt{4\xi^2\omega_n^2 - 4\omega_n^2}}{2} = -\xi\omega_n + \omega_n\sqrt{\xi^2 - 1}$

- ✓ $s_2 = \frac{-2\xi\omega_n - \sqrt{4\xi^2\omega_n^2 - 4\omega_n^2}}{2} = -\xi\omega_n - \omega_n\sqrt{\xi^2 - 1}$

- ❖ If **ξ = 0**, in this case $\sqrt{0 - 1} = \pm j$, the poles $s_1 and\ s_2$ are pure imaginary. The transient response does not die out and the response is undamped**.** The system is **undamped**.

$$s_1 = j\omega_n \text{ and } s_2 = -j\omega_n$$

ω_n representsthe **undamped natural frequency** of the system.

Classify second-order systems according to damping ratio

- ❖ If $\mathbf{0 < \xi < 1}$, in this case $\xi^2 - 1 < 0$ and the term becomes $\omega_n\sqrt{\xi^2 - 1} = \pm j\omega_n\sqrt{1 - \xi^2} = \pm j\omega_d$, the two closed-loop poles are complex, conjugates and lie in the left-half of s plane. The system is **underdamped**, and the **transient response is oscillatory**.

✓$s_1 = -\xi\omega_n - j\omega_n\sqrt{1 - \xi^2}$ and $s_2 = -\xi\omega_n + j\omega_n\sqrt{1 - \xi^2}$

✓$s_1 = -\xi\omega_n - j\omega_d$ and $s_2 = -\xi\omega_n + j\omega_d$,

✓ $\omega_d = \omega_n\sqrt{1 - \xi^2}$ is called the **damped natural frequency**.

Classify second-order systems according to damping ratio

- If $\boldsymbol{\xi}$ **=1**, in this case $\sqrt{\xi^2 - 1} = 0$ and the poles $s_1 and\ s_2$ are real, negative and equal or repeated. The system is **critically damped**.

$$s_1 = s_2 = -\omega_n$$

- If $\boldsymbol{\xi} > \mathbf{1}$ **,** in this case $\sqrt{\xi^2 - 1} > 0$, the poles s_1 and s_2 are real, negative and distinct. **The system is then Overdamped**.

$$s_1 = -\xi\omega_n + \omega_n\sqrt{\xi^2 - 1} \text{ and } s_2 = -\xi\omega_n - \omega_n\sqrt{\xi^2 - 1}$$

Determine a unit step response of second-order system

- ❖ Using the standard equation of the second order systems, we shall now solve for **the response of the system to a unit-step input**:

$$\boldsymbol{\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}}$$

- ❖ **Undamped systems (ξ = 0)**:
  - ✓The system transfer function becomes: $\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2+\omega_n^2}$
  - ✓For a unit step input, $\mathrm{R(s)} = \frac{1}{\mathrm{s}}$
  - ✓$C(s) = \frac{\omega_n^2}{s(s^2+\omega_n^2)}$

Determine a unit step response of second-order system

- ❖ Expand C(s) into partial fractions:

  - ✓$C(s) = \frac{\omega_n^2}{s(s^2+\omega_n^2)} = \frac{K_0}{s} + \frac{K_1 s+K_2}{s^2+\omega_n^2}$, because of pure imaginary poles

  - ✓$C(s) = \frac{\omega_n^2}{s(s^2+\omega_n^2)} = \frac{s^2(K_0+K_1)+K_2 s+K_0\omega_n^2)}{s(s^2+\omega_n^2)}$
  - ✓ Equating the coefficients of s of both sides of the above equation, we get: $K_0 = 1$, $K_1 = -1$, *and* K_2 =*0*

  - ✓ $C(s) = \frac{1}{s} - \frac{s}{s^2+\omega_n^2}$

Determine a unit step response of second-order system

- $\boldsymbol{c(t) = 1 - \cos(\omega_n t), t \geq 0}$
- When the damping ratio ξ is equal to zero, the response becomes undamped and oscillations continue indefinitely.
- **Critically damped systems (ξ = 1**):
  - ✓The system transfer function becomes: $\frac{C(s)}{R(s)} = \frac{\omega_n^2}{(s+\omega_n)^2}$
  - ✓For unit step input, R(s) $= \frac{1}{s}$
  - ✓$C(s) = \frac{\omega_n^2}{S(s+\omega_n)^2}$

Determine a unit step response of second-order system

✓Expand $C(s)$ into partial fractions:

✓$C(s) = \frac{\omega_n^2}{S(s+\omega_n)^2} = \frac{A_0}{s} + \frac{A_1}{s+\omega_n} + \frac{A_2}{(s+\omega_n)^2}$, all poles are real

$$A_0 = \left.\frac{{\omega_n}^2}{(s+\omega_n)^2}\right|_{s=0} = 1$$

$$A_2 = \left.\frac{{\omega_n}^2}{s}\right|_{s=-\omega_n} = -\omega_n$$

$$A_1 = -\left.\frac{{\omega_n}^2}{s^2}\right|_{s=-\omega_n} = -1$$

Determine a unit step response of second-order system

✓$c(t) = \frac{1}{s} - \frac{1}{s+\omega_n} + \frac{\omega_n}{(s+\omega_n)^2}$

✓$c(t) = 1 - e^{\omega_n t} - \omega_n t e^{\omega_n t}, t \geq 0$

✓ $c(t) = 1 - e^{-\omega_n t}(1 + \omega_n t), t \geq 0$

❖ **Underdamped system $(\mathbf{0} < \boldsymbol{\xi} < \mathbf{1})$:**

✓ The system transfer function is :$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}$

✓For unit step input, $\mathrm{R(s)} = \frac{1}{\mathrm{s}}$ and $C(s) = \frac{\omega_n^2}{s(s^2+2\xi\omega_n s+\omega_n^2)}$

Determine a unit Step Response of Second-Order System

✓$C(s) = \frac{\omega_n^2}{s(s^2+2\xi\omega_n s+\omega_n^2)}$

✓ And as demonstrated on slide 7, for $\mathbf{0 < \xi < 1}$, the transfer function of a second order system has two complex, conjugates poles and thus, $C(s)$ has too.

✓Decompose $C(s)$ into partial fractions:

✓$C(s) = \frac{\omega_n^2}{s(s^2+2\xi\omega_n s+\omega_n^2)} = \frac{K_0}{s} + \frac{K_1 s+K_2}{s(s^2+2\xi\omega_n s+\omega_n^2}$

✓$\omega_n^2 = K_0(s^2+2\xi\omega_n s + \omega_n^2) \ + (K_1 s + K_2) * s$

Determine a unit Step Response of Second-Order System

- ✓Rearranging the previous equation, we get:

$$\omega_n^2 = (K_0 + K_1)s^2 + (2\xi\omega_n K_0 + K_2)s \ + K_0\omega_n^2$$

- ✓ Equating the coefficients of s of both sides of the above equation, we get: $K_0 = 1$, $K_1 = -1$, and $K_2 = -2\xi\omega_n$

- ✓ $C(s) = \frac{1}{s} + \frac{-1s-2\xi\omega_n}{s^2+2\xi\omega_n s+\omega_n^2} = \frac{1}{s} - \frac{s+2\xi\omega_n}{s^2+2\xi\omega_n s+\omega_n^2}$

- ✓ The denominator and numerator of the second term of C(s) can be rearranged to get the Laplace transforms of cosine and sine as follows:

- ✓ $C(s) = \frac{1}{s} - \frac{s+2\xi\omega_n}{s^2+2\xi\omega_n s+\omega_n^2} = \frac{1}{s} - \frac{s+2\xi\omega_n}{(s+\xi\omega_n)^2-(\xi\omega_n)^2+\omega_n^2}$

Determine a unit Step Response of Second-Order System

- $C(s) = \frac{1}{s} - \frac{s+2\xi\omega_n}{(s+\xi\omega_n)^2-(\xi\omega_n)^2+\omega_n^2} = \frac{1}{s} - \frac{s+2\xi\omega_n}{(s+\xi\omega_n)^2+\left(\omega_n\sqrt{1-\xi^2}\right)^2}$
- $C(s) = \frac{1}{s} - \frac{s+2\xi\omega_n}{(s+\xi\omega_n)^2+\omega_d{}^2} = \frac{1}{s} - \frac{s+\xi\omega_n}{(s+\xi\omega_n)^2+\omega_d{}^2} - \frac{\xi\omega_n}{(s+\xi\omega_n)^2+\omega_d{}^2}$
- $C(s) = \frac{1}{s} - \frac{s+2\xi\omega_n}{(s+\xi\omega_n)^2+\omega_d{}^2} = \frac{1}{s} - \frac{s+\xi\omega_n}{(s+\xi\omega_n)^2+\omega_d{}^2} - \frac{\xi\omega_n * \frac{\omega_d}{\omega_d}}{(s+\xi\omega_n)^2+\omega_d{}^2}$
- Then, the inverse Laplace transform c(t) of C(s) is:
- $c(t) = 1 - e^{-\xi\omega_n t}\cos\omega_d t - \frac{\xi\omega_n}{\omega_d}e^{-\xi\omega_n t}\sin\omega_d t\ , for\ t \geq 0$

Determine a unit Step Response of Second-Order System

✓ c(t) can also be rearranged into the following form:

✓ $c(t) = 1 - e^{-\xi\omega_n t}\cos\omega_d t - \frac{\xi\omega_n}{\omega_n\sqrt{1-\xi^2}}e^{-\xi\omega_n t}\sin\omega_d t\,, for\ t \geq 0$

✓ $(c(t) = 1 - e^{-\xi\omega_n t}(\cos\omega_d t + \frac{\xi\omega_n}{\omega_n\sqrt{1-\xi^2}}\sin\omega_d t)\,, for\ t \geq 0$

✓ $(c(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}\ (\sqrt{1-\xi^2}\cos\omega_d t + \xi\sin\omega_d t)\,, for\ t \geq 0$

