Week 3

Chapter 2
Kinematics of Particles – Curvilinear Motion

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Contents

By the end of this lecture, you are able to:

1. Understand the scope of Curvilinear Motion
2. Understand Rectangular Components
3. Understand Normal and Tangential Components
4. Understand Cylindrical Components
5. Solve problems and Understand how to use the equations

2.1 Understand curvilinear motion

- **Involves a particle moving along a curved path, resulting in continuous changes in direction and magnitude[1].**
- Here, both the **magnitude** and **direction** of velocity and acceleration vary, necessitating a vectoral analysis to fully describe the motion**.**

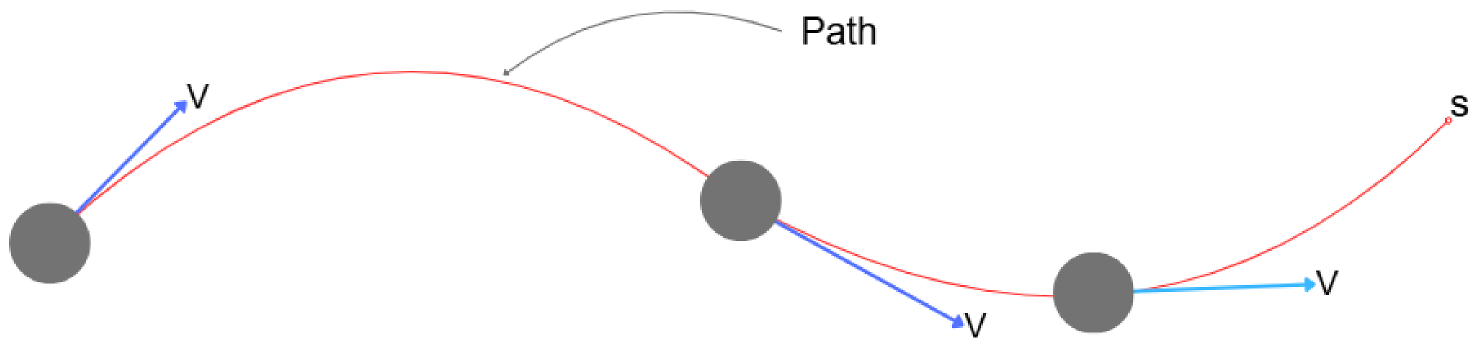


Figure 1. curvilinear motion

Unlike rectilinear motion where the direction is constant, in curvilinear motion, the ***velocity's direction is constantly changing*** as the ball follows the curve.

NB: To fully describe this type of motion, we need to use a different method than the one we used for rectilinear motion

Describing Direction in Curvilinear Motion

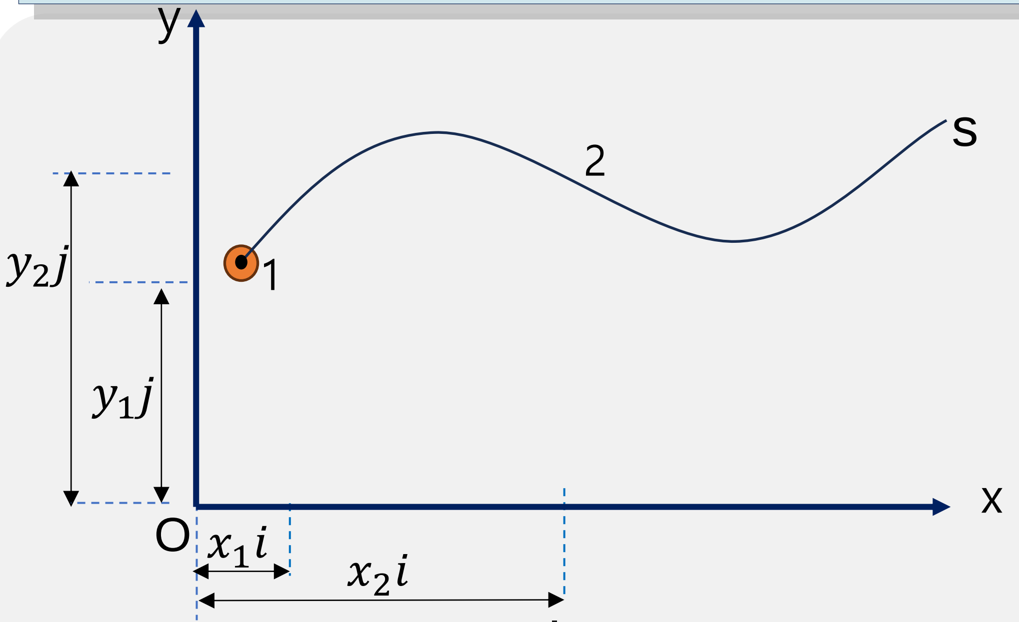
- To effectively analyze curvilinear motion, it's essential to describe the particle's direction using appropriate coordinate systems [1]. Three commonly used systems are:

