Course: Regulation and control

Lecture 1: Modelling physical systems I

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Objectives of this lecture

At the end of this lecture the student able to

- Define what is regulation and control
- Understand the purpose of control system
- Types of control system
- Understand the objectives of the course
- Model simple mechanical and electrical systems

Introduction: Regulation and Control

- **Regulation and Control** refers to the process of **maintaining a desired state or outcome** in a system by automatically adjusting its inputs based on feedback about its outputs or predetermined relationship.
- **Regulation:** The *goal* (e.g., maintain a room at 72°F, keep a drone at a specific altitude).
- **Control:** The *action* taken to achieve that goal (e.g., turning the heater on/off, adjusting the drone's propeller speed).

Why we build control system?

- **Power amplification**: amplifies human effort.
  - A tiny input signal (turning a steering wheel) controls a large output power (turning tires of a heavy car).
- **Remote control**: to operate a system from a safe or convenient distance
  - Controlling a Mars rover from Earth
  - Controlling robot cars inside nuclear reactor destruction.

Why we build control system? Cont….

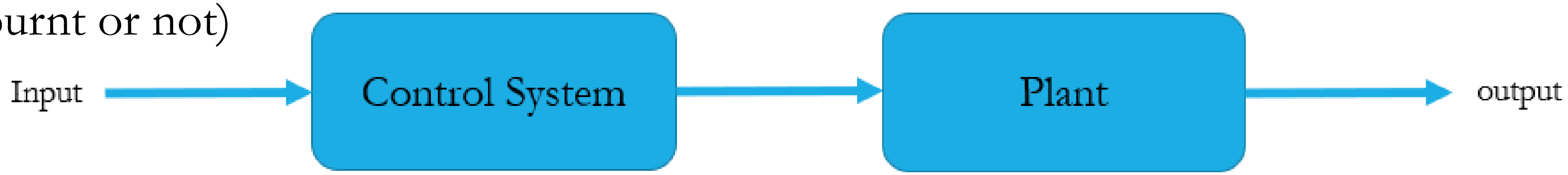
- **Convenience of input form**: to allow a simple, user-friendly input to control a complex internal process.
  - Setting a desired temperature on a thermostat instead of manually turning a furnace on and off.
- **Compensation for disturbances**: to automatically maintain performance despite of external disruptions.
  - Cruise control maintaining speed up a hill, a drone staying stable in gusty wind.

Examples of control application areas

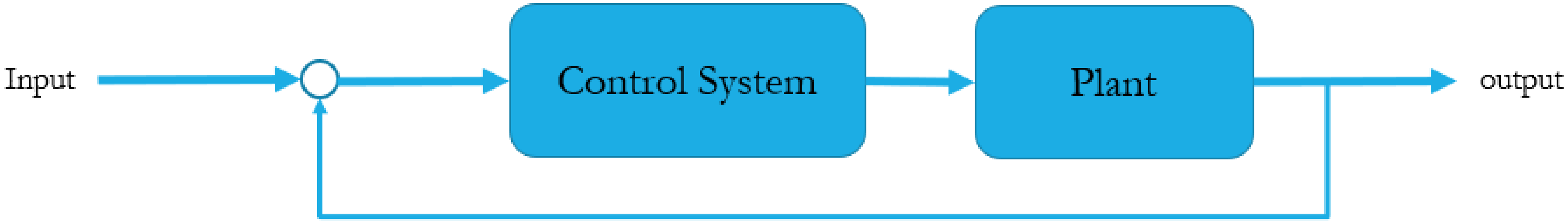
Application	Process Variable	Controller	Actuator
Cruise Control	Vehicle Speed	PID Controller	Throttle
Aircraft Autopilot	Altitude, Heading	Advanced Control Laws	Control Surfaces
Chemical Plant	Temperature, Pressure, Flow	PLC / DCS	Control Valves, Pumps
Thermostat	Room Temperature	On/Off or PID	HVAC System
Refrigerator	Internal Temperature	On/Off	Compressor
Washing Machine	Motor Speed, Water Level	Microcontroller	Motor, Water Valve

Classifications of control loops

- **Open-Loop Control:** A system **without feedback**. It executes a **pre-determined** action regardless of the output. (e.g., a toaster that runs for a set time, whether the bread is burnt or not)



- **Closed-Loop Control:** A system that uses feedback to adjust its operation automatically. (e.g., cruise control in a car).



Components of a control system

- **Process (or Plant):** The system or object to be controlled

e.g., a motor, a chemical reactor, an airplane, a room.

- **Process Variable (PV):** The current, measured value of the quantity being controlled.
- **Sensor (or Measurer):** A device that measures the current value of the output variable.

e.g., a thermometer measures temperature, a tachometer measures speed

Components of a control system Cont….

- **Setpoint (SP):** The desired target value for the output (e.g., 72°F for a thermostat).
- **Error (e):** The difference between the Setpoint and the Measured value of the Process Variable

$$\mathbf{e = SP - MV.}$$

- The controller's job is to drive this error to zero.
- **Controller:** The "brain" of the system. It compares the measured value from the sensor to the desired setpoint and calculates a corrective action.

Components of a control system Cont….

- **Actuator:** The device that executes the command from the controller to **physically** change the process

e.g., a valve that opens to allow more steam, a motor that adjusts its speed.

- **Disturbance :** An external input that can cause the process variable to deviate from the setpoint

e.g., a door opening on a cold day lets cold air into a temperature-controlled room.

Open loop process

- **Input (Command):** The user or a program provides a specific input or command. This is the "do this" instruction.
- **Controller:** The controller receives this input. Based on its pre-programmed design or calibration, it calculates the necessary signal to send to the actuator.
- **Actuator:** The actuator (e.g., a motor, heater, or valve) converts the controller's signal into a physical action.
- **Process / System**: The process receives this action and produces an output.
- **Output (Result**): The system achieves a result. This output is never measured or fed back to the controller for comparison.

Closed loop process

- **Sense:** A sensor measures the current output of the system This is the **Measured Variable**.
- **Compare:** The measured value is compared to the desired value (the **Setpoint** or **Reference**). The difference between them is called the **Error**.
- **Compute:** A controller calculates the necessary action needed to reduce the error to zero. This is where control strategies (like PID control) are applied.
- **Actuate:** An actuator (e.g., a valve, motor, heater) executes the command from the controller, adjusting the system's input.
- **Repeat:** This process happens continuously, keeping the system regulated at the setpoint despite external disturbances.

