Week 2

Chapter 2
Kinematics of Particles – Plane Rectilinear Motion

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Contents

By the end of this lecture, you are able to:

1. Understand the kinematics particles and its scope
2. Define and explain rectilinear motion of particles
3. Define motion parameters such us position , velocity and acceleration
4. Rectilinear motion equations for erratic and continuous motion

2.1 Understand the kinematics particles and its scope

- Kinematics is often described as the “geometry of motion. The principles relate the displacement, velocity, acceleration, and time of a body’s motion, without reference to the cause of the motion [1].
- Recall that a particle has a mass but negligible size, shape and rotation.
- The use of the word particles does not mean that our study is restricted to small objects; rather, it indicates that we are treating an object (like a car or a plane) as a single point with mass, ignoring its size ,shape and rotation.

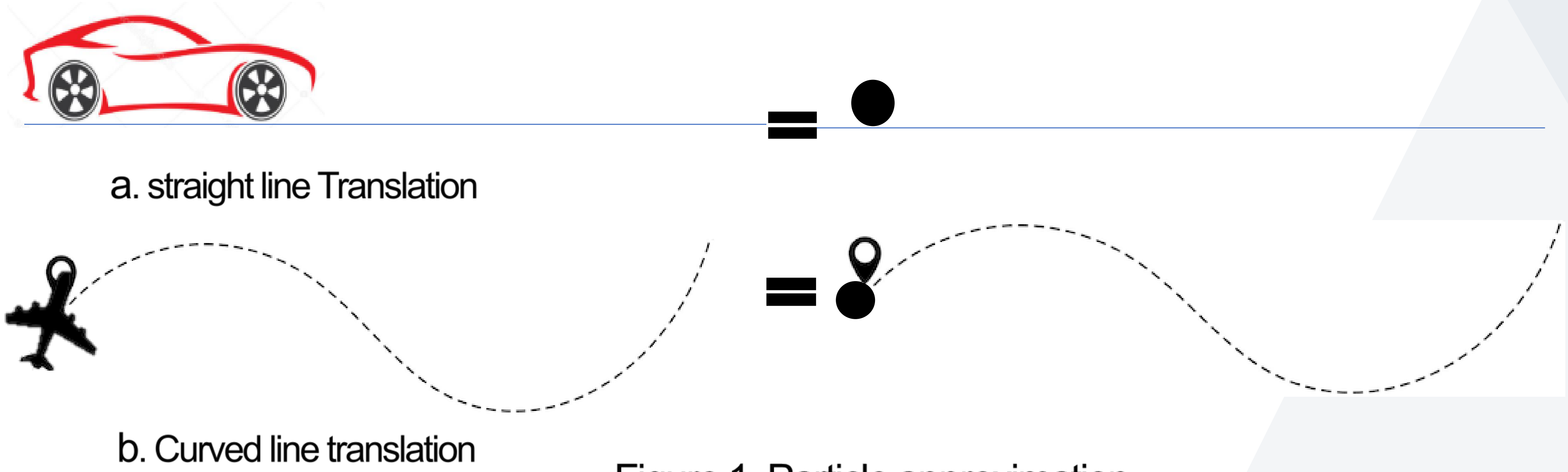


Figure 1. Particle approximation

- Kinematics of particle is defined as the study the motion of bodies, possibly as large as cars, rockets, or airplanes—without regard to their size, shape, rotation and causing force [1].
- In simple in this chapter we study translation motion of solid rigid bodies ignoring other type of motions and force**.**
- Translational motion is the movement of a rigid body where every point on the body moves the same distance and in the same direction over a given time interval**.** It can be classified as rectilinear and curvilinear.
- Thus, this chapter is covered under two big topics., namely Kinematics of Particles – ***Rectilinear Motion*** and Kinematics of Particles- ***curvilinear Motion*****.**