Rectangular Coordinates X, y, and z	Describe the motion in Standard Cartesian coordinates
Normal and Tangential Coordinates (n-t)	This system aligns with the particle's path
Polar Coordinates (r-θ)	Ideal for motions involving circular or rotational paths, utilizing radial and angular components

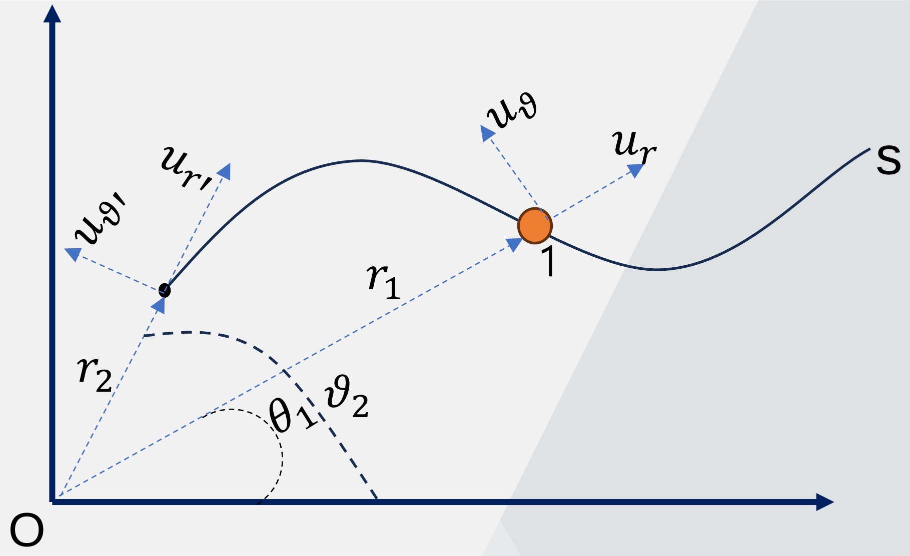
By employing this 2D and 3D coordinates, we can capture changes in both magnitude and direction of motion in different directions at the same time

The methods and descriptions of motion differ across coordinate systems, but the magnitudes of velocity and acceleration remain the same in each system

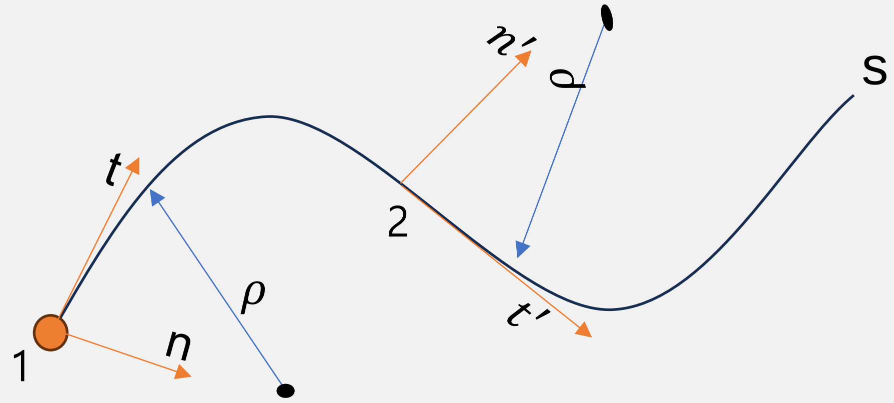
Coordinate systems



Rectangular



Polar Coordinates



Normal and Tangential Coordinates

Figure 2. Coordinate systems

2.2 Rectangular coordinate

Kinematic description of the path

Position

If a particle is at point (x, y, z) on a curved path, its location is given by the position vector.