Comparisons of open loop and closed loop

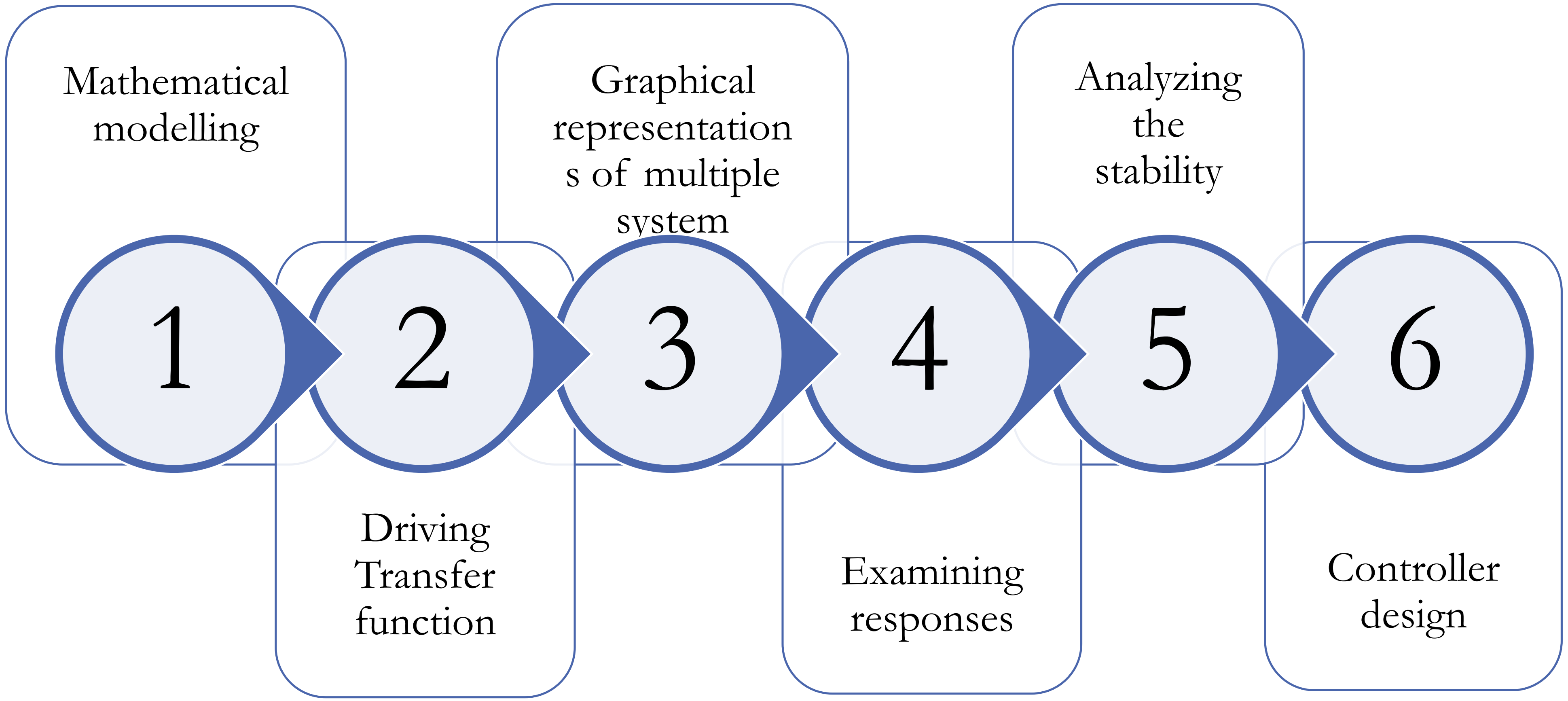
Feature	Open-Loop Control System	Closed-Loop Control System
Key Components	Controller, Actuator, Process.	Controller, Actuator, Process, Sensor, Error Detector (Comparator).
Effect of Disturbances	Cannot compensate.	Can compensate.
Accuracy	Less accurate.	Highly accurate.
Stability	Inherently stable.	Can be unstable.
Design Complexity	Simple to design and build.	More complex to design
Cost	Generally cheaper	More expensive
Calibration	Requires precise calibration	Calibration is less critical
Applications	Toasters, washing machines (timed cycles), traffic light controllers, fan speed switches.	Thermostats, cruise control, autopilot, robot navigation, human body (homeostasis).

When to use open loop and closed loop

- Use an **Open-Loop** system when:
  - The input-output relationship is **known precisely** and is constant.
  - There are **few or no external disturbances**.
  - **Cost is a major factor**, and high accuracy is not critical.
  - The task is simple and repetitive.

- Use a **Closed-Loop** system when:
  - **High accuracy** and **precision** are required.
  - **Unpredictable disturbances** are expected.
  - The system parameters might **change over time**.
  - The task is complex and the outcome must be guaranteed.

Layout of the course



- Application and case studies
- Simulation of mechanical and electrical systems

Mathematical modelling of physical systems

Mathematical modelling

- **Mathematical model**: is a representation of a physical system using a mathematical equation is called mathematical modelling.
- We create mathematical representations of physical systems to predict their behavior without building them first.
- **Why we model?**
  - Analyze performance.
  - Predict response.
  - Design controllers before building the real system.

Basic physical systems

Physical systems can be

- Mechanical (Spring-Mass-Damper systems)

  Governing equation newton 2nd law: $\sum F = ma$

- Electrical (R-L-C circuits) and

  Governing equation KVL and KCL

- Electro-Mechanical systems

  Electrical and mechanical systems related by $e_b = k_b \omega$ *and* $T_m = k_t i_a$

Modelling physical systems I:

Spring-Mass-Damper(translational)

- **Spring (Stiffness, K):**

Follows **Hooke's Law**. The force it exerts is proportional to its displacement (x) from equilibrium.

$$\mathbf{F_{spring} = F_s = -Kx}$$

(The negative sign indicates it's a *restoring* force).

Any elastic materials can be modelled as spring.

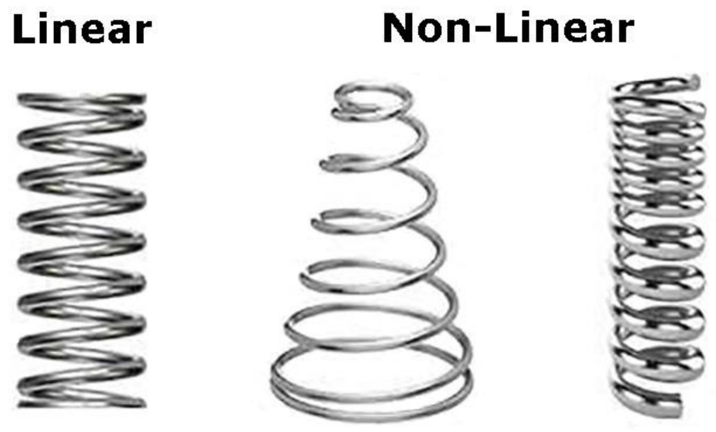


Figure 1. linear spring follows Hooke's law [1]

url: https://www.motioncontroltips.com/wp-content/uploads/2020/08/Linear-and-Non-linear-Springs-Feature.jpg

Modelling physical systems I:

Spring-Mass-Damper(translational)

- **Damper: Dashpot (Damping coefficient, B):** Provides a force proportional to **velocity** ($v = \dot{x}$).