- **Translational motion can be classified as rectilinear and curvilinear**

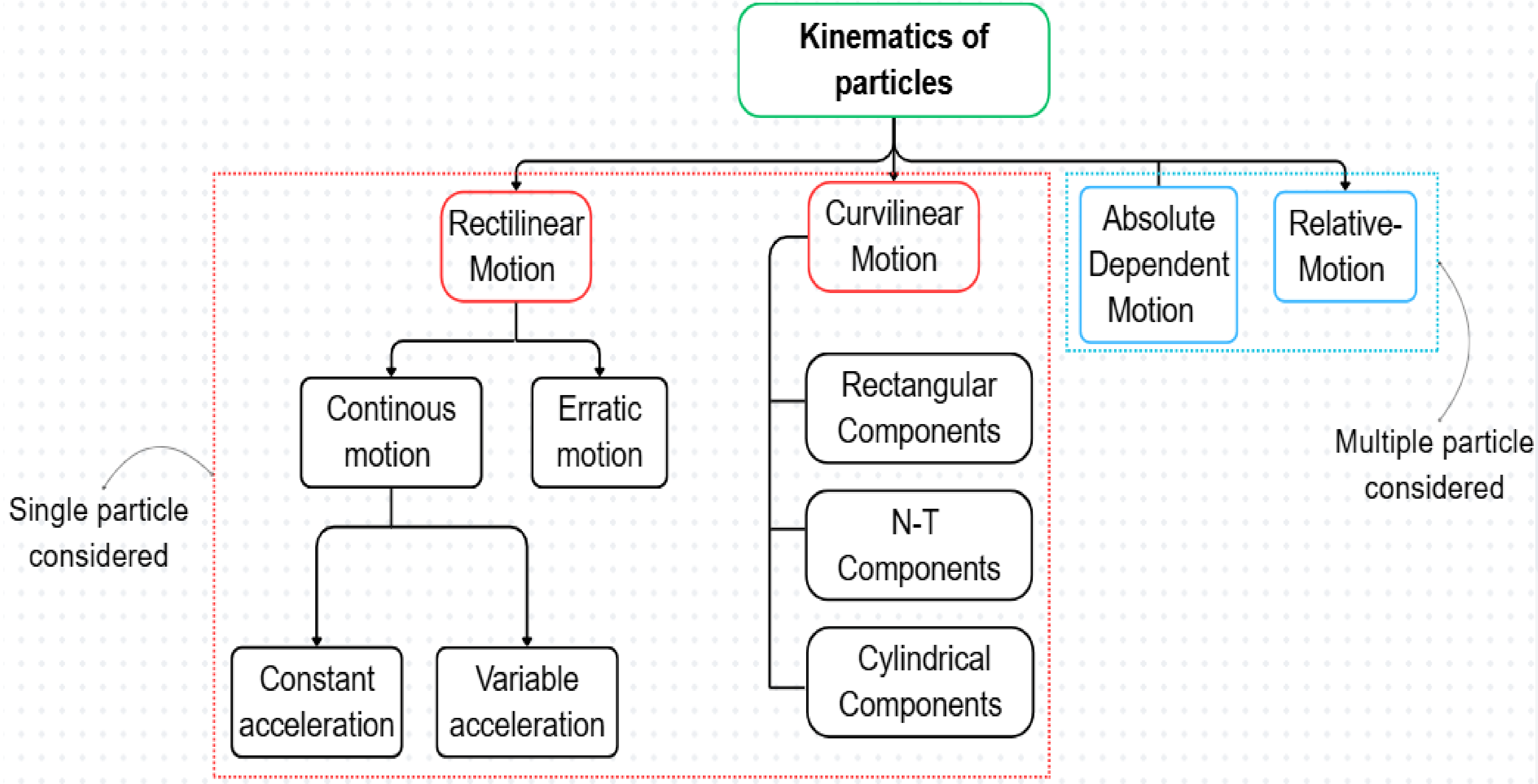


Figure 2. Chapter structure

Rectilinear motion Vs Curvilinear motion

- The key difference between rectilinear and curvilinear motion of particles is summarized below.

Translation motion categories

Rectilinear motion	Curvilinear motion
• All points move in parallel straight lines • scalar form is often enough • 1D or typically x, y, or z- axis is enough to specify the motion at time E.g. Simple lifting mechanisms	• All points move in the parallel curved path direction • Vector approach is crucial • 2D or 3D (rectangular, polar, and cylindrical coordinates are used) E.g. car turning, projectile motion

Figure 3. Rectilinear Vs Curvilinear

- NB : Today's lecture mainly focuses on the rectilinear motion of a particle.

2.2 Rectilinear motion analysis

- **In rectilinear motion, the particle just moves on a straight path. Since the direction doesn’t change, we only focus on how fast or slow it moves [2].**
- This scalar approach simplifies calculations, as there's no need to account for directional changes
- This following image is a perfect example of rectilinear motion

Figure 4. car moving on straight path

- You are simply measuring how far (displacement) the car has gone and how fast (Velocity and acceleration) it is moving along that single straight line, without concern for any changes in direction.
- we only need to focus on its magnitude of position, velocity, and acceleration.
- The rectilinear motion of particle can be studied as continuous motion and Erratic motion.

Continuous motion vs erratic motion

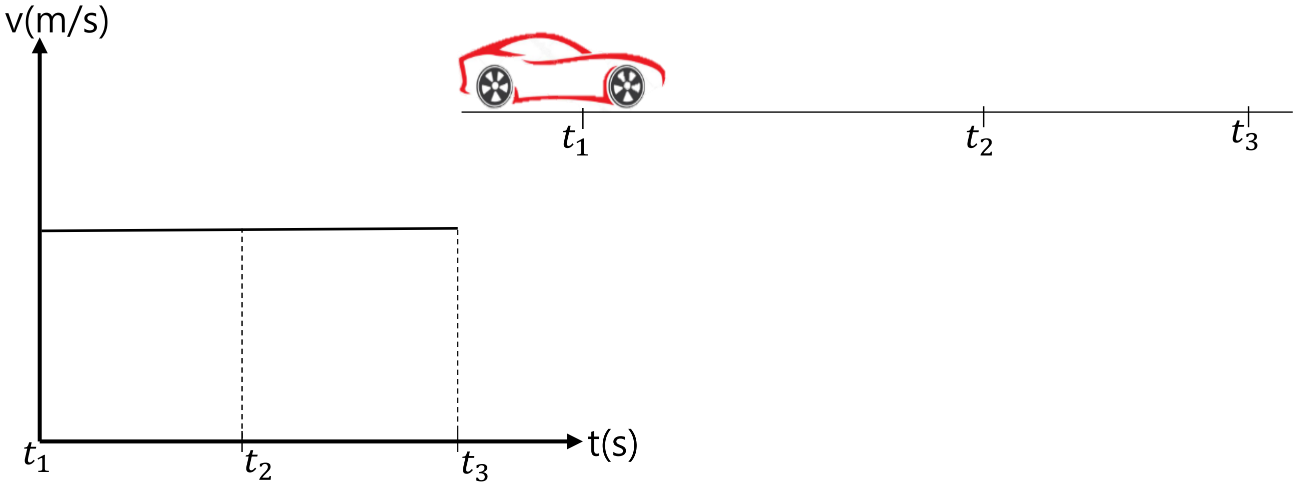
- **The equations are applied differently depending on whether the motion is continuous or erratic.**

Continuous motion

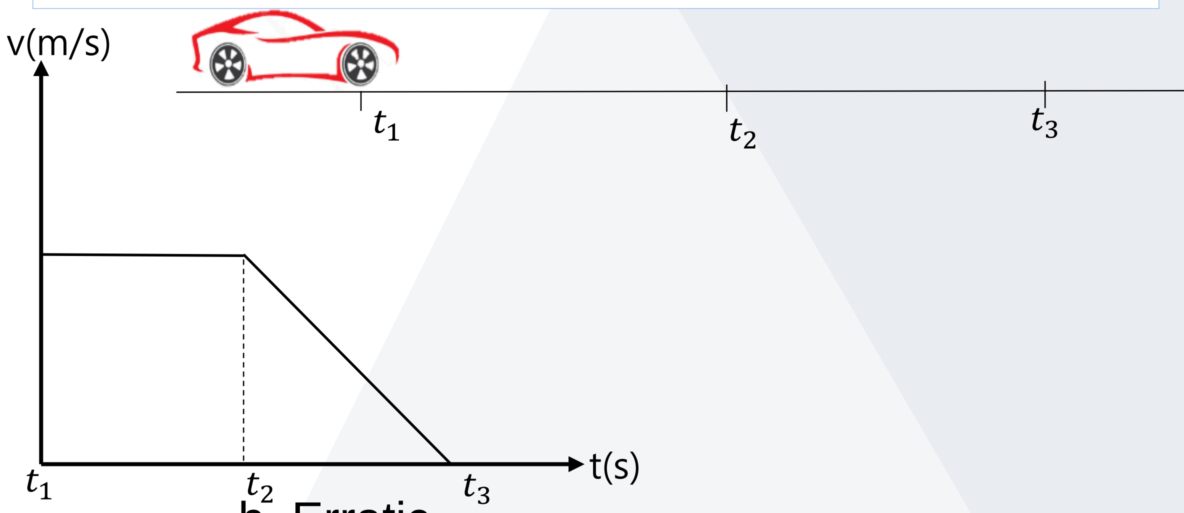
- The object’s motion is smooth and consistent, without sudden stops, starts, or jumps.
- can be described by a single, continuous mathematical function for its entire duration.
- eg A car moving smoothly on a straight highway at **steady** speed