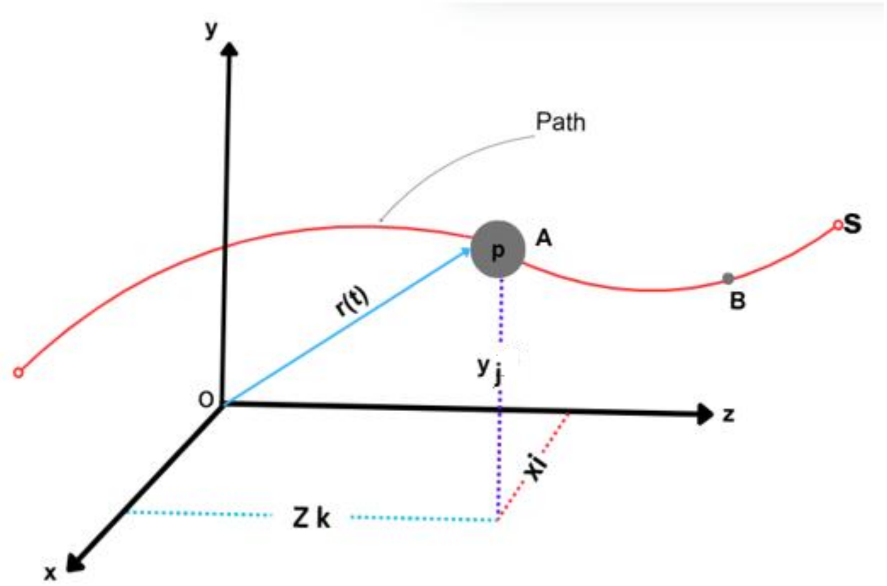


Figure 2. 3D Cartesian coordinate system

$$\boldsymbol{r} = x\boldsymbol{i} + y\boldsymbol{j} + z\boldsymbol{k}$$

and the Magnitude

$$\mathbf{r} = \sqrt{(x\boldsymbol{i})^2 + (y\boldsymbol{j})^2 + (z\boldsymbol{k})^2}$$

Velocity

The first time derivative of r yields the velocity of the particle. Hence

$$\boldsymbol{v} = \frac{\boldsymbol{dr}}{\boldsymbol{dt}} = \frac{\boldsymbol{d}}{\boldsymbol{dt}}(\boldsymbol{xi} + \boldsymbol{yj} + \boldsymbol{zk})$$

- Since x, y, z reference frame is fixed, and therefore the derivative of i, j and k is zero.

$$\mathbf{v} = \frac{\boldsymbol{dr}}{\boldsymbol{dt}} = \frac{\boldsymbol{dx}}{\boldsymbol{dt}}\boldsymbol{i} + \frac{\boldsymbol{dy}}{\boldsymbol{dt}}\boldsymbol{j} + \frac{\boldsymbol{dz}}{\boldsymbol{dt}}\boldsymbol{k} \quad \boldsymbol{or} \quad \boldsymbol{v} = \dot{\boldsymbol{x}}\boldsymbol{i} + \dot{\boldsymbol{y}}\boldsymbol{j} + \dot{\boldsymbol{z}}\mathrm{k} \quad \text{where} \quad V_X = \dot{\boldsymbol{x}}\boldsymbol{i} \quad V_y = \dot{\boldsymbol{y}}\boldsymbol{j} \quad V_z = \dot{\boldsymbol{z}}\boldsymbol{k}$$

The velocity has a magnitude and a direction that is found from

$$v = \sqrt{(\boldsymbol{v_x})^2 + (\boldsymbol{v_y})^2 + (\boldsymbol{v_z})^2} \qquad \boldsymbol{U_v} = \frac{\boldsymbol{V}}{\boldsymbol{v}}$$

Acceleration

It is particle is obtained by taking the first time derivative of velocity. We have:

$$\mathbf{a} = \frac{\boldsymbol{dV}}{\boldsymbol{dt}} = \frac{V_X}{\boldsymbol{dt}}\boldsymbol{i} + \frac{V_y}{\boldsymbol{dt}}\boldsymbol{j} + \frac{V_z}{\boldsymbol{dt}}\boldsymbol{k} = a_X i + a_y j + a_z k$$