✓ $c(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}\ \sin(\omega_d t + \beta), \sin\beta = \sqrt{1-\xi^2}$ and $cos\,\beta = \xi$

✓ $c(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}\ \sin\left(\omega_d t + tan^{-1}(\frac{\sqrt{1-\xi^2}}{\xi})\right), for\ t \geq 0$

Determine a unit Step Response of Second-Order System

- ✓ $c(t)$ is a unit step response of a second-order system
- ✓ $c(t)$ displays two components: the transient response, $\boldsymbol{c_{tr}(t)}$, which is the part that goes to zero after large interval of time, and the steady state response, $\boldsymbol{c_{ss}(t)}$, which is the part of the response that remains constant when the time becomes very large or simply when t tends to infinity.

- $\boldsymbol{c_{tr}(t)} = -\frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} \sin\left(\omega_d t + tan^{-1}(\frac{\sqrt{1-\xi^2}}{\xi})\right), t \geq 0$
- $\boldsymbol{c_{ss}(t)} = 1 \ , t \geq 0$

Determine a unit Step Response of Second-Order System

- ✓ It can be seen that the frequency of transient oscillation is the damped natural frequency $\boldsymbol{\omega_d} = \omega_n\sqrt{1-\xi^2}$ and thus, it varies with the damping ratio ξ.
- ✓ The error signal for this system is the difference between the input and output: $e(t) = r(t) - c(t)$

$$e(t) = e^{-\xi\omega_n t}(cos\omega_d t + \frac{\xi}{\sqrt{1-\xi^2}}\sin\omega_d t)\ ,\ t \geq 0$$

- ✓ This error signal exhibits a damped sinusoidal oscillation. At steady state, i.e: at $t = +\infty$, no error exists between the input and output; i.e $\boldsymbol{e_{ss}} = \boldsymbol{e}(\infty) = \mathbf{0}$

Determine a unit Step Response of Second-Order System

- ❖ **Overdamped system ($\xi > 1$):**
  - ✓ The system transfer function is :$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}$
  - ✓ For unit step input, $\mathrm{R(s)} = \frac{1}{\mathrm{s}}$ and $C(s) = \frac{\omega_n^2}{s(s^2+2\xi\omega_n \mathrm{s}+\omega_n^2)}$
  - ✓ We have shown on slide 8 that for $\boldsymbol{\xi > 1}$, the transfer function of a second order system has two poles s_1 and s_2 which are real, negative and distinct. And thus, $C(s)$ has too.
  - ✓ $s_1 = -\xi\omega_n + \omega_n\sqrt{\xi^2 - 1}$ and $s_2 = -\xi\omega_n - \omega_n\sqrt{\xi^2 - 1}$

Determine a unit Step Response of Second-Order System

- ✓Decompose $C(s)$ into partial fractions:
- ✓$C(s) = \frac{\omega_n^2}{s(s^2+2\xi\omega_n s+\omega_n^2)} = \frac{\omega_n^2}{\mathrm{s}(\mathrm{s}-s_1)(\mathrm{s}-s_2)} = \frac{K_0}{s} + \frac{K_1}{s-s_1} + \frac{K_2}{s-s_2}$
- ✓$K_O = \left.\frac{\omega_n^2}{(\mathrm{s}-s_1)(\mathrm{s}-s_2)}\right|_{s=0} = \frac{\omega_n^2}{s_1 s_2} = \frac{\omega_n^2}{(-\xi\omega_n+\omega_n\sqrt{\xi^2-1})(-\xi\omega_n-\omega_n\sqrt{\xi^2-1})}$
- ✓$K_O = \frac{\omega_n^2}{s_1 s_2} = \frac{\omega_n^2}{(-\xi\omega_n)^2-(\omega_n\sqrt{\xi^2-1})^2} = \frac{\omega_n^2}{\omega_n^2} = 1$
- ✓ $K_1 = \left.\frac{\omega_n^2}{\mathrm{s}(\mathrm{s}-s_2)}\right|_{s=s_1} = \frac{\omega_n^2}{s_1(s_1-s_2)}$
- ✓$K_2 = \left.\frac{\omega_n^2}{\mathrm{s}(\mathrm{s}-s_1)}\right|_{s=s_2} = \frac{\omega_n^2}{s_2(s_2-s_1)}$

Determine a unit Step Response of Second-Order System

✓Therefore: $C(s) = \frac{1}{s} + \frac{\omega_n^2}{s_1(s_1 - s_2)} \frac{1}{s - s_1} + \frac{\omega_n^2}{s_2(s_2 - s_1)} \frac{1}{s - s_2}$

✓$c(t) = 1 + \frac{\omega_n^2}{s_1(s_1 - s_2)} e^{s_1 t} + \frac{\omega_n^2}{s_2(s_2 - s_1)} e^{s_2 t}, t \geq 0$

✓$c(t) = 1 + \frac{\omega_n^2}{(s_1 - s_2)} \left(\frac{e^{s_1 t}}{s_1} - \frac{e^{s_2 t}}{s_2} \right), t \geq 0$