Erratic motion

- The movement is inconsistent, with no regular or predictable pattern, making it difficult to forecast.
- different functions are needed for entire duration
- it is easier to analyze using graphs
- A car in city traffic, stopping suddenly, speeding up, turning, or swerving.



a. Continuous



b. Erratic

Figure 6. Continuous vs erratic Motion

Basics Equations of rectilinear motions

- The parameters are used to characterize and quantify the motion of a particle along a straight-line path are position, displacement, velocity, and acceleration [3].

position

- Describes the exact location of an object, usually described by coordinates or distance from a fixed point. It is a vector quantity
- For example, Consider a particle P located along a straight line axis called S

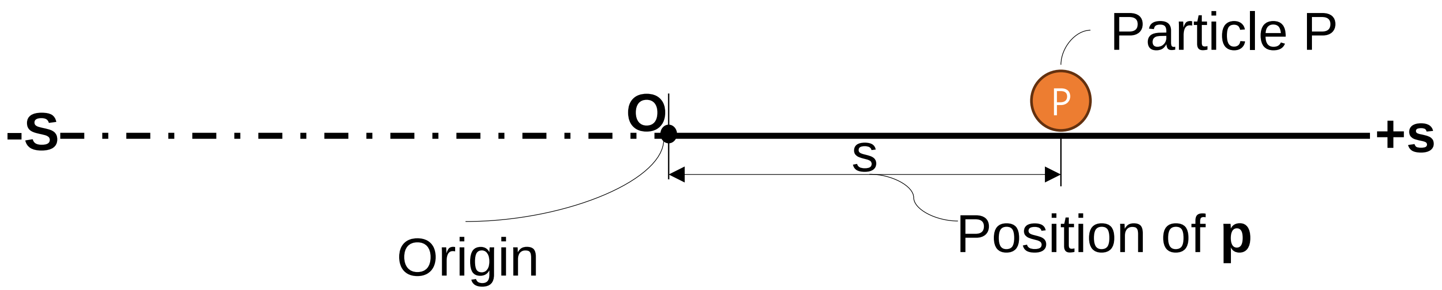


Figure 7. Position of particle

- The position of P at any instant of time t can be specified by its distance s measured from some convenient reference point O fixed on the line.
- The magnitude of s is the distance from O to the particle, usually measured in meters (m) or feet (ft), and the sense of direction is defined by the algebraic sign on s.

Displacement

- The displacement of the particle is defined as the change in its position.

- For example, if the particle moves from S_1 point to another S_2 , Fig.8 , the displacement (Δs) is: $\Delta s = S_2 - S_1$

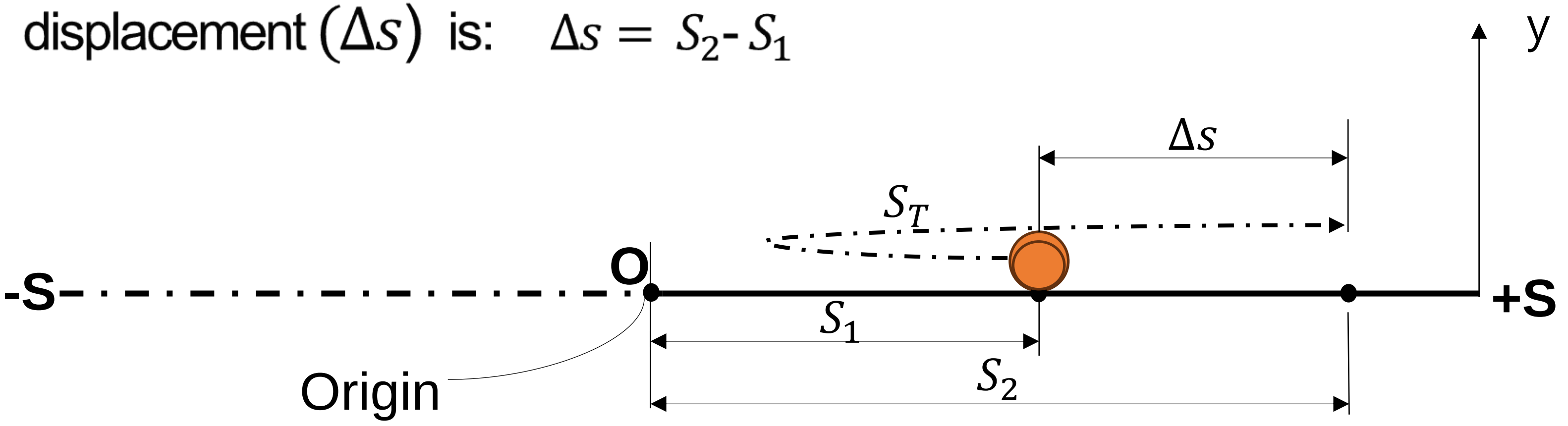


Figure 8. Displacement of particle

- It is a vector quantity, and it should be distinguished from the total distance (S_T) the particle travels which is positive scalar that represents the total length of path over which the particle travels.

Velocity

- If the particle moves through a displacement Δs during the time interval Δt, the average velocity of the particle during this time interval is : $V= \frac{\Delta s}{\Delta t}$

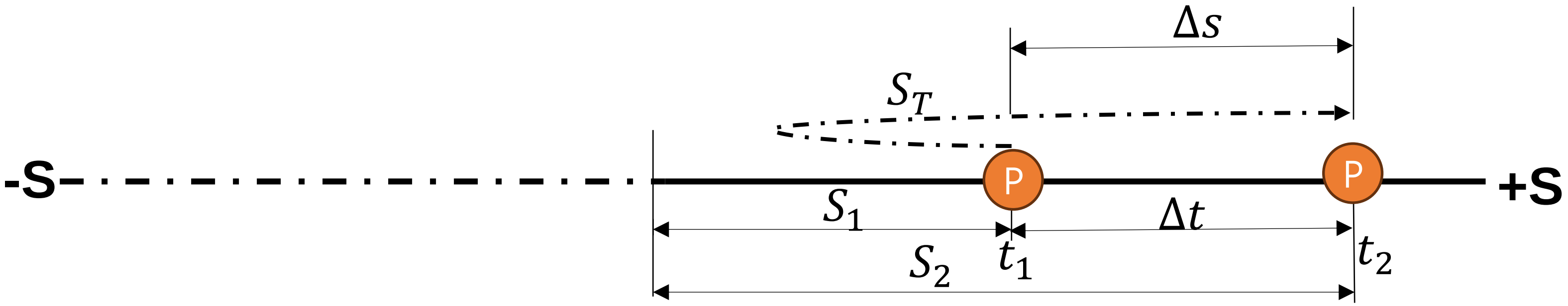


Figure 9. velocity of particle

- As Δt becomes smaller and approaches zero in the limit, the average velocity approaches the instantaneous velocity of the particle, $v = lim\ (\frac{\Delta s}{\Delta t})$, or

$$V= \frac{ds}{dt} \quad \text{(m/s)}$$

- The magnitude of the velocity is known as the speed.