- The acceleration has a magnitude:

$$\boldsymbol{a} = \sqrt{(\boldsymbol{a_x})^2 + (\boldsymbol{a_y})^2 + (\boldsymbol{a_z})^2} \qquad \boldsymbol{U_a} = \frac{\mathbf{a}}{\boldsymbol{a}}$$

NB. The "dot" notation X, y, z r the first time derivative

Important Points

- The is ***no directional change*** only their is ***magnitude*** and sense (algebraic sign) will change.
- It measure magnitude change in 2D and in 3D
- The velocity vector is always directed tangent to the path.
- Two types of problems generally occur:

1. If the coordinates are specified as time parametric equations, $x = x(t)$ *and* $y = y(t)$ then the time derivatives can be found directly.

2. If the time-parametric equations are not given, then the path $y = f(x)$ must be known. Using the chain rule of calculus we can then find the relation between V_X and V_y , and between a_X and a_y.

- Once the x, y, z components of **v** and **a** have been determined, the magnitudes of these vectors are found from the Pythagorean theorem

2.2.1) Motion of a Projectile

It is a special case studied in rectangular coordinates, where the particle moves with constant acceleration in two directions [2].

When air resistance is neglected, the projectile to have a constant downward acceleration of approximately the a=g=9.81 $^{m}/_{s^2}$ (or 32.2 $^{ft}/_{s^2}$).