$\boldsymbol{F}_{\mathbf{damper}} = \boldsymbol{F}_{\mathbf{d}} = \mathbf{Bv} = -\boldsymbol{B}\dot{\boldsymbol{x}}$. (It opposes motion, like shock absorbers).

Frictional surface and linear bearing are modelled as dampers.

- **Mass (Inertia, M):** Obeys **Newton's Second Law**. The sum of all forces equals mass times acceleration.

$\mathbf{\Sigma F = ma} = -\boldsymbol{m}\dot{\boldsymbol{v}} = \mathbf{m}\ddot{\boldsymbol{x}}$

Modelling physical systems I: Cont….

Spring-Mass-Damper(translational)

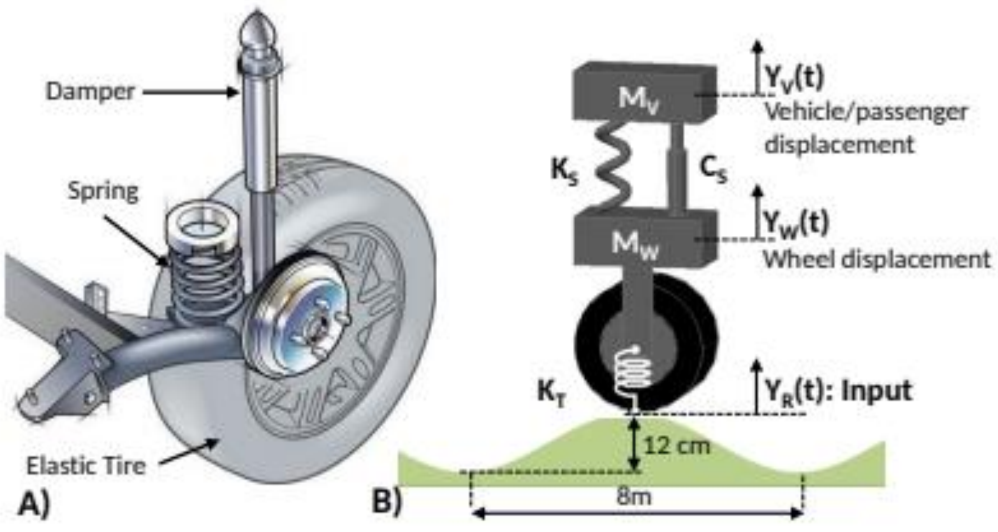


Figure 2: A typical suspension system of a vehicle; typical example of spring-mass-damper system [2]

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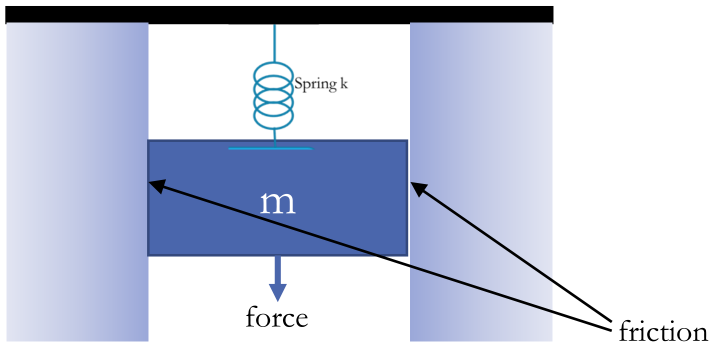
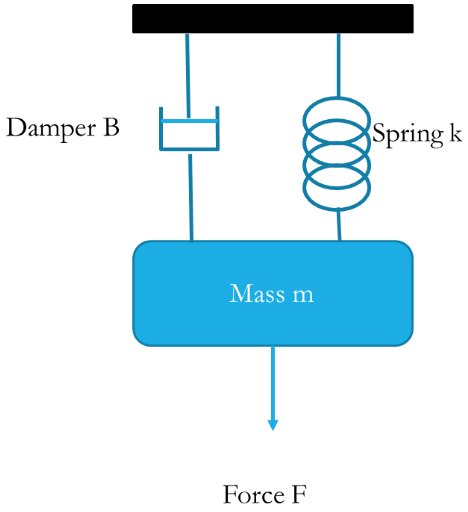
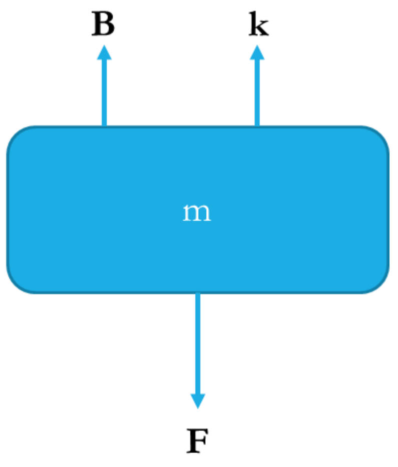




Modelling physical systems I: Cont….

Spring-Mass-Damper(translational)



$$\sum F = ma$$

$F - F_b - F_s = ma$

$F = ma + F_b + F_s$

$F = ma + Bv + kx$

$$F = m\frac{d^2x}{dt^2} + B\frac{dx}{dt} + kx$$

$$F = m\frac{dv}{dt} + Bv + k\int v\,dt$$

$$a = \frac{dv}{dt} = \frac{d^2x}{dt^2}$$

Or

$$x = \iint a\,d^2t = \int v\,dt$$

Where:

F-applied force

F_b-force by damper

F_s-force of spring

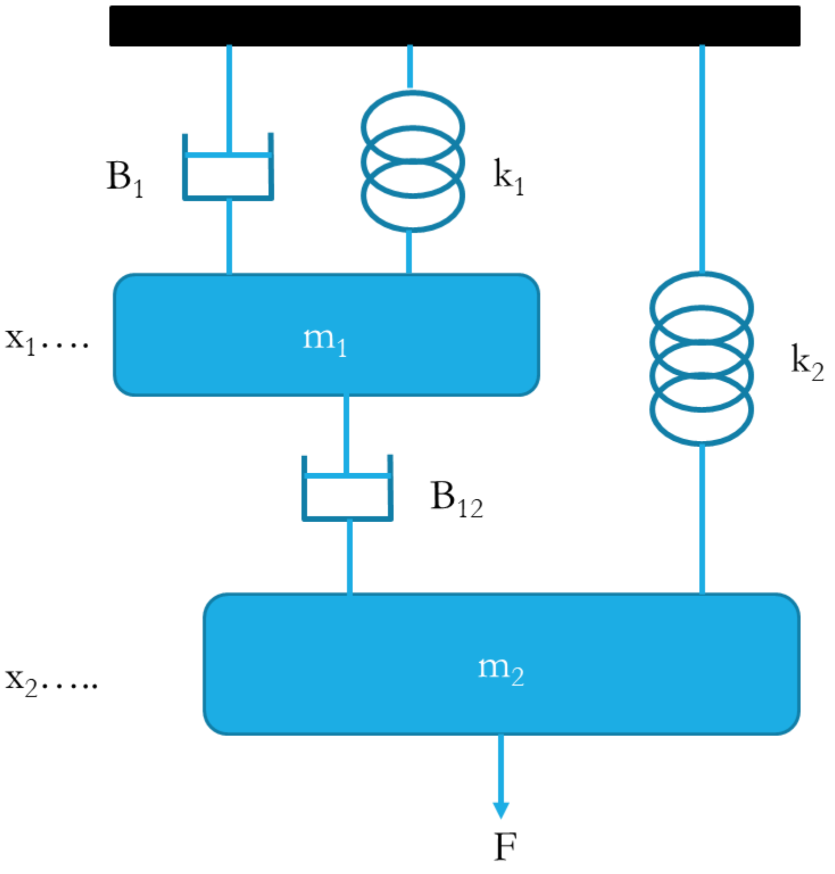
a-acceleration

v- velocity

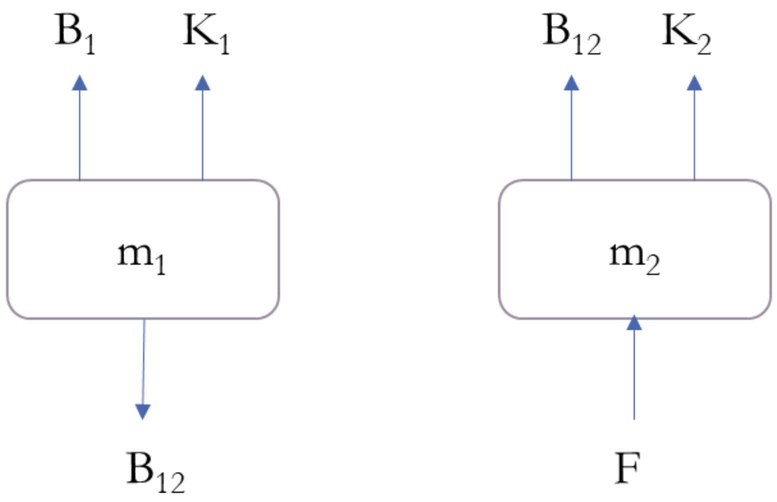
x- displacement

Modelling physical systems I: Cont….

Spring-Mass-Damper(translational) *example*



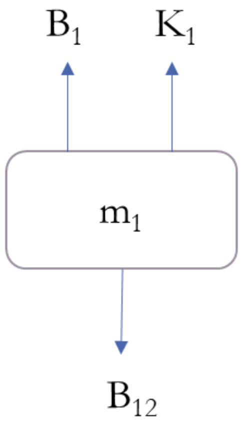
- We have two masses let's start by sketching free body diagram for each mass



Modelling physical systems I: Cont....

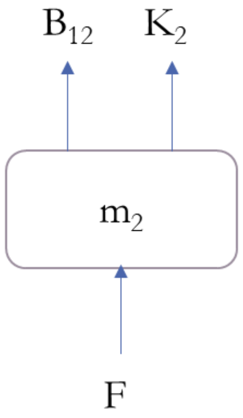
Spring-Mass-Damper(translational) *example*

- *Assuming acceleration due to the force applied is downward*
- $\sum F = ma$ @ *mass 1*



$$-B_1 v_1 - K_1 x_1 - B_{12}\, v_{12} = m_1 a_1$$

$\sum F = ma$ @ *mass 2*

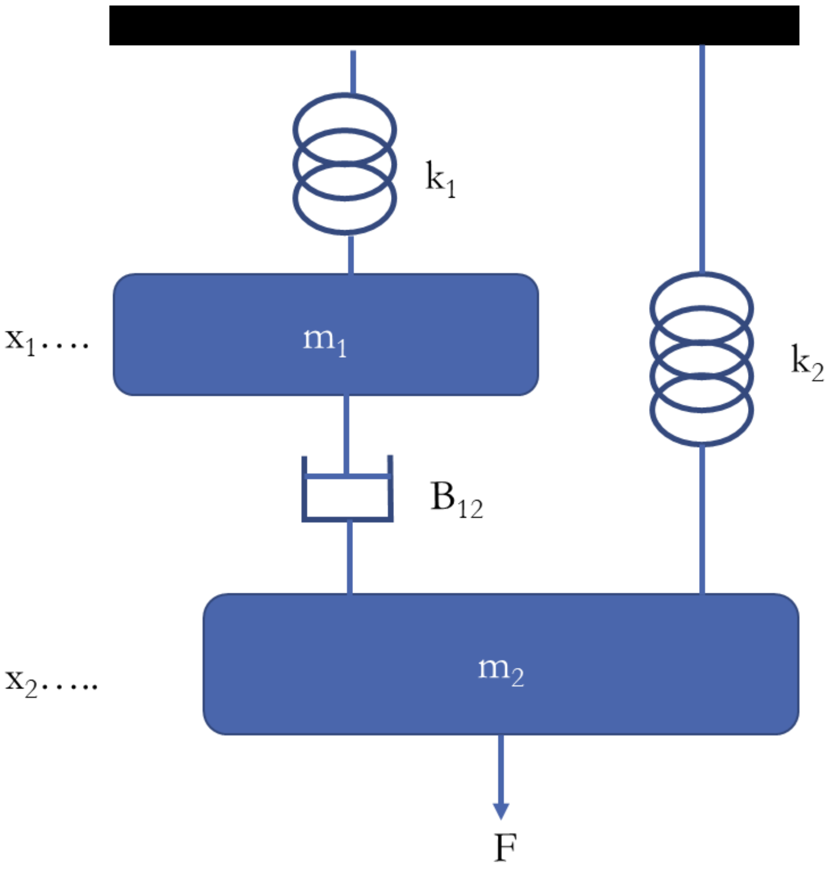


$$F - K_2 x_2 - B_{12}\, v_{12} = m_2 a_2$$

Where v_{12} relative velocity between mass 1 and mass 2

Modelling physical systems I: Cont....

