

Answer key

Final Exam: Regulation and Control - Answer Key

Section A: Fundamental Concepts & Multiple Choice (25 Marks)

A1. Multiple Choice (10 Marks)

1. c) Measure the current value of the output variable. (1 mark)
2. d) Open-loop system (1 mark)
3. b) Damping Ratio (ζ) (1 mark)
4. d) There are no sign changes in the first column of the Routh array. (1 mark)
5. c) The additional phase lag required to make the system marginally stable, measured at the gain crossover frequency. (1 mark)
6. c) Increase the phase margin and speed up the transient response. (1 mark)
7. c) The magnitude plot crosses 0 dB. (1 mark)
8. d) Open-loop poles. (1 mark)
9. c) Eliminate steady-state error. (1 mark)
10. c) The number of poles and zeros to its right is odd. (1 mark)

A2. Short Answer Questions (15 Marks)

1. Definitions and Examples (5 marks)

- **Regulation:** Maintaining a process variable at a constant setpoint despite disturbances (1 mark). Example: Thermostat maintaining room temperature (1 mark).
- **Control:** Making a process variable follow a changing setpoint or trajectory (1 mark). Example: Cruise control maintaining vehicle speed (1 mark).
- **Clear distinction between concepts** (1 mark).

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2. Phase Margin Effects (5 marks)

- **Too much PM:** System becomes overly sluggish with slow response (2.5 marks)
- **Too little PM:** Excessive oscillations and poor stability (2.5 marks)

3. Angle Condition (5 marks)

- For a point s to lie on the root locus: $\sum \angle(s - \text{poles}) - \sum \angle(s - \text{zeros}) = 180^\circ + 360^\circ k$ (5 marks)
 - Where k is any integer (included in full marks)
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Section B: Analytical Problems (50 Marks)

B1. Mathematical Modeling & Transfer Function (10 Marks)

a) Differential Equation (5 marks)

- Apply KVL: $v_i(t) = v_{R1} + v_{L1} + v_c + v_{R2}$ (2 marks)
- Substitute component relationships:

$$v_{R1} = iR_1; v_{L1} = L_1 \frac{di}{dt}; v_c = \frac{1}{C_1} \int i dt; v_{R2} = iR_2 \quad (2 \text{ marks})$$

- Final equation: $v_i = L_1 C_1 \frac{d^2 v_c}{dt^2} + (R_1 + R_2) C_1 \frac{dv_c}{dt} + v_c$ (1 mark)

b) Transfer Function (5 marks)

- Take Laplace transform with zero initial conditions (2 marks)
- $G(s) = \frac{V_c(s)}{V_i(s)} = \frac{1}{s^2 L_1 C_1 + s(R_1 + R_2) C_1 + 1}$ (3 marks)

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B2. Time Domain Analysis (10 Marks)

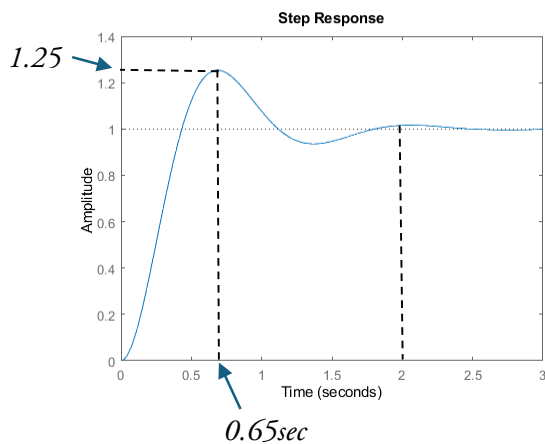
a) Parameters (3 marks)

- $\omega_n = 5 \text{ rad/s}$ (1 mark)
- $\zeta = 0.4$ (1 mark)
- $\omega_d = \omega_n \sqrt{1 - \zeta^2} = 4.583 \text{ rad/s}$ (1 mark)

b) Performance Metrics (4 marks)

- $T_p = \frac{\pi}{\omega_d} = 0.685 \text{ s}$ (1 mark)
- $\%OS = e^{-\frac{\zeta\pi}{\sqrt{1-\zeta^2}}} \times 100\% = 25.4\%$ (1.5 marks)
- $T_s = \frac{4}{\zeta\omega_n} = 2.0 \text{ s}$ (1.5 marks)

c) Sketch (3 marks)



- Underdamped response shape (1 mark)
- Label peak overshoot, peak time, settling time (1 mark each)

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B3. Stability Analysis (10 Marks)

a) Routh Array (5 marks)

text

$$s^4: 1 \quad 11 \quad K$$

$$s^3: 6 \quad 6 \quad 0$$

$$s^2: 10 \quad K$$

$$s^1: \frac{60-6K}{10}$$

$$s^0: K$$

(3 marks for correct array, 2 marks for calculations)

b) K Range (5 marks)

- From s^1 row: $\frac{60-6K}{10} > 0 \Rightarrow K < 10$ (2.5 marks)
- From s^0 row: $K > 0$ (2.5 marks)
- **Final answer: $0 < K < 10$**

B4. Bode Plot & Stability Analysis (10 Marks)

a) PM and GM Calculation (6 marks)

- Convert to standard form: $G(s) = \frac{10}{(s+1)(0.2s+1)}$ (1 mark)
- Find ω_{gc} where $|G(j\omega)| = 1 \Rightarrow \omega_{gc} \approx 6.2$ rad/s (2 marks)
- Phase at ω_{gc} : $\varphi \approx -132^\circ \Rightarrow PM = 48^\circ$ (2 marks)
- Phase never reaches $-180^\circ \Rightarrow GM = \infty$ (1 mark)

b) Stability (2 marks)