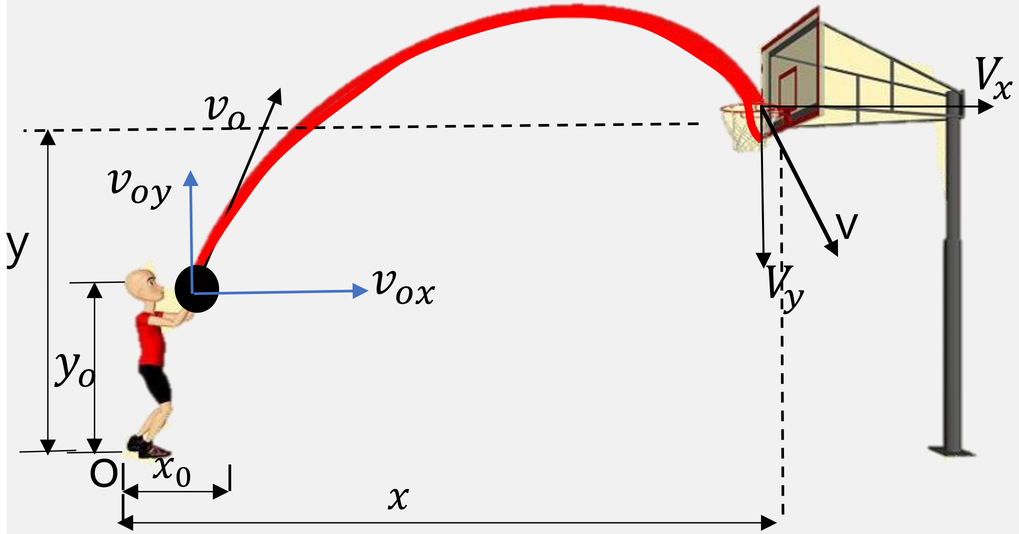


Figure 4. Trajectory of a Projectile

https://www.basic-mathematics.com/projectile-motion.html

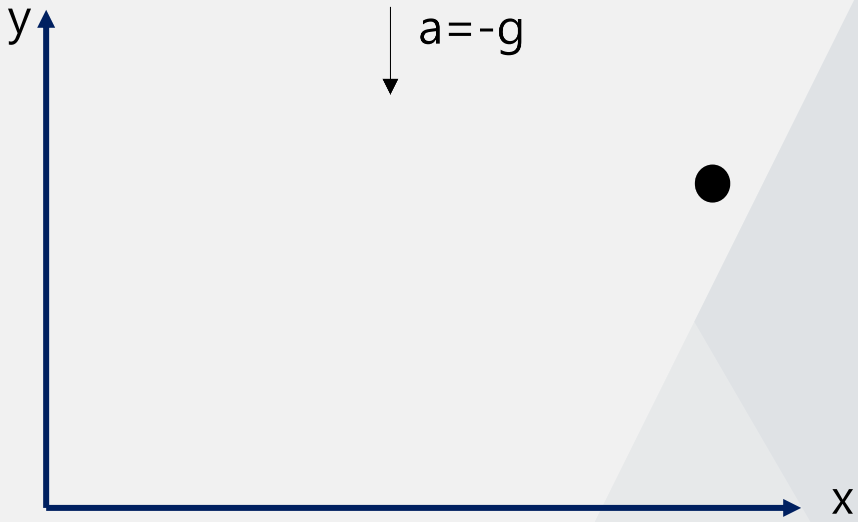
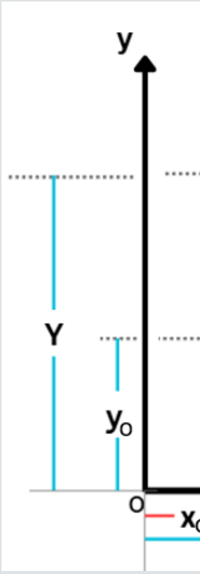


Figure 5. Projectile kinematics



Rectilinear motion

$V = V_O + \boldsymbol{a_c t}$

$S = s_O + v_O t + \frac{1}{2}(a_c t^2)$

$V^2 = (V_O)^2 + 2\boldsymbol{a_c}\,(S - s_O)$

Horizontal Motion ($a_x = 0$)

$V_x = (v_o)_x$

$X = X_O + (v_o)_x t$

$V_y{}^2 = ((v_o)_x)^2$

Vertical Motion (a_y) $= -g$

$V_y = (v_o)_y - \boldsymbol{gt}$

$Y = y_O + (v_o)_y t - \frac{1}{2} g t^2)$

$V_y{}^2 = (V_O)^2 - 2\boldsymbol{g}\,(Y - y_O)$

Important Points

- The equations $V_x = (v_o)_x$ indicate that the horizontal component of velocity always remains constant during the motion.

- we can have at most *three unknowns* since only *three independent equations* can be written; that is, *one equation in the horizontal direction* and *two in the vertical direction*.

$$Y = y_O + (v_o)_y t - \frac{1}{2} g t^2)$$

$$X = X_O + (v_o)_x t$$

$$V_y = (v_o)_y - \boldsymbol{gt}$$

$${V_y}^2 = (V_O)^2 - 2\boldsymbol{g}\,(Y - y_O)$$

- Establish the fixed x, y coordinate axes and sketch the trajectory of the particle. Between any two points on the path specify the given problem data and identify the three unknowns

- Depending upon the known data and what is to be determined, a choice should be made as to which three of the following four equations should be applied between the two points

2.3 Normal and Tangential Components

Kinematic description of the path

- If a particle's path is known, motion is often described using tangent (t) and normal (n) coordinates.
- The origin of the coordinate is located at the particle and moves together with the particle.
- Consider A particle moves along a curved path (red line) between point 1 and 2. At each point on the path:

The t-axis is tangent to the curve at the point, positive along the direction of increasing path length s, and represented by the unit vector u_t.

The normal vector (n) is always perpendicular to the tange nt and points toward the center of curvature on the concave side of the path, represented by the unit vector u_n

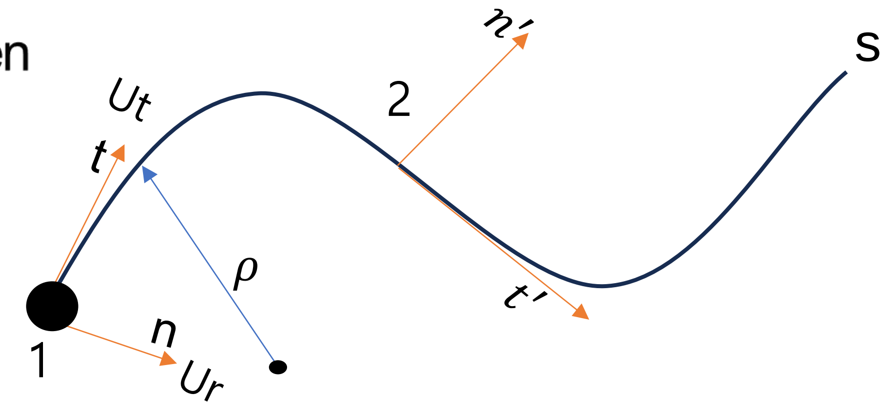


Figure 6. normal tangent

- NB: Here we are considering plane motion

velocity

- since the particle moves, s is a function of time, as indicated in Fig 7, the particle's velocity v has a direction that is always tangent to the path,.