Spring-Mass-Damper(translational) *exercise*



For mass 1

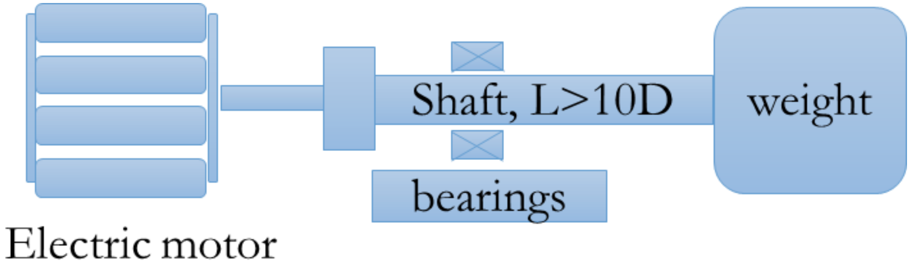
$-K_1x_1-B_{12}\, v_{12}=m_1a_1$

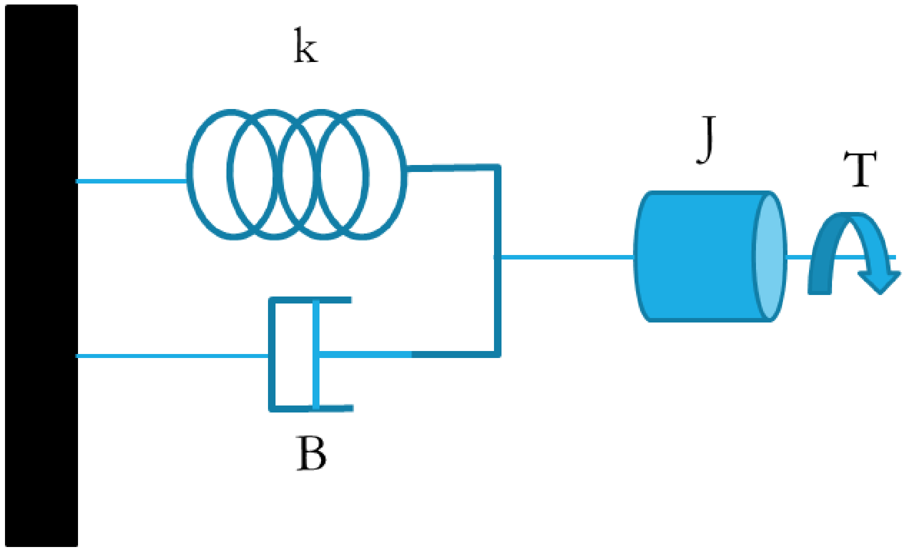
For mass 2

- $F-K_2x_2-B_{12}\, v_{12}=m_2a_2$

Modelling physical systems I:

Spring-Inertia-Damper(rotational)





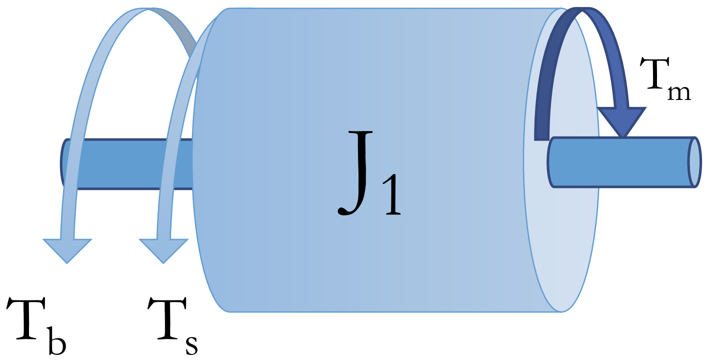
An electric motor source of torque (T) and angular speed (ω) connected directly to inertial load (J)

K- **Torsional stiffness** of the shaft connecting the motor and the load.

B-**Damping coefficient** of a bearing on which the shaft is mounted on.

Modelling physical systems I: cont….

Spring-Inertia-Damper(rotational)



$\sum T = J\alpha$

T-Tb-Ts=$J\alpha$

$T = J\alpha + T_b + T_s$

$T = J\alpha + B\omega + k\theta$

$$T = J\frac{d^2\theta}{dt^2} + B\frac{d\theta}{dt} + k\theta$$

$$\alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$$

Or

$$\theta = \iint \alpha\, d^2t = \int \omega\, dt$$

Where:

T-applied torque

T_b-torque resistance by damper

T_s-torque resistance by spring

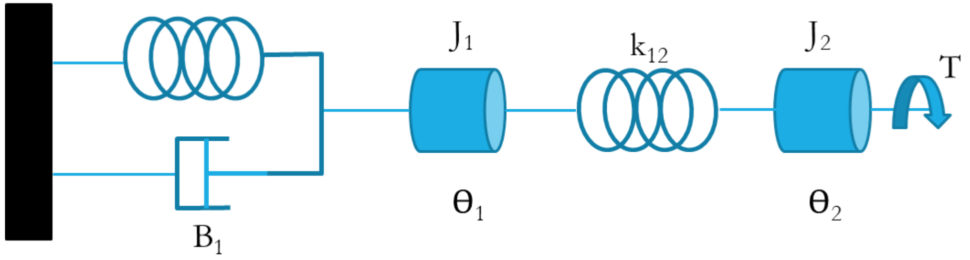
α-angular acceleration

ω- angular velocity

θ- angular displacement

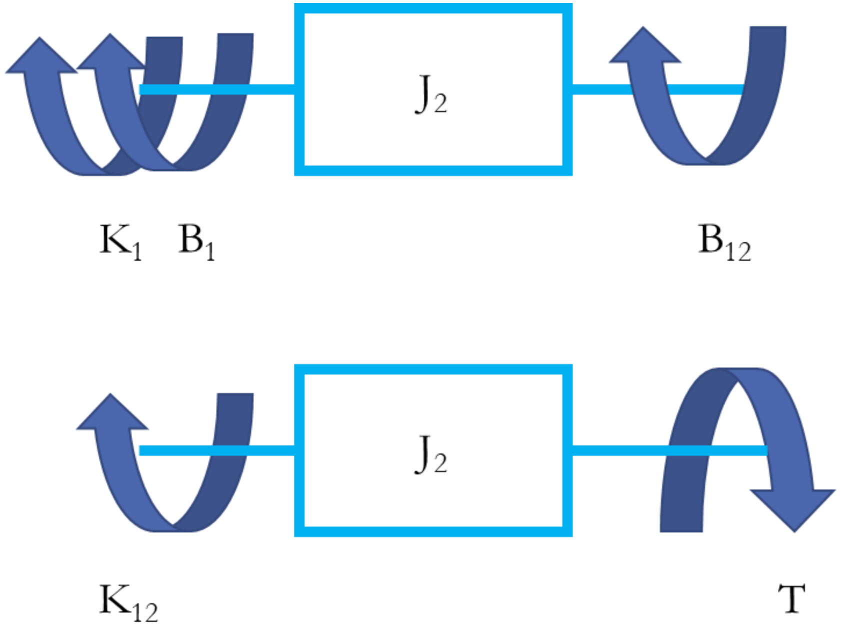
Modelling physical systems I:

Spring-Inertia-Damper(rotational) *example*



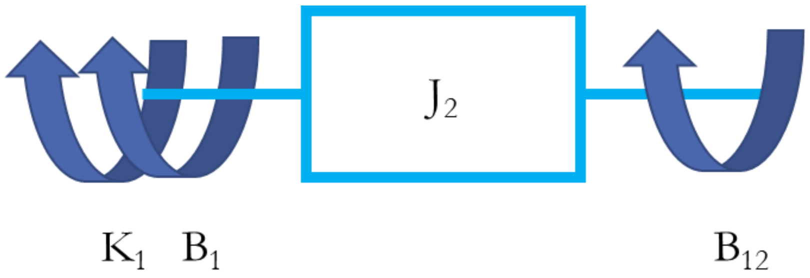
In this mechanical rotational system there are two masses(inertial) which are connected by a shaft subjected to a torsional torque.

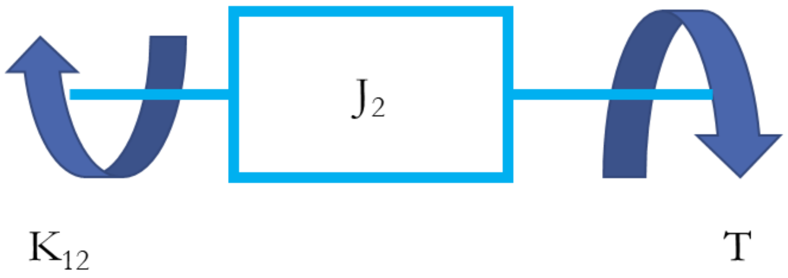
Lets start by sketching FBD



Modelling physical systems I: cont….

Spring-Inertia-Damper(rotational) *example*





$\sum T = J\alpha$

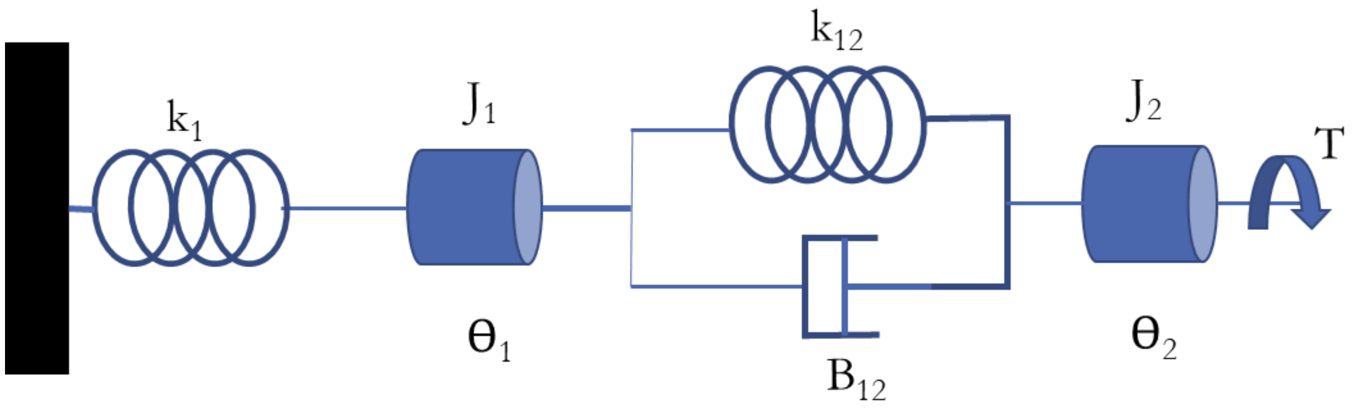
$-K_1\theta_1 - B_1\omega_1 - K_{12}\theta_{12} = J_1\alpha_1$

$\sum T = J\alpha$

$-K_{12}\theta_{12} = J_2\alpha_2$

Modelling physical systems I:

Spring-Inertia-Damper(rotational) *exercise*



For inertial mass 1

$-K_1\theta_1-K_{12}\theta_{12}-B_{12}\omega_{12}=J_1\alpha_1$

For inertial mass 2

$T-K_{12}\theta_{12}-B_{12}\omega_{12}=J_2\alpha_2$

Modelling physical systems I:
basic elements in Resistor(R)-Inductor(L)-Capacitor(C) circuits

- **Resistor (R):** The voltage drop across it is proportional to current (i). $V_R = iR$
- **Inductor (L):** Opposes change in current. Voltage drop is proportional to the *derivative* of current. $V_L = L\frac{\mathrm{di}}{\mathrm{d}t}$
- **Capacitor (C):** Stores charge (q). Voltage drop is proportional to the integral of current (or charge divided by capacitance). $V_c = \frac{1}{C}\int \mathrm{i}\,\mathrm{d}t = \frac{q}{C}$

Modelling physical systems I:

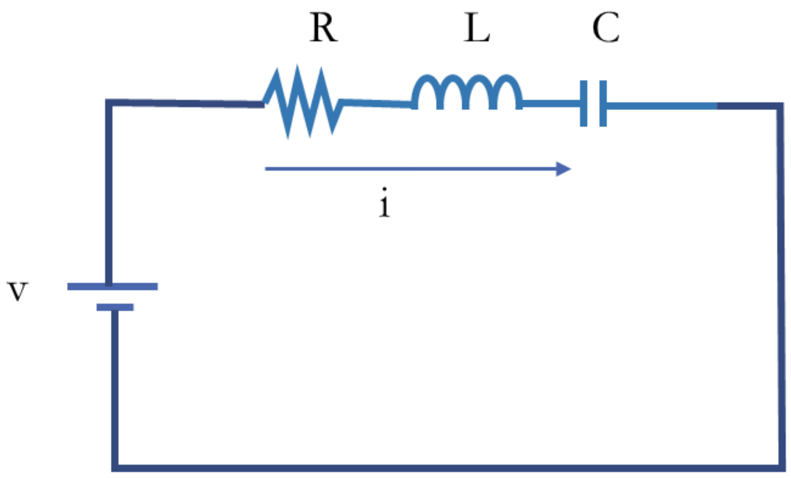
Series vs Parallel electrical circuits

- **Kirchhoff's Voltage Law (KVL)** - "The Law of Conservation of Energy“
- The sum of all voltage drops around any closed loop in a circuit is zero. This works for elements in series.
- **Kirchhoff's Current Law (KCL)** - "The Law of Conservation of Charge"
- The sum of all currents entering any node (junction) in a circuit equals the sum of all currents leaving that node. This is for elements in parallel.

Modelling physical systems I:

Series vs Parallel electrical circuits

- Series circuit

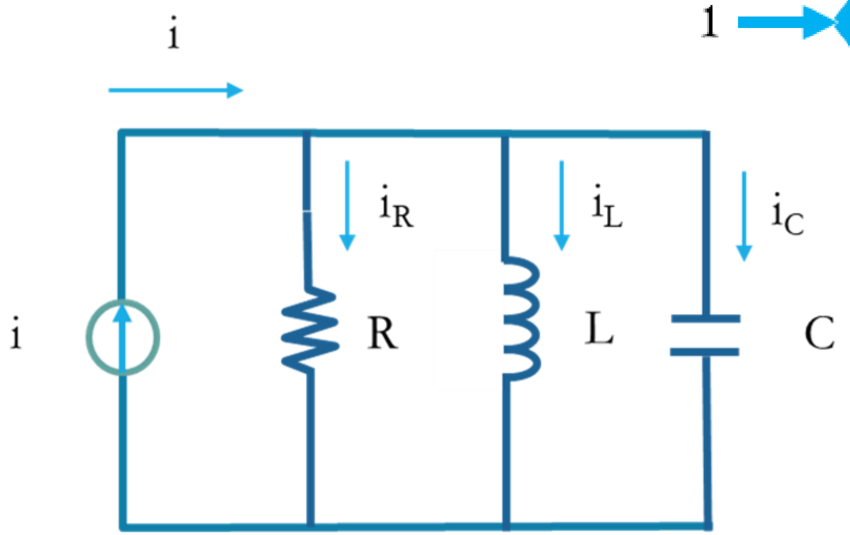


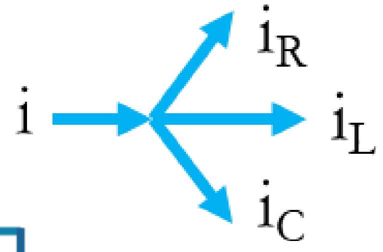
$$v_s - v_R - v_L - v_C = 0$$

$$v_s = v_R + v_L + v_C$$

$$v_s = iR + L\frac{di}{dt} + \frac{1}{C}\int i\,dt$$

- Parallel circuit



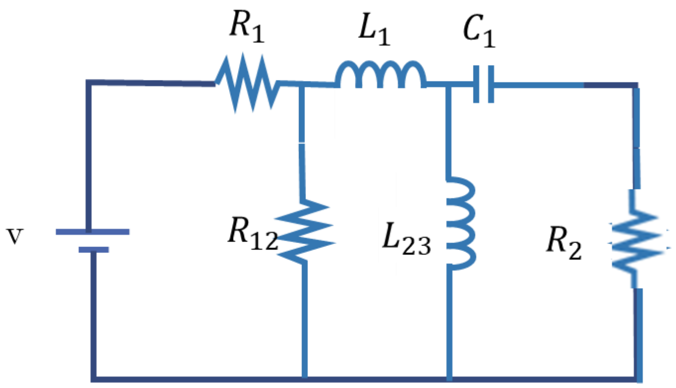


$$i = i_R + i_L + i_C$$

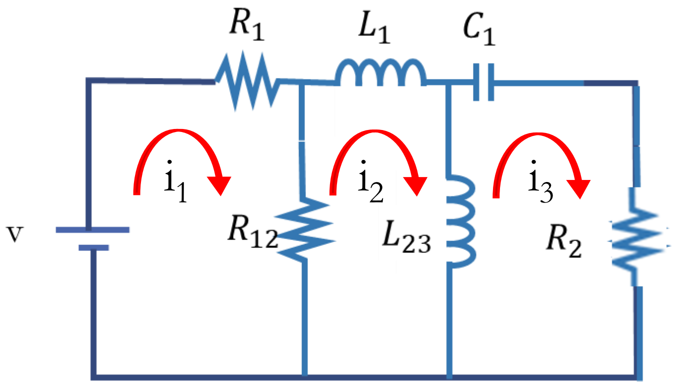
$$i = v\frac{1}{R} + \frac{1}{L}\int v\,dt + c\frac{dv}{dt}$$

Modelling physical systems I:

Resistor(R)-Inductor(L)-Capacitor(C) circuits *example*



For this circuit with multiple loops, we apply KVL to each loop independently



Loop 1: $v - v_{R1} - v_{R12} = 0$

$$v = v_{R1} + v_{R12}$$

$$v = R_1 i_1 + R_{12} i_{12}$$

Loop 2: $-v_{R12} - v_{L1} - v_{L1} = 0$

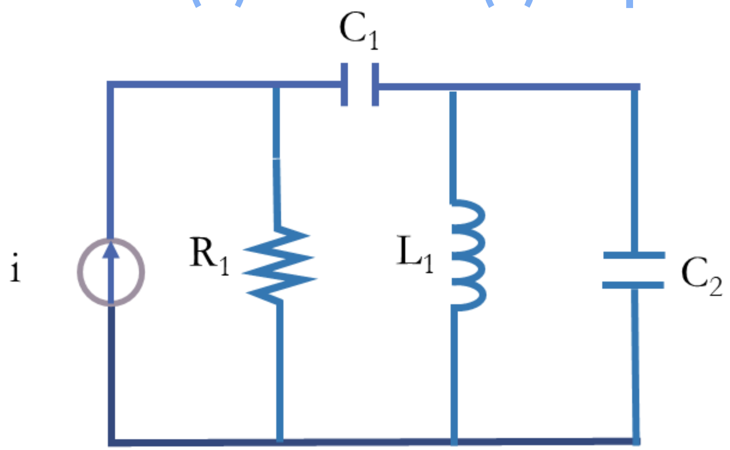
$$-R_{12} i_{12} - L_1 \frac{di_2}{dt} - L_{23} \frac{di_{23}}{dt} = 0$$