- **Stable** because $PM > 0$ and $GM > 0$ (2 marks)

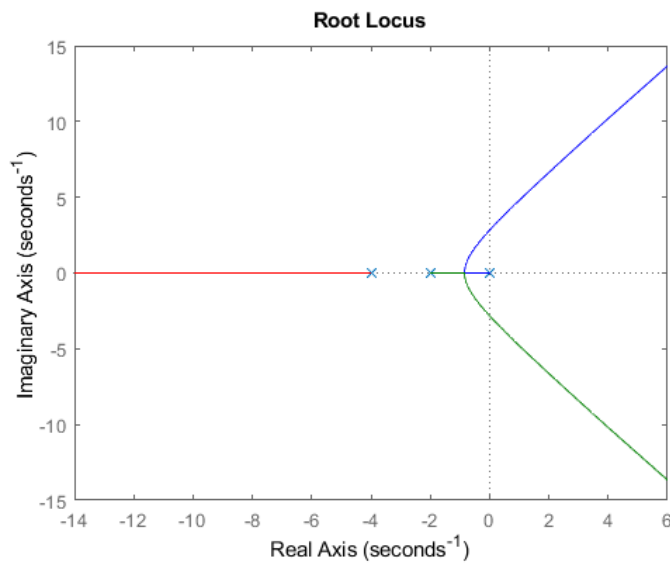
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c) K Range (2 marks)

- $0 < K < \infty$ since phase never crosses -180° (2 marks)

B5. Root Locus Construction (10 Marks)

a) Sketch (7 marks)



- Poles at $s = 0, -2, -4$; No zeros (1 mark)
- Real-axis segments: $(-\infty, -4)$ and $(-2, 0)$ (1 mark)
- Asymptotes: $\sigma = -2$, angles = $\pm 60^\circ, 180^\circ$ (2 marks)
- Breakaway point at $s \approx -0.845$ (2 marks)
- $j\omega$ -axis crossing at $s = \pm j2\sqrt{2}$ (1 mark)

b) K Value (3 marks)

- $K = 48$ at crossing point (2 marks)
 - Represents stability boundary (1 mark)
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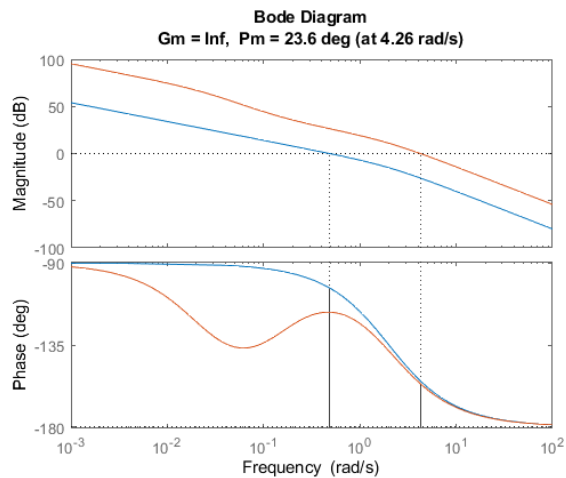
Section C: Design & Synthesis (25 Marks)

C1. Compensator Design (12 Marks)

a) Lag Compensator Design (10 marks)

- Determine K for K_v requirement: $K_v = \lim_{s \rightarrow 0} sG(s) = K/2 \geq 10 \Rightarrow K \geq 20$ (2 marks)
- Choose $K = 20$ (1 mark)
- Check uncompensated PM at ω where phase = $-180^\circ + 45^\circ + 5^\circ = -130^\circ$ (2 marks)
- Find $\omega \approx 1.4$ rad/s where $|G(j\omega)| \approx 15.3$ dB (2 marks)
- Design lag: $\beta = 10^{-\frac{15.3}{20}} \approx 0.17$, zero at $\frac{\omega}{10} = 0.14$, pole at $\beta \times \text{zero} \approx 0.024$ (2 marks)
- Final compensator: $G_c(s) = 20 \frac{(s+0.14)}{s+0.024}$ (1 mark)

b) Bode Sketch (2 marks)



- Show increased low-frequency gain (1 mark)
- Show reduced gain at crossover, maintained phase (1 mark)

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C2. Controller Selection & Real-World Application (13 Marks)

System A: DC Motor for Conveyor Belt (4 marks)

1. **PI controller** (1 mark)
2. **Justification:** Eliminates steady-state error from load variations while providing proportional response (1 mark)
3. **Trade-off:** Potential integral windup requiring anti-windup protection (1 mark)
4. **Clarity and completeness** (1 mark)

System B: Drone Pitch Control (5 marks)

1. **PID or Lead-Lag compensator** (1 mark)
2. **Justification:** Unstable system requires phase lead for stabilization; integral action eliminates steady-state error (2 marks)
3. **Trade-off:** Complex tuning; sensitivity to parameter variations (1 mark)
4. **Clarity and completeness** (1 mark)

System C: Liquid Level Control (4 marks)

1. **ON/OFF or P controller** (1 mark)
2. **Justification:** Simple control adequate for limit-based requirements; minimal complexity (1 mark)
3. **Trade-off:** ON/OFF causes cycling; P control has steady-state error (1 mark)
4. **Clarity and completeness** (1 mark)