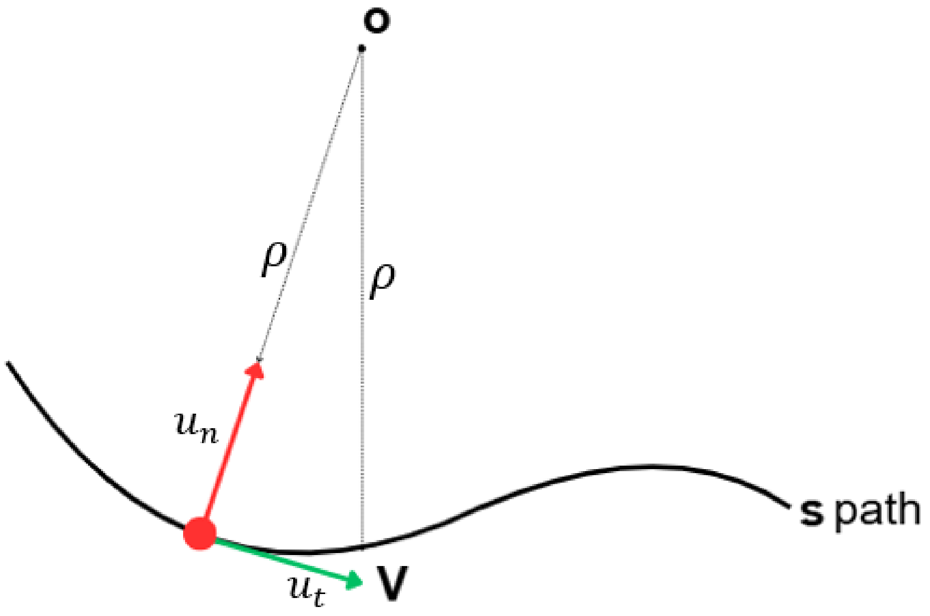


Figure 7. Velocity vector

- The magnitude that is determined by taking the time derivative of the path function

$s = s(t)$ $\quad$ $\mathbf{V} = v\,\boldsymbol{u_t}$ $\quad$ where $v = \frac{ds}{dt} = \dot{s}$

cont'd…

Acceleration

- The acceleration of the particle is the time rate of change of the velocity. Thus,

$$\boldsymbol{a} = \frac{dv}{dt} = \frac{d}{dt}(v\,\boldsymbol{u}_t) = \frac{dv}{dt}\boldsymbol{u}_t + v\,\frac{d\boldsymbol{u}_t}{dt}$$