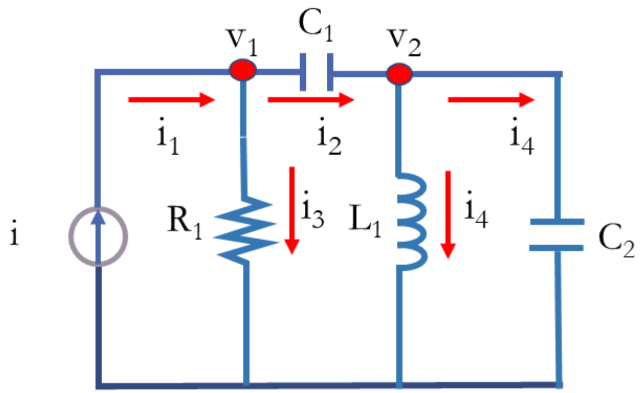
Loop 3: $-v_{L23} - v_{C1} - v_{R2} = 0$

$$- L_{23} \frac{di_{23}}{dt} - \frac{1}{C_1} \int i_3 dt - i_3 R_2 = 0$$

Modelling physical systems I:

Resistor(R)-Inductor(L)-Capacitor(C) circuits *example*





If the input to a circuit is a current source, it's often easier to use KCL instead of KVL. We define nodes—points where elements connect and apply KCL at each node

Node 1: $i = i_1 = i_2 + i_3$

$$i = c_1 \frac{dv_{21}}{dt} + \frac{v_1}{R_1}$$

Node 2: $i2 = i4 + i5$

$$C_1 \frac{dv_{21}}{\alpha_t} = C_1 \frac{dv_{21}}{\alpha_t} + \frac{1}{L_1} \int v_2 \, dt$$

Modelling physical systems I:

Electro-mechanical system (DC-motor)

- The real world isn't just electrical or just mechanical. It's both. A DC motor is a perfect example—it converts electrical energy into mechanical motion. Our job is to model this coupling.
- Here we are going to model a DC motor as an electromechanical system that combines the RLC circuit and spring-mass-damper analogies into a single, coupled system.

Modelling physical systems I: Cont….

Electro-mechanical system (DC-motor)

- **Electrical Side:** The input is a voltage (V). The motor's armature has R and L.
- R-resistance of motor armature winding wire, L- inductance caused by a the coil(winding) of the motor armature.
- **Mechanical Side:** The output is the shaft's angular position (θ) or speed (ω). It has rotational inertia J and B.
- J-inertia caused by the mass of the armature or external load connected to the motor directly. B-damping coefficient which can be bearing or friction between armature shaft and the motor casing.

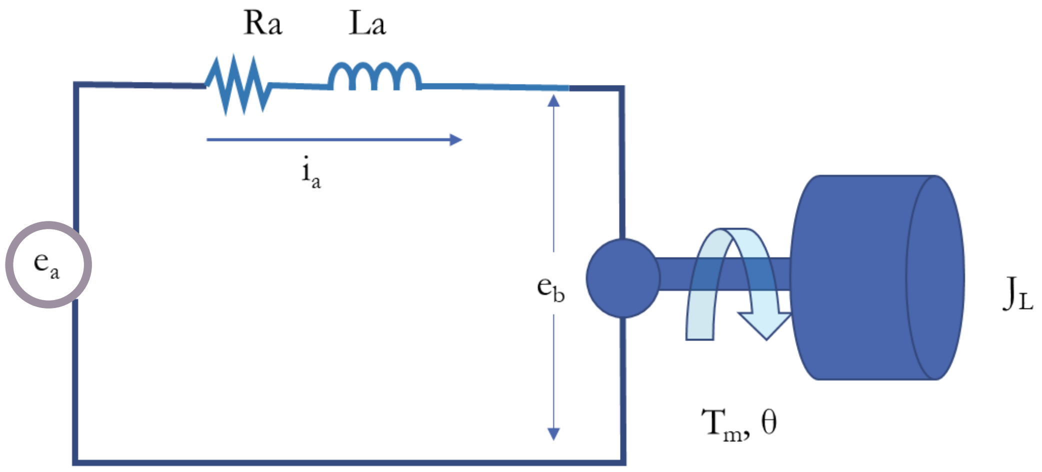
Modelling physical systems I: Cont.…

Electro-mechanical system (DC-motor)

- **Coupling mechanical and electrical parts**
  - **Motor Torque (Electrical -> Mechanical):** When current (i) flows through the armature in a magnetic field, it generates a torque.
  - $T_m = k_t$i. (k_t is the motor torque constant).
  - **Back-EMF (Mechanical -> Electrical):** When the motor spins, it acts like a generator, creating a voltage that opposes the applied voltage.
  - $e_\mathrm{b} = k_b\omega$. (k_b is the back-emf constant). (Note: In SI units, $k_b = k_t$).

Modelling physical systems I:

Electro-mechanical system (DC-motor)



A DC motor has electrical and mechanical systems.

For the electrical part we have

$$\sum v=0$$

$$e_a - R_a i_a - L_a \frac{di_a}{dt} - e_b = 0$$

- For the mechanical part

$$\sum T = Ja$$

$$T_m - B_m \omega_m - K_m \theta_m = J_m a_m + J_L a_L$$

- But $\alpha_m = \alpha_L$ (load directly connected to motor shaft)

$$T_m - B_m \omega_m - K_m \theta_m = a_m (J_m + J_L)$$

- To relate the electrical and mechanical part

$$e_b = k_b \omega \quad and\ T_m = K_t i_a$$

Summary

- Regulation and control is a process of maintaining a desired output of a system by automatically adjusting the input based on predetermined relationship (open loop) or by using a feedback from the output(closed loop).
- We need to represent physical system by a mathematical equation so that we can compute or simulate its response without a physical system. The process of deriving an equation that represent a physical system is called mathematical modelling.
- While modelling mechanical system we use Newton second law as a governing equation and we use KVL or KCL for electrical systems based on the source (i.e. voltage or current)

Home work

- Identify a control system—it could be your oven, your car's cruise control, an automatic door or any system with a control mechanism.
- Try to identify its
  - Process variable
  - Sensor,
  - Controller,
  - Actuator and Possible disturbances

Next class

- Mathematical modelling of gear and lever system
- Electrical and mechanical analogies will be discussed.

References

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Thankyou

for your attention