- Occasionally, the term "average speed" is used. The average speed is always a positive scalar and is defined as: $(v_{sp})avg = \frac{S_T}{\Delta t}$

Acceleration

- Provided the velocity of the particle is known at two points, the average acceleration of the particle during the time interval Δt is defined as:

$$(a_{avg}) = \frac{\Delta v}{\Delta t}$$

- Here ΔV represents the difference in the velocity during the time interval Δt, i.e., $\Delta v = v_2 - v_1$

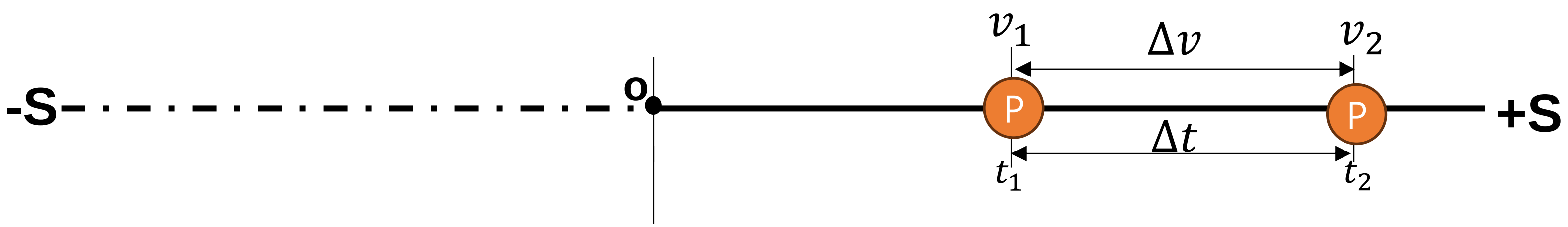


Figure 10. Acceleration of particle

- As becomes Δt smaller and approaches zero in the limit, the average acceleration approaches the instantaneous acceleration of the particle, which is: $a = lim\ (\frac{\Delta V}{\Delta t})$

$$a = \frac{dv}{dt} \quad \text{or} \quad a = \frac{d^2s}{dt^2} \ \ldots \ (\frac{m^2}{s^2})$$

- Acceleration can be positive or negative.

2.2.1 Equation of continuous motion

- Continuous rectilinear motion is described by six main equations: three for motion with constant acceleration and three for motion with variable acceleration [4].

1. Equation for variable acceleration motion

- When acceleration, velocity, or position vary as functions of time (or other parameters), the three quantities are related by the following equations:

$$v = \frac{ds}{dt}$$ → Velocity is the rate of change of position.

$$a = \frac{dv}{dt}$$ → Acceleration is the rate of change of velocity.

- Finally, an important differential relation involving the $ds, v \; and \; a$, along the path may be obtained by eliminating the time differential dt between Eqs. 12-1 and 12-2, which gives:

$$a = v\frac{dv}{ds} \quad or \quad a\,ds = v\,dv$$ → Acceleration relates velocity directly with displacement.

2 .Equation for constant acceleration motion

If the acceleration is known to be constant $\boldsymbol{a} = \boldsymbol{a_c}$, then the differential equations relating time, posi tion, velocity, and acceleration can be integrated to obtain formulas that relate $\boldsymbol{a_c}$, v, s, and t [1].

Velocity as a Function of Time.

- Integrate $\boldsymbol{a}$ = dv/dt, assuming that initially $V = V_O$ when t = 0.

$$\int_{V_O}^{V} dv = \int_0^t a dt$$

$$V = V_O + \boldsymbol{a_c t}$$

Position as a Function of Time.

- Integrate v = ds/dt = $V_O + a_c t$, assuming that initially s = $\boldsymbol{s_0}$ when t = 0.

$$\int_{s_O}^{s} ds = \int_0^t (\quad v \quad) dt$$

$$S = s_O + v_O t + \frac{1}{2}(a_c t^2)$$

Velocity as a Function of Position .

- Either solve for t in Eq. 12-4 and substitute into Eq. 12-5, or integrate $a\, ds = v\, dv$, assuming that initially $V = V_O$ at s = s_0 .

$$\int_{s_O}^{s} \boldsymbol{a_c}\, ds = \int_{V_O}^{V} V\, dv \qquad V^2 = (V_O)^2 + 2\boldsymbol{a_c}(S - s_O)$$

Summery on continuous motion

Continuous

Variable acceleration

- When acceleration varies with time, calculus becomes essential
- acceleration is obtained by differentiating Velocity over time, and Velocity is found by differentiating position.

time dependent

$$v = \frac{ds}{dt} \qquad a = \frac{dv}{dt}$$

$$a = v\frac{dv}{ds}$$

$$a\,ds = v\,dv$$

Constant acceleration

- The following three equations, often referred o as the "kinematic equations," are valid only when acceleration is constant
- Consists of 2 time dependent and 1 time independent equation

time dependent:

$$V = V_O + \boldsymbol{a_c}t$$

$$S = s_O + v_O t + \frac{1}{2}(\boldsymbol{a_c}t^2)$$

$$V^2 = (V_O)^2 + 2\boldsymbol{a_c}\,(S - s_O)$$

The first two equations are Useful when you want know or if you want to calculate time, **and are truly independent of each other.**

Most of the time, two independent equations are enough for two unknowns.

2.2.2 Equation of Erratic motion

- When the motion is erratic (changing motion),the motion of the particle is described by using 2 or more mathematical functions [3] .
  - We typically use graphs that relate position, velocity, acceleration and time
  - We have 5 general Cases: **s-t, V-t, a-t, v-s, a-s.**
  - When you have a motion graph that connects any two variables (position, velocity, acceleration, or time), you can use it to create other graphs.
  - This is possible because the variables are linked by fundamental calculus relationships like: **v=ds/dt, a=dv/dt, and ads=vdv**.