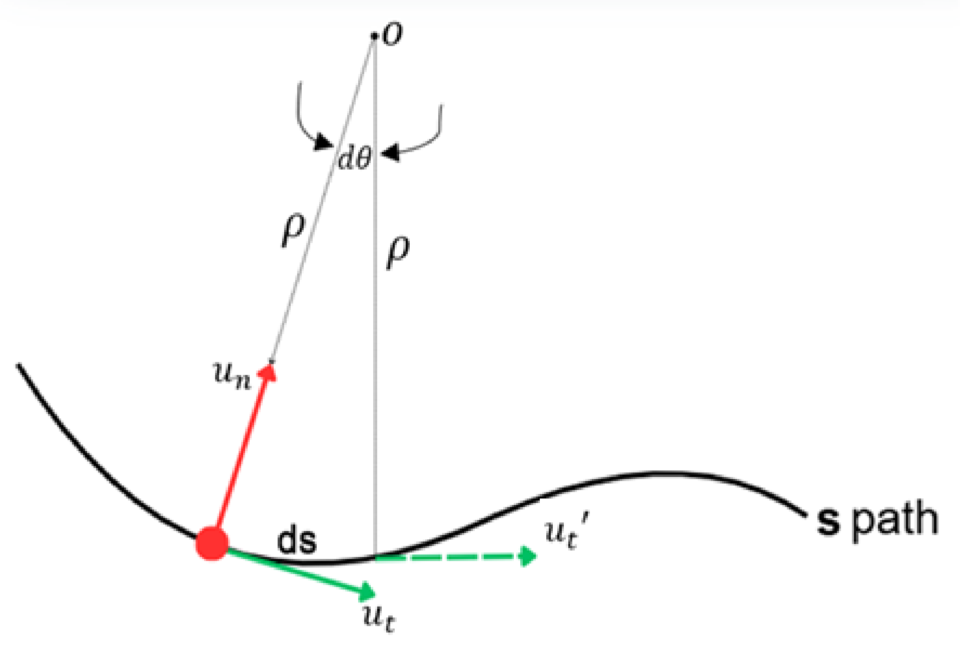


Figure. 8 particle direction change

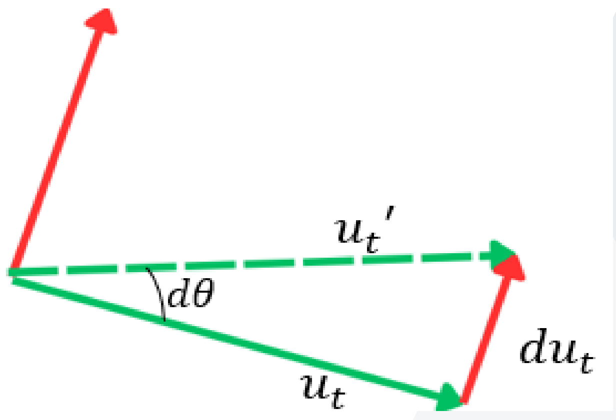


Figure 9 . Change in Unit vectors

The change of tangent direction over time will be :

$$\frac{d\boldsymbol{u}_t}{dt} = \frac{v}{\rho}\,\boldsymbol{u}_n$$

Substituting into acceleration equation, a can be written as the sum of its two components,

$$\boldsymbol{a} = \frac{\mathrm{d}v}{dt}\boldsymbol{u}_t + v\,\frac{\mathrm{d}u_t}{dt} \qquad and\ ,\frac{\mathrm{d}u_t}{dt} = \frac{v}{\rho}\boldsymbol{u}_n$$

$$\boldsymbol{a} = \frac{\mathrm{d}v}{dt}\boldsymbol{u}_t + \frac{v^2}{\rho}\boldsymbol{u}_n$$

or

$$\boldsymbol{a} = a_t\,\boldsymbol{u}_t + a_n\boldsymbol{u}_n$$

where

$$a_t = \frac{dv}{dt} \qquad a_n = \frac{v^2}{\rho}$$

These two mutually perpendicular components are shown in Figure 10. Therefore, the magnitude of acceleration is the positive value of

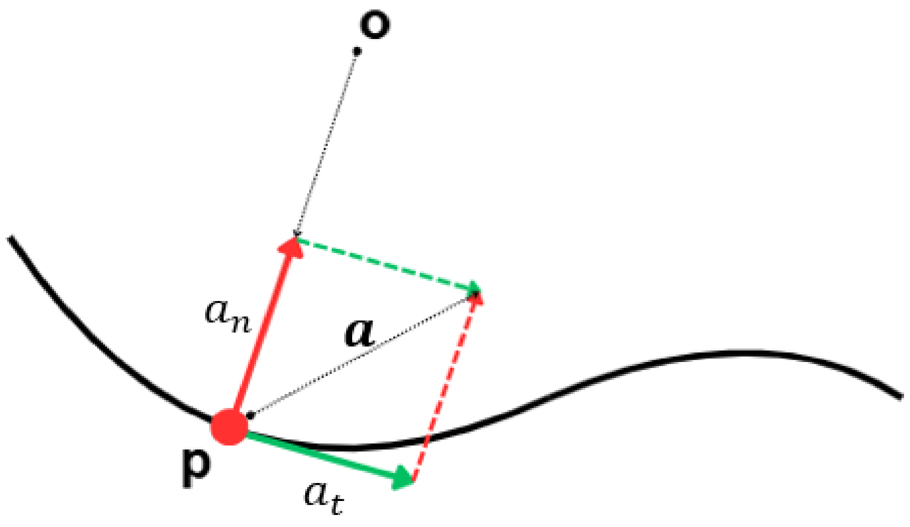


Figure. 10 Magnitude of acceleration

$$a = \sqrt{(a_n)^2 + (a_t)^2}$$

$$tan(\theta) = \frac{a_n}{a_t}$$

The tangential component of acceleration (at)