✓$s_1 - s_2 = -\xi\omega_n + \omega_n\sqrt{\xi^2 - 1} - \left(-\xi\omega_n - \omega_n\sqrt{\xi^2 - 1} \right)$

✓$s_1 - s_2 = 2\omega_n\sqrt{\xi^2 - 1}$ and $\frac{\omega_n^2}{s_1 - s_2} = \frac{\omega_n}{2\sqrt{\xi^2 - 1}}$

✓Thus: $c(t) = 1 + \frac{1}{2\sqrt{\xi^2 - 1}} \left(\frac{e^{s_1 t}}{s_1} - \frac{e^{s_2 t}}{s_2} \right), t \geq 0$

Determine a unit Step Response of Second-Order System

✓A family of unit-step response curves c(t) with various values of **ξ** is shown in figure 2.These curves are functions only of **ξ** .

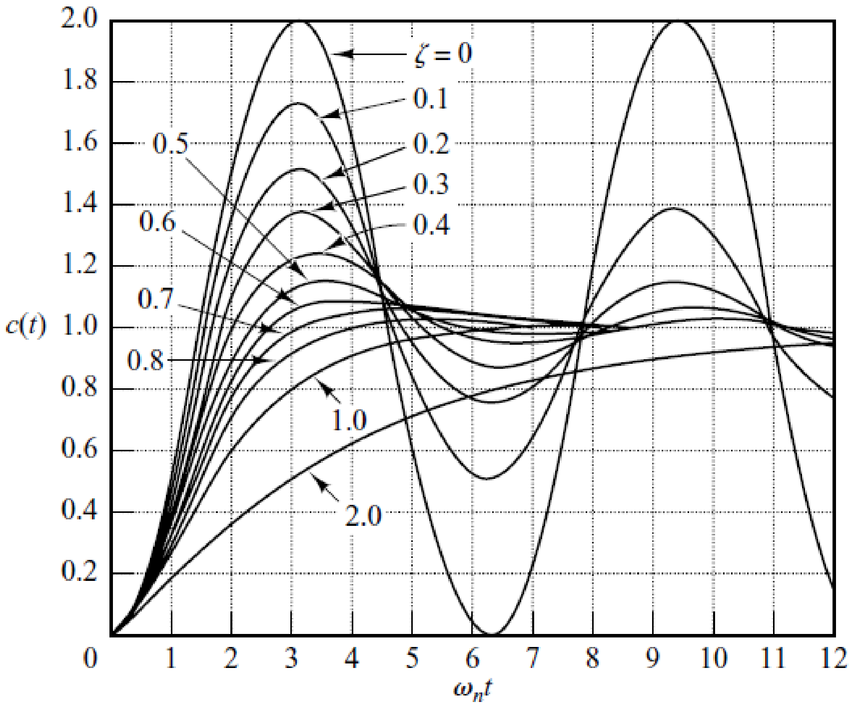

Figure 2. Unit-step response curves of the system

Katsuhiko Ogata(2009), Modern Control Engineering,5th Edition, Prentice Hall, page 169.

Determine a unit Step Response of Second-Order System

- The curves in figure 2 are obtained from equations for undamped system, underdamped system, critically damped and overdamped system as demonstrated in the previous slides.
- Note that two second-order systems having the same **ξ** but different ω_n will exhibit the same overshoot and the same oscillatory pattern.
- Such systems are said to have the same relative stability.

Determine a unit Step Response of Second-Order System

- From figure 2, we see that an underdamped system with **ξ** between 0.5 and 0.8 gets close to the final value more rapidly than a critically damped or overdamped system.
- Among the systems responding without oscillation, a critically damped system exhibits the fastest response.
- An overdamped system is always sluggish in responding to any inputs.

Represent poles and unit step responses of a second order system

- The positions of poles in s-plane and a unit step second-order system response is shown below:

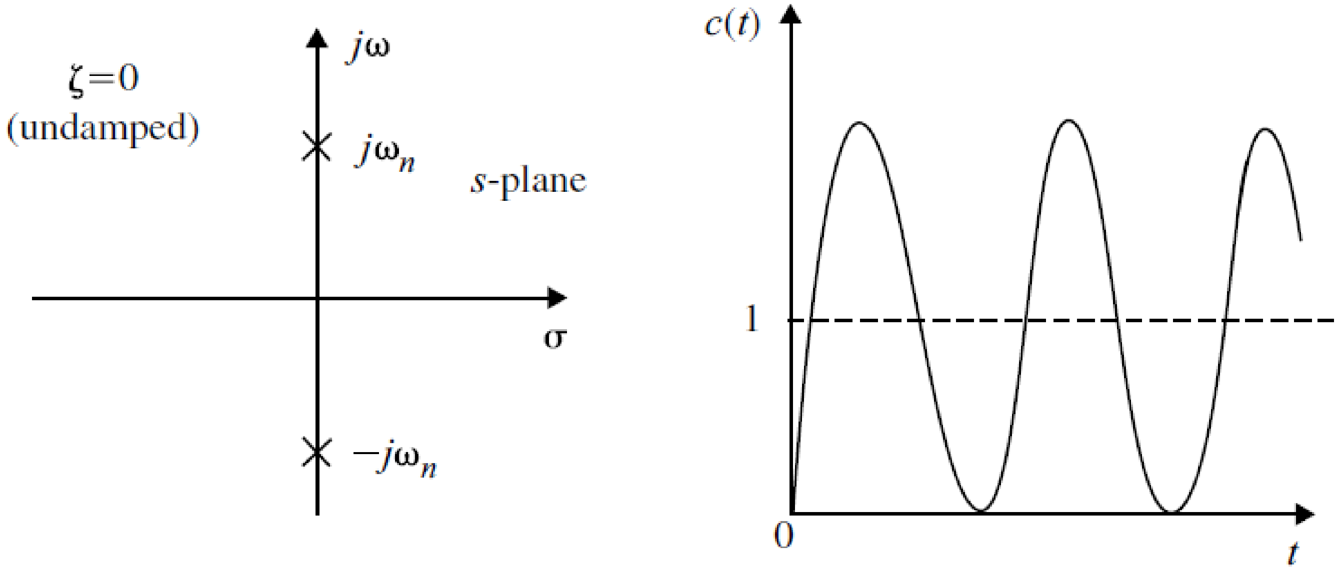

Figure 3. Position of poles in s-plane and unit step response of a second-order system. (a) undamped system.

Palani S. (2022), Automatic Control Systems: With MATLAB, 2nd Edition, Springer, page 184.

Represent poles and unit step responses of a second order system

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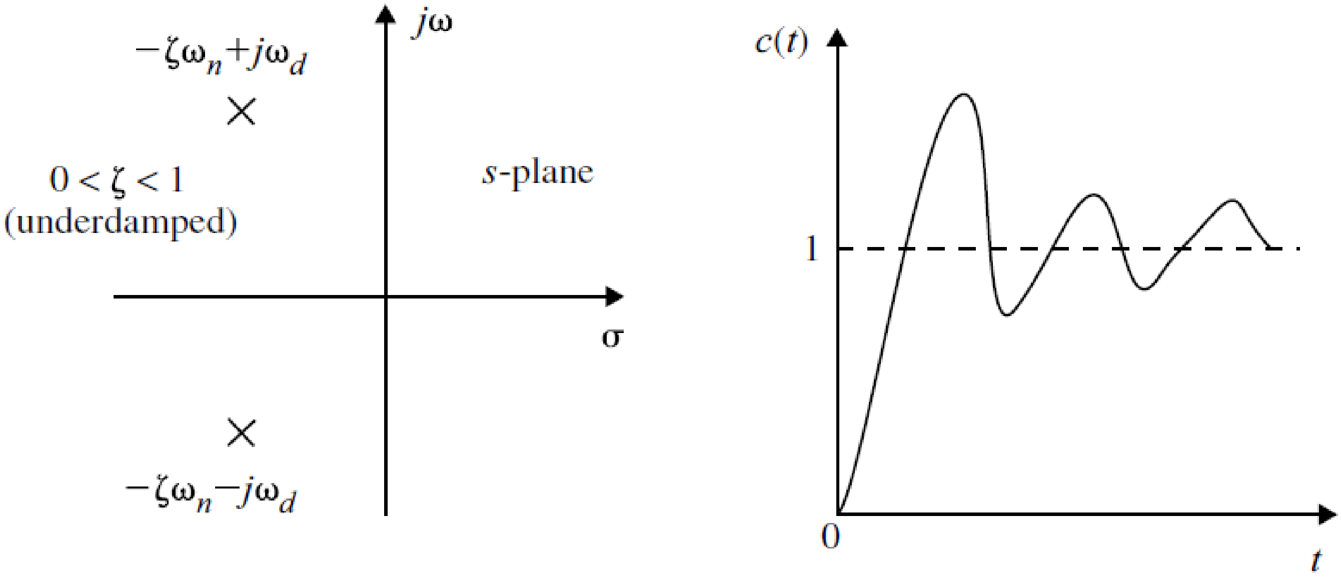

Figure 3. Position of poles in s-plane and unit step response of a second-order system. (b) underdamped system.

Palani S. (2022), Automatic Control Systems: With MATLAB, 2nd Edition, Springer, page 184.