Case 1. s-t graph given

- To construct the v-t and a-t graphs, integrations are employed

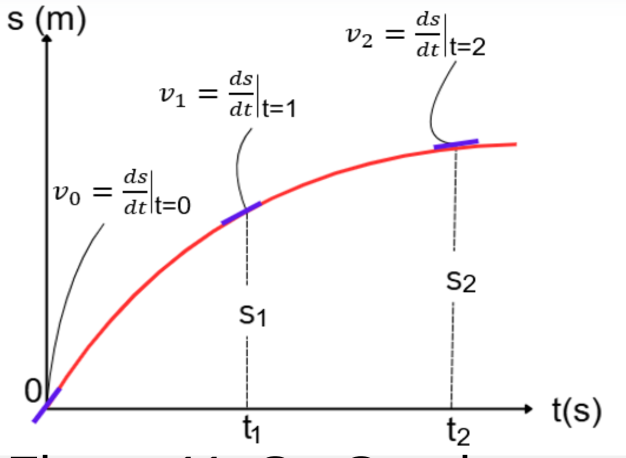


Figure 11. S-t Graph

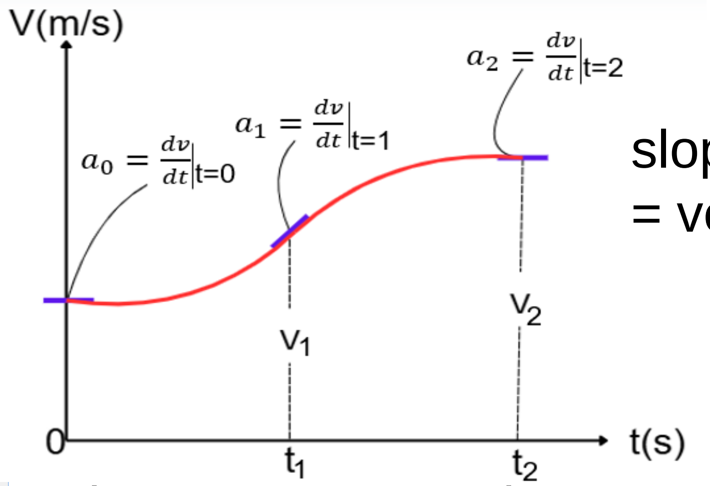
To construct the v-t graph, $v = \frac{ds}{dt}$ is used and This equation states that :

$$\frac{ds}{dt} = v$$ **or** slope of s-t = velocity

Then a-t graph is constructed from the v-t graph, in the same manner:

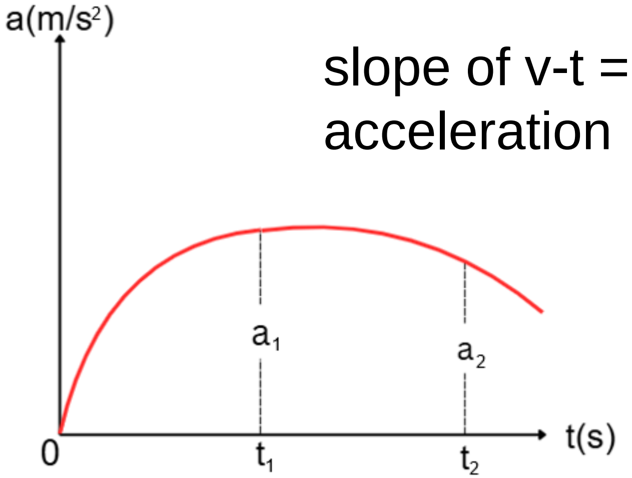
$$\frac{dv}{dt} = a$$ **or** slope of v-t = acceleration

NB. If s-t is parabolic, then v-t is a straight line, and a-t is a constant line



slope of s-t = velocity

Figure 12. v-t Graph



slope of v-t = acceleration

Figure 13. a-t Graph

Case 2. a-t graph given

- If the **a–t graph** is given, the v–t graph may be constructed using the integral form of equation $a=dv/dt$ or $adt=dv$, which give as:

$$\Delta \mathbf{v} = \int_0^t a\,dt$$

• i.e V=Vo+Δv

- If a velocity-time (v-t) graph is provided, you can determine the **s-t graph** by using the integral of the velocity function:

$$\Delta s = \int_0^t v\,dt$$

• i.e S=So+Δs

NB. If a-t is straight line, then v-t is a parabolic, and s-t is cubic

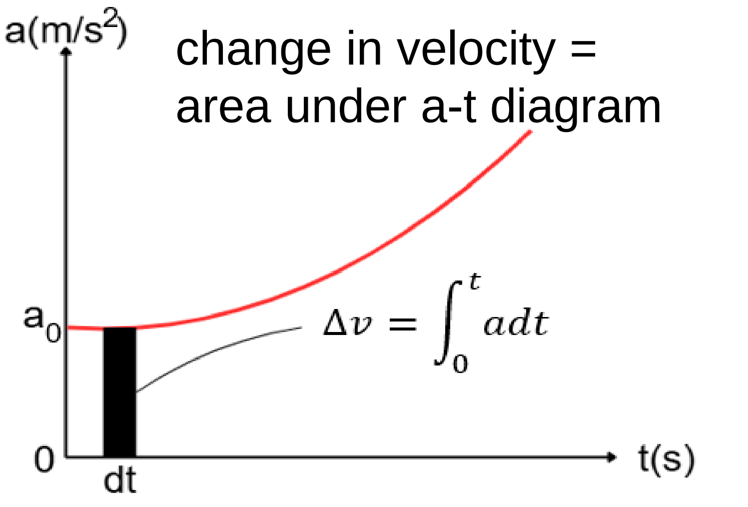


Figure 14. a-t Graph

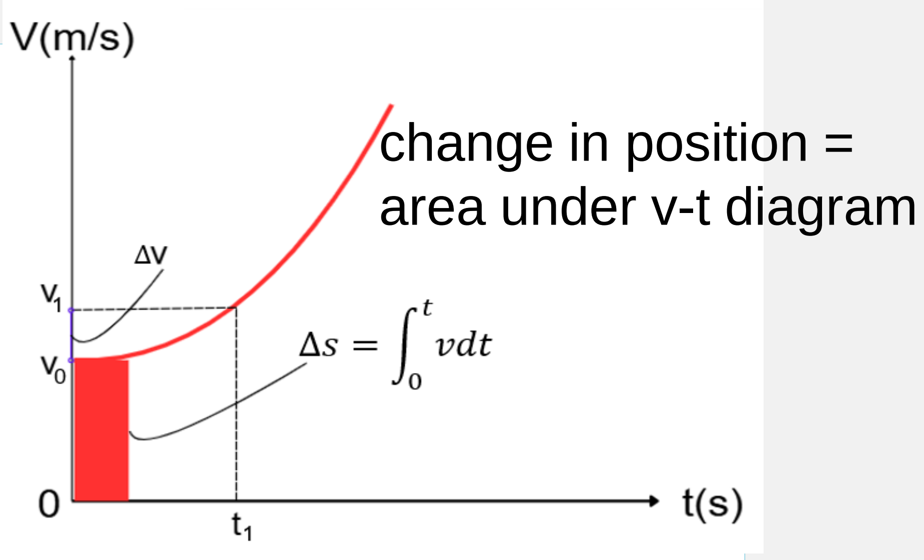


Figure 15. v-t Graph

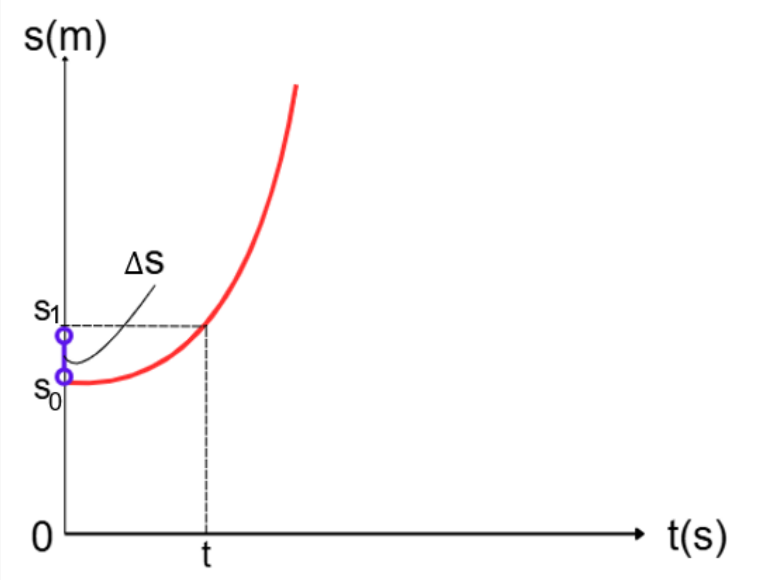


Figure 16. s-t Graph

Case 4 and 5. a-s or v-s graph given

a-s graph given

- integrating the equation **Vdv=ads** from an initial state (V= $v = v_0$ at s= S= s_0) to the final state $v = V_0$ at $s = S_0$, gives us construct a velocity-position (v-s) graph.

v-s graph given

- the acceleration (a) at any position (s) can be determined using the equation ads= Vdv. .
- Rearranging this equation, we get: $a = v\frac{dv}{ds}$

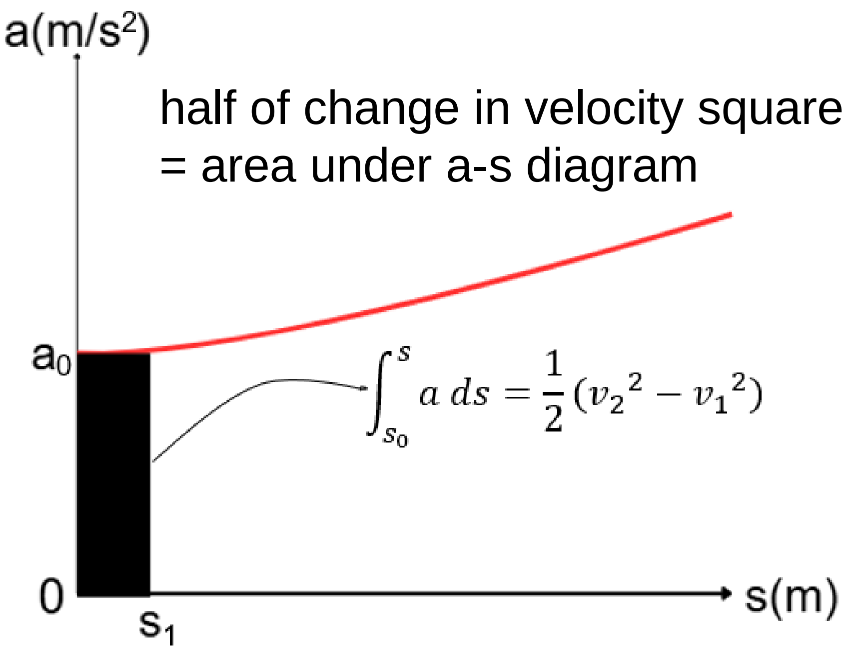


Figure 17. a-s Graph

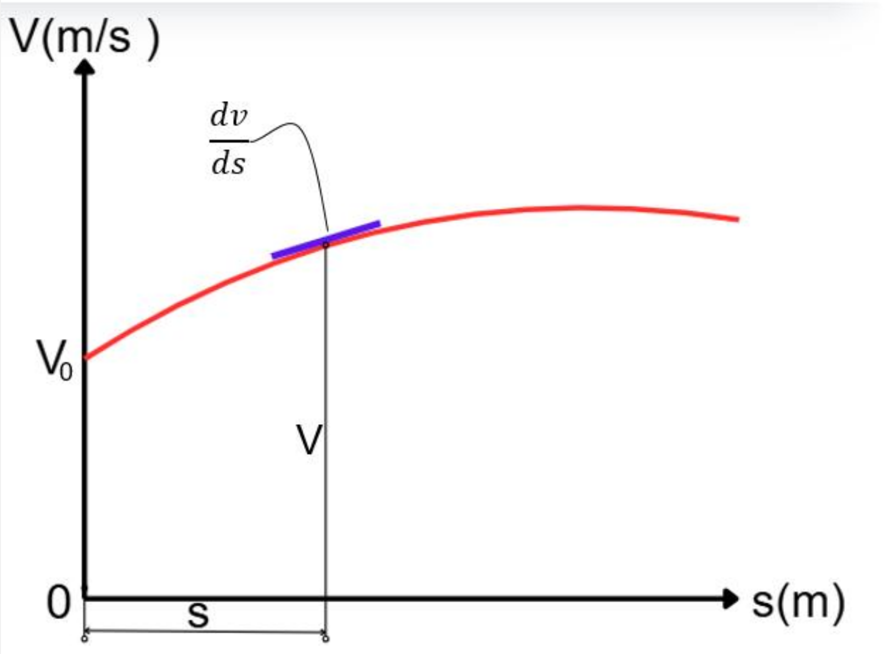


Figure 18. v-s Graph

Summary on erratic

- The 5 cases are summarized below

S-t given	• Velocity from slope of position-time graph S-t • acceleration from slope of velocity -time graph V-t
V-t Given	• Displacement from area under v-t graph • acceleration from slope of V-t graph
a-t Given	• velocity from area of a-t graph • Displacement from area under v-t graph

V-s Given	• Acceleration from slope of v-s diagram multiplied by velocity
a-s Given	• half of change in velocity square from area under a-s diagram.