Represent poles and unit step responses of a second order system

- The positions of poles in s-plane and a unit step second-order system response is shown below:

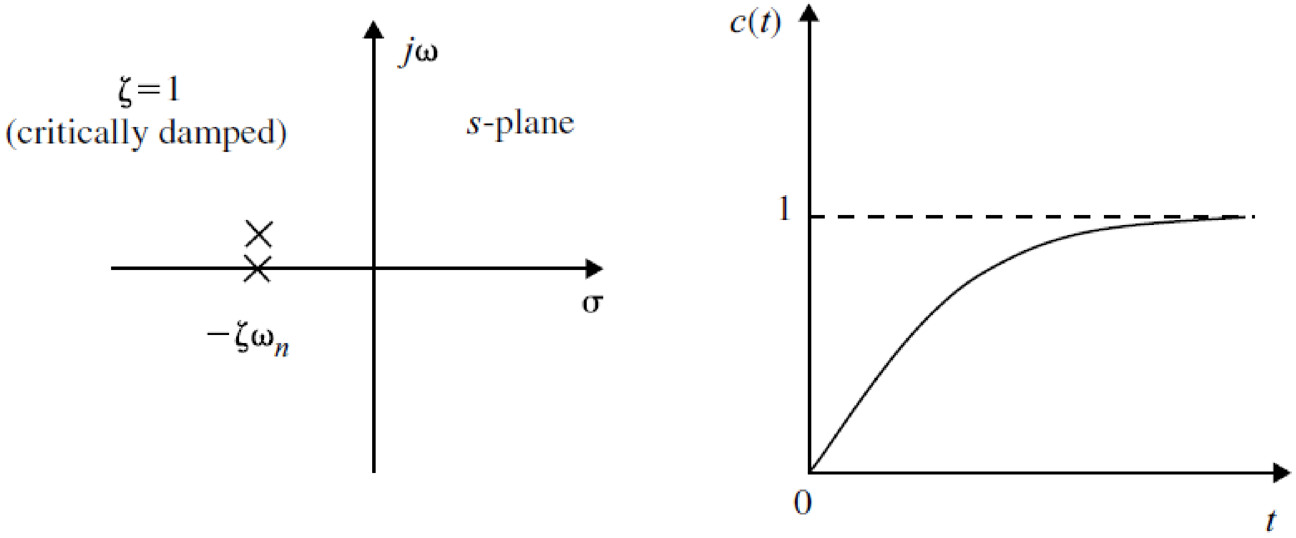

Figure 3. Position of poles in s-plane and unit step response of a second-order system. (c) critically damped system.

Palani S. (2022), Automatic Control Systems: With MATLAB, 2nd Edition, Springer, page 184.

Represent poles and unit step responses of a second order system

- ❖ The positions of poles in s-plane and a unit step second-order system response is shown below:

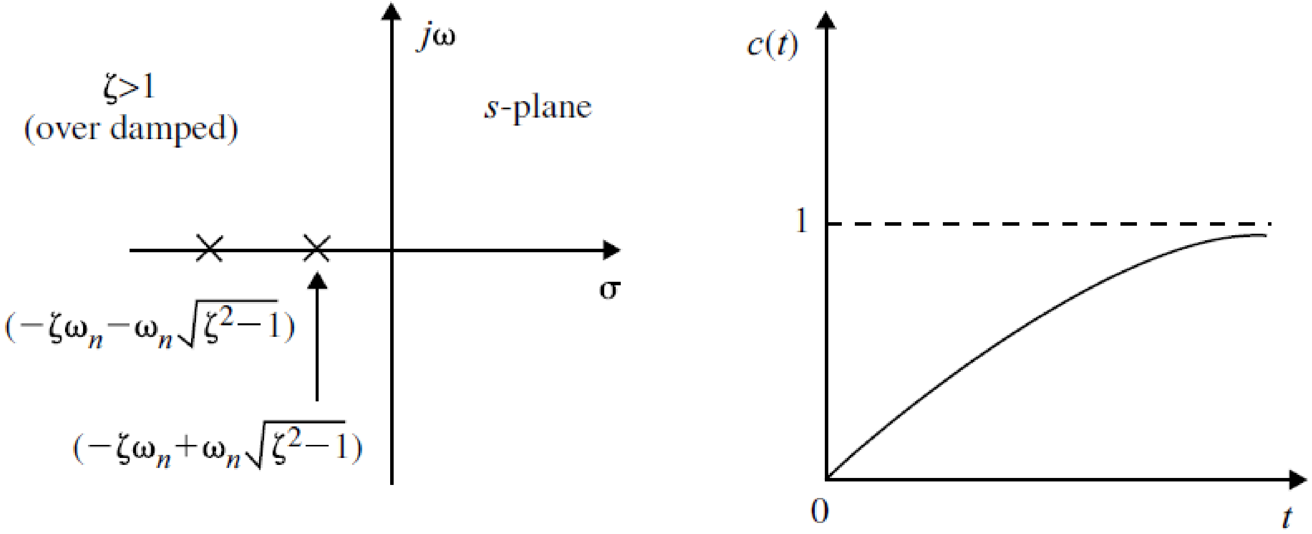

Figure 3. Position of poles in s-plane and unit step response of a second-order system. (d) overdamped system.

Palani S. (2022), Automatic Control Systems: With MATLAB, 2nd Edition, Springer, page 184.

Determine time response specifications of a second order system

- The performance of a second-order system is measured by the following specifications:

    1. Delay time, t_d
    2. Rise time, t_r
    3. Peak time, t_p
    4. Maximum overshoot (in percentage), M_p
    5. Time constant T
    6. Settling time, t_s

- These specifications are defined, for underdamped systems, in next slides and shown graphically in figure 4.

Determine time response specifications of a second order system

- ✓ **Delay time, $\boldsymbol{t_d}$**: The delay time is the time required for response to reach 50% (half) of its final value the very first time. It is given by this expression: $\boldsymbol{t_d = \frac{1+0.7\xi}{\omega_n}}$
- ✓ **Rise time,$\boldsymbol{t_r}$**: The rise time is the time required for the response to rise from 10% to 90%, 5% to 95%, or 0% to 100% of its final value.
- ✓ For **underdamped** second order systems, the **0% to 100%** rise time is normally used and it is determined by this expression : $\boldsymbol{t_r = \frac{\pi-\beta}{\omega_d}}$, where $\boldsymbol{\beta = \tan^{-1}(\frac{\omega_d}{\sigma})}$ with $\boldsymbol{\sigma = \xi\omega_n}$

Determine time response specifications of a second order system

- ✓ **Peak time, $\boldsymbol{t_p}$**: The peak time is the time required for the response to reach its first maximum overshoot. It is expressed as: $\boldsymbol{t_p = \frac{\pi}{\omega_d}}$
- ✓ **Maximum overshoot, $\boldsymbol{M_p}$** : The maximum overshoot is the maximum peak value of the response curve measured from the final value. It is expressed as: $\boldsymbol{M_p = c(t_p) - c}(\infty)$

Determine time response specifications of a second order system

- ✓ The **maximum percent overshoot** $M_p(\%)$ is expressed as:

$$\boldsymbol{M_p}(\%) = \frac{\boldsymbol{c(t_p) - c(\infty)}}{\boldsymbol{c(\infty)}} \times \mathbf{100} \quad \text{or} \quad \boldsymbol{M_p}(\%) = \boldsymbol{e}^{\boldsymbol{-(\frac{\sigma}{\omega_d})\pi}} \times \mathbf{100}$$

- ✓ The amount of the maximum (percent) overshoot directly indicates the **relative stability** of the system.
- ✓ **Time constant, *T*:** The time required for the response to reach 63.2% of its final value for the first time. It is expressed as: $\boldsymbol{T = \frac{\pi}{\sigma}}$, with $\boldsymbol{\sigma = \xi\omega_n}$, $\boldsymbol{\sigma}$ is called damping factor or attenuation.

Determine time response specifications of a second order system

- ✓ **Settling time, t_s :** The settling time is the time required for the response curve to reach and stay within a certain percentage of its final value (usually 2% or 5%).

$$t_s = 4T = \frac{4}{\sigma} = \frac{4}{\xi\omega_n} \quad \textit{(2 \% criterion)}$$

$$t_s = 3T = \frac{3}{\sigma} = \frac{3}{\xi\omega_n} \quad \textit{(5 \% criterion)}$$

- ✓ Practically, the settling time is the time required to achieve target.
- ✓ And for any control system, the settling time must be kept minimum.