Summery of Important in rectilinear motion

Continuous

Variable acceleration

$\boldsymbol{v} = \frac{\boldsymbol{ds}}{\boldsymbol{dt}}$

$a = \frac{\boldsymbol{dv}}{\boldsymbol{dt}}$

$a = v\frac{dv}{ds}$ Or $a\,ds = v\,dv$

Constant acceleration

$V = V_O + \boldsymbol{a_c t}$

$S = s_O + v_O t + \frac{1}{2}(a_c t^2)$

$V^2 = (V_O)^2 + 2\boldsymbol{a_c}(S - s_O)$

Erratic motion

Given Graph	Required Graph
S-t	• V-t • a-t
V-t	• s-t • a-t
a-t	• s-t • v-t
a-s	• v-s
V-s	• a-s

2.3 Understanding & Solving Rectilinear Motion

- The list of series of steps followed to solve kinematics of particle is listed below.

Identify given data and Decide type of motion

- Determine initial velocity, final velocity, acceleration, displacement, time.
- Identify whether the Type of Motion
  - Constant Acceleration (Uniform a)
  - Variable Acceleration (a changes with t, v, or s)
  - Erratic / Irregular Motion

Choose suitable equation

- Constant Acceleration (Uniform a) → Use 3 standard equations
- Variable Acceleration(a changes with t, v, or s) → Use 3 calculus equations.
- Erratic / Irregular Motion → Use graphs (s–t, v–t, a–t)

1 Understanding & Solving Rectilinear Motion

Simplify using tips

- When a vehicle stops and starts from rest → velocity = 0.
- At maximum height (for vertical motion) → velocity = 0.
- When acceleration = 0 → velocity is constant (no speeding up or slowing down).
- Acceleration(speeding up)= →+a and deceleration(slowing down) = -a
- If the object falls under gravity only → a= g = 9.81m/s
- Displacement vs. distance: displacement can be positive/negative, while distance is always positive.

Solve step by step

- check units and reasonableness of answer.

Problem 1

A particle travels along a straight-line path such that in 4 s it moves from an initial position s_A = - 8 m to a position s_B = +3 m. Then in another 5 s it moves from s_B to s_c = -6 m. Determine the particle's average velocity and average speed during the 9-s time interval.

Solution

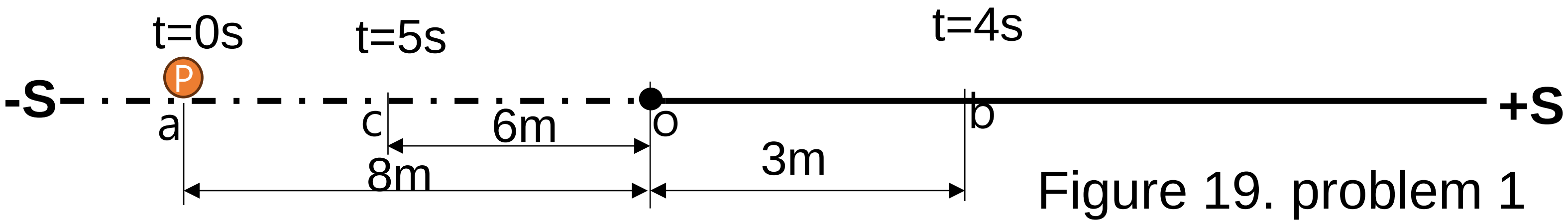


Figure 19. problem 1

Given

- $s_A = -8m$
- $s_B = 3m$
- $s_c = -6m$
- $t_{ab} = 4s$
- $t_{bc} = 5s$

Required

$a)\ v_{(vag)} = ?$

$b)\ v_{sp(avg} = ?$

a) Average Velocity ($v_{(vag)}$)= $\frac{\Delta s_{cA}}{\Delta t}$

The displacement from A to C is

$$\Delta s_{Ac} = s_c - s_a = (-6 - (-8) = 2m$$

$$v_{(vag)} = \frac{2}{9} = 0.222m/s$$

ans

b) Average Speed

The Total distances traveled from A to B and B to C are:

$s_{AB} = 8 + 3 = 11m$ and $s_{BC} = 3 + 6 = 9m$

Then The Total distances traveled

$$s_{tot} = 11 + 9 = 20m$$

Thus the average speed will be:

$$v_{sp(avg)} = \frac{s_{tot}}{\Delta t} = \frac{20m}{9s} = 2.22m/s$$

ans

Problem 2

- A car has an initial speed of 25 m/s and a constant deceleration of 3 m/ s^2. Determine the velocity of the car when t= 4s. What is the displacement of the car during the 4-s time interval? How much time is needed to stop the car?

Given

- $v_0 = 25m/s$
- a= -3 m/ s^2
- t= $4s$
- $v_f = 0(stops)$

Required

- a) $v_{(@t=4s)}$=?
- $b)\ \Delta s_{(@t=4s)}$ =?
- $c)\ t_{v_f=0}$ =?

Solution

a) velocity of the car when t= 4s

v= $v_o - at = (25 + (-3(4) = 13m/s$

ans

b) Displacement (Δs) of the car during the 4-s

$$-s \quad = s_o + v_o t + \frac{1}{2}at^2$$

$$\Delta s = (25(4) + \frac{1}{2}(-3)4^2 = 76\text{m}$$

ans

C) time stop the car

$$v_f = v_o - at$$

$$0 = 25 + (-3(t)$$

$$(t) = 8.33s$$

ans

Problem 3.

- A particle travels along a straight line with a velocity $v = (12 - 3t^2)$ m/s, where t is in seconds. When t = 1 s, the particle is located 10 m to the left of the origin. Determine the acceleration when t = 4 s, and the displacement from t = 0 to t = 10 s.

Solution

Given

- $v = (12 - 3t^2)$ m/s
- $s_{(@t=1s)} = -10m$
- $t = 4s$

Required

- a) $a_{(@t=4s)} = ?$
- $b)\ \Delta s_{(t=0\ and\ 10s)} = ?$

a) acceleration when t= 4s, using the relation, $a = \frac{dv}{dt}$

$$a = \frac{dv}{dt} = \frac{d}{dt}\left(12 - (3t^2)\right) = -6t$$

$$a_{(@t=4s)} = -24\ {}^{m}/{}_{s^2}$$

ans

b) displacement during the t=0s to t = 10 s

using the relation, ds = vdt, and boundary conditions t = 0, s = −10m, we have:

$$\int_{-10}^{s} ds = \int_{0}^{t} v\ dt = \int_{0}^{t} (12 + 3\ t^2) dt$$

$$s(t) = 12t - t^3 - 21$$

$$s_{(@t=0)} = -21m \quad and \quad s_{(@t=10)} = -901m$$

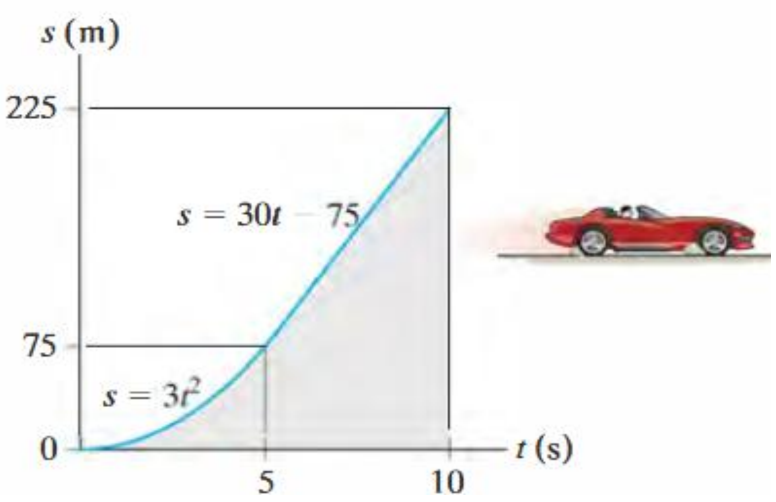
so

$$\Delta s_{(@t=0-10s)} = -901 - (-21) = -880m$$

ans

Problem 4

The sports car travels along a straight road such that its position is described by the graph. Construct the v-t and a-t graphs for the time interval $0 \le t \le 10s$.



Given

- $0 \le t \le 5s$

$S = (3t^2)$ m

- $5 \le t \le 10s$

$S = (30t - 75)$ m

Required

- a) v − t graph =?
- b) a − t graph =?

Solution

For the time interval:	$0 \le t \le 5s$	$5 \le t \le 10s$
Using $\mathbf{v} = \frac{ds}{dt}$ we have	V=6t	$v = 30$
Using $a = \frac{dv}{dt}$,we have	a=6	$a = 0$

$v - t\ graph$ a$-t\ graph$

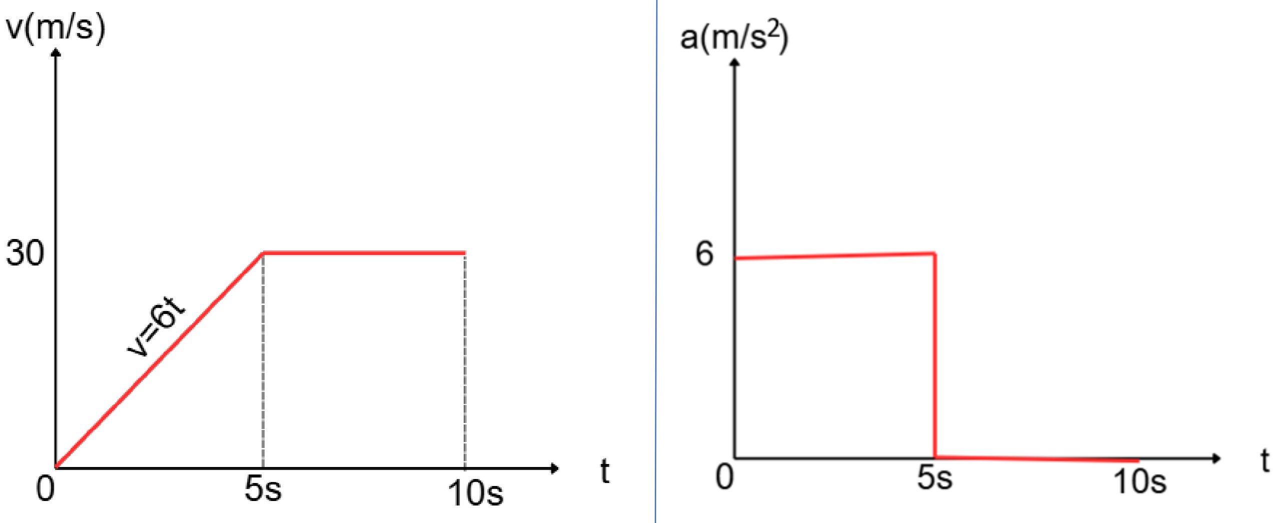


Figure 19. problem 4

Activities

1) particle moving along a straight line is subjected to a deceleration a =($-2v^3$) m / s^2, where v is in m/s. If it has a velocity v = 8 m/s and a position s = 1 0 m when t = 0, determine its velocity and position when t = 4 s.

2) A car starts from rest and travels along a straight road with a velocity described by the graph. Determine the total distance traveled until the car stops. Construct the s-t and a-t graphs.

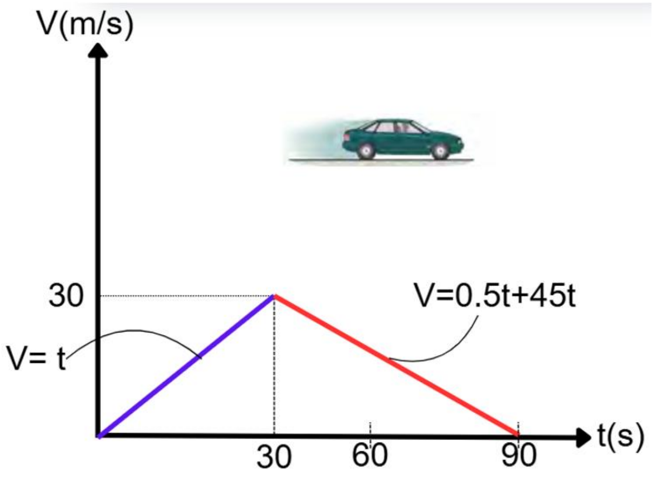


Figure 19. v-t Graph

Summary

In This Lecture We Covered:

1. Introduction to Kinematics of Particles→ Scope and definition of kinematics
2. Rectilinear Motion of Particles→ continuous motion and erratic motion
3. Continuous Rectilinear Motion → variable acceleration and constant acceleration
4. Erratic Rectilinear Motion→ motion graphs (s–t, v–t, a–t, v–s, a–s)
5. Steps to Solve Rectilinear Motion Problems→ description
6. Solve Rectilinear Motion Problems

References

[1] Dynamics, Hibbeler, Rusel M.,Prentice Hall,10th ed., 2003

[2] Cengel, Yunus, and John Cimbala. *Ebook: Fluid mechanics fundamentals and applications (si units)*. McGraw Hill, 2013.

[3] Engineering Mechanics - Dynamics, Meriam J.L.,John Wiley & Sons, 9th ed., 2020.,

[4] Vector Mechanics for Engineers: Dynamics, Johnston, E. R., & Clausen, W. E., McGraw-Hill, 11th ed., 2015