Determine time response specifications of a second order system

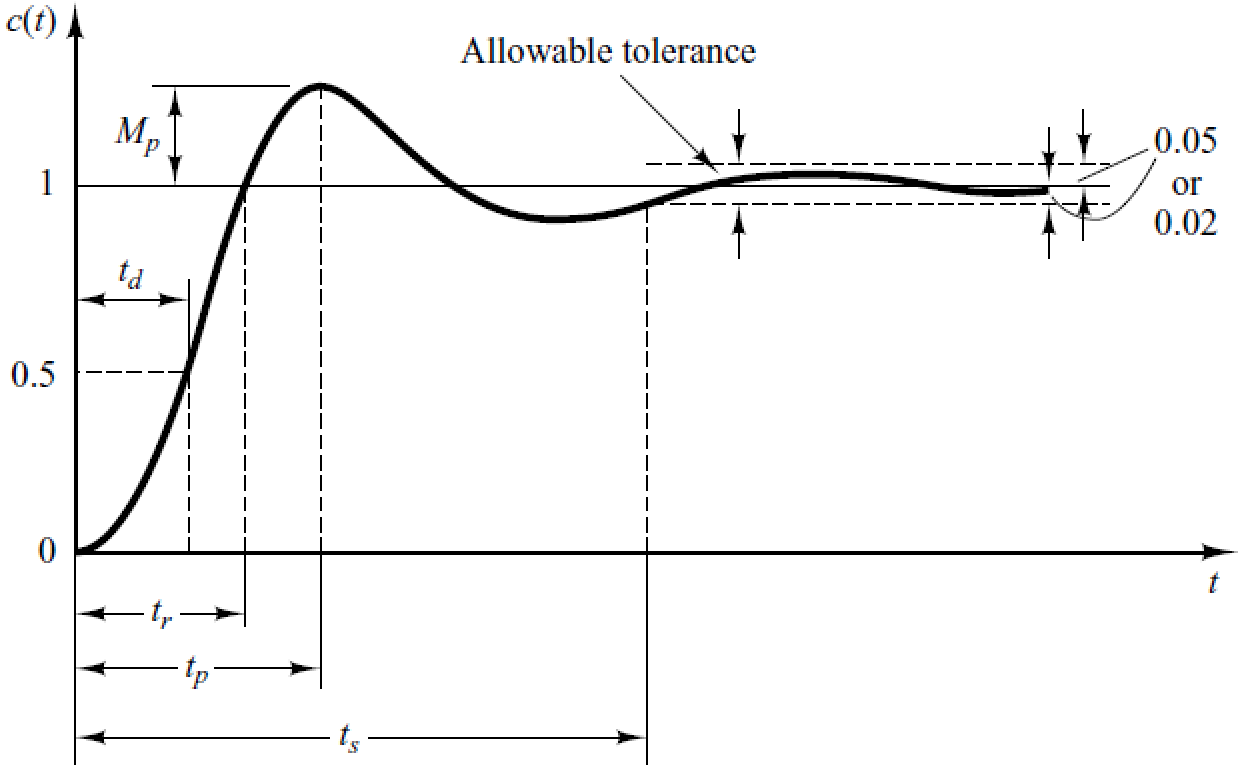

Figure 4. Unit-step response curve showing t_d, t_r, t_p, M_p,and t_s

Katsuhiko Ogata(2009), Modern Control Engineering,5th Edition, Prentice Hall, page 170.

Determine time response specifications of a second order system

❖ **Comments on time response specifications:**

- ✓ Note that not all these specifications necessarily apply to any given control system.
- ✓ For example, for overdamped system, the terms peak time and maximum overshoot do not apply.
- ✓ For systems that yield steady state errors for step inputs, this error must be kept within a specified percentage level (usually 2% or 5%).

Determine time response specifications of a second order system

- **Comments on time response specifications *(cont.)*:**

✓ Except for certain applications where oscillations can not be tolerated, it is desirable that the transient response be sufficiently fast and be sufficiently damped.

✓ Thus, the desirable transient response of a second order system, the damping ratio must be between 0.4 and 0.8.

  - Systems with small values of ξ (that is $\xi < 0.4$) yield excessive overshoot in transient response ,
  - And system with large values of ξ (that is $\xi > 0.8$) respond sluggishly.

Determine time response specifications of a second order system

❖ **Comments on time response specifications *(cont.)*:**

- ✓ The maximum overshoot and the rise time conflict with each other.
- ✓ In other words, both the maximum overshoot and the rise time can not be made smaller simultaneously.
- ✓ If one of them is made smaller, the other necessarily becomes larger.

Determine time response specifications of a second order system

- **Example:** Consider a second order control system that has parameter $\xi = 0.6$ and $\omega_n = 5\, rad/sec$. Obtain the rise time t_r , peak time t_p , maximum overshoot M_p and settling time t_s when the system is subjected to a unit-step input.

  Katsuhiko Ogata(2009), Modern Control Engineering,5th Edition, Prentice Hall, page 175.

- **Solution:**

  ✓From the given values of ξ and ω_n, we obtain:

  $\omega_d = \omega_n\sqrt{1-\xi^2}$ =4 and $\sigma = \xi\omega_n = 3$

Determine time response specifications of a second order system

- **Solution *(cont.)*:**

  ✓ **Rise time $\boldsymbol{t_r}$**: The rise time is $t_r = \frac{\pi - \beta}{\omega_d} = \frac{3.14 - \beta}{4}$,

  where β is given by $\beta = tan^{-1}\left(\frac{\omega_d}{\sigma}\right) = tan^{-1}\left(\frac{4}{3}\right) = 0.93$ rad

  The rise time t_r is this $t_r = \frac{\pi - \beta}{\omega_d} = \frac{3.14 - 0.93}{4} = 0.55$ sec

  ✓ **Peak time $\boldsymbol{t_p}$:**

  The peak time is $t_p = \frac{\pi}{\omega_d} = \frac{3.14}{4} = 0.785\ sec$

Determine time response specifications of a second order system

- **Solution *(cont.):***

  ✓ **Maximum overshoot $\boldsymbol{M_p}$:**

  The maximum overshoot is $M_p = e^{-\pi(\frac{\sigma}{\omega_d})} = e^{-\pi(\frac{3}{4})} = 0.095$

  The maximum percent overshoot is thus 9.5%

  ✓ **Settling time $\boldsymbol{t_s}$:**

  For the 2% criterion the settling time is $t_s = \frac{4}{\sigma} = \frac{4}{\xi\omega_n} = \frac{4}{3} = 1.33s$

  For the 5% criterion, the settling time is $t_s = \frac{3}{\sigma} = \frac{3}{\xi\omega_n} = \frac{3}{3} = 1s$

References

1. Palani S. (2022), Automatic Control Systems: With MATLAB, 2nd Edition, Springer.
2. Katsuhiko Ogata(2009), Modern Control Engineering, 5th Edition, Prentice Hall.

THANK YOU