Important Points

- It is the result of the time rate of change in the magnitude of velocity.
- Acts in the positive s direction and measures the increasing or decrease in speed of particle's.
- The relations between $\boldsymbol{a_t}$, v, t and s are the same as for rectilinear motion, namely,

If $\boldsymbol{a_t}$ is varies, $\boldsymbol{a_t}\, ds = v\, dv$ $\quad \boldsymbol{a_t} = \frac{\boldsymbol{dv}}{\boldsymbol{dt}}$

If $\boldsymbol{a_t}$ is constant,

$$V = V_O + \boldsymbol{a_t t}$$

$$S = s_O + v_O t + \frac{1}{2}(a_t t^2)$$

$$V^2 = (V_O)^2 + 2\boldsymbol{a_t}(S - s_O)$$

- If the particle moves along a curve with a constant speed, then $\boldsymbol{a_t}$ = 0 and a = $\boldsymbol{a_n}$

Important Points

The normal component of acceleration

- It is the result of the time rate of change in the direction of the velocity.
- it is always directed toward the center of curvature of the path, i.e. along the positive n axis.
- The magnitude of this component is determined from:

$$\boldsymbol{a_n} = \frac{\boldsymbol{v}^2}{\boldsymbol{\rho}}$$

- If the path is expressed as y = f(x), the radius of curvature ρ at any point on the path is determined from the equation:

$$\rho = \frac{[1 + (\frac{dy}{dx})^2]^{\frac{3}{2}}}{\frac{d^2y}{dx^2}}$$

- If the particle moves along a straight line, then $\rho = \infty$ and $\boldsymbol{a_n}$ = 0. Thus a = $\boldsymbol{a_t}$

2.4 Cylindrical coordinates

Kinematic description of the path

- Sometimes the motion of the particle is constrained on a path that is best described using cylindrical (3D) or polar coordinates(2D).
- Consider specifying the location of the particle shown in Fig. 9 using polar coordinate (r,θ)..

a radial coordinate r, extends outward from the fixed origin o to the particle, and represented by the unit vector u_r.

a transverse coordinate θ**,**which is the counterclockwise angl e between a fixed reference line and the r axis**,** and represented by the unit vector u_θ

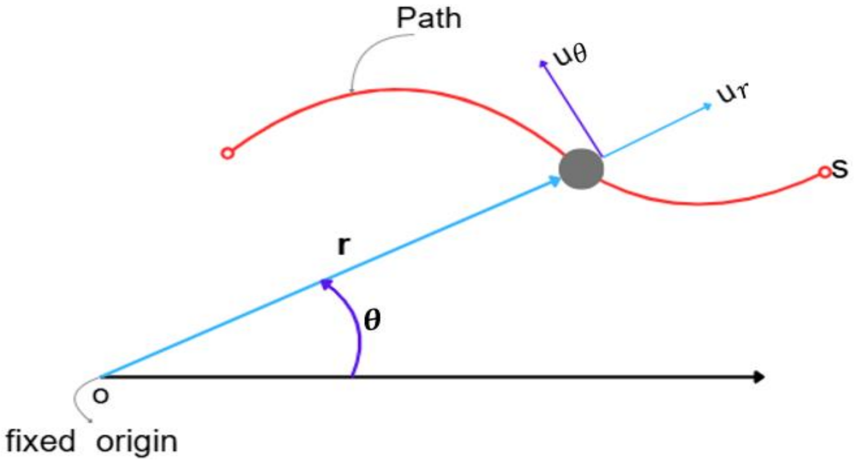


Figure 11. polar coordinate system

- Here $\boldsymbol{u_r}$ is in the direction of increasing r when θ is held fixed, and $\boldsymbol{u_\theta}$ is in a direction of increasing θ when r is held fixed.

- NB: These directions are perpendicular to one another.

position

- At any instant the position of the particle, Fig. 10, is defined by the position vector

$$r = ru_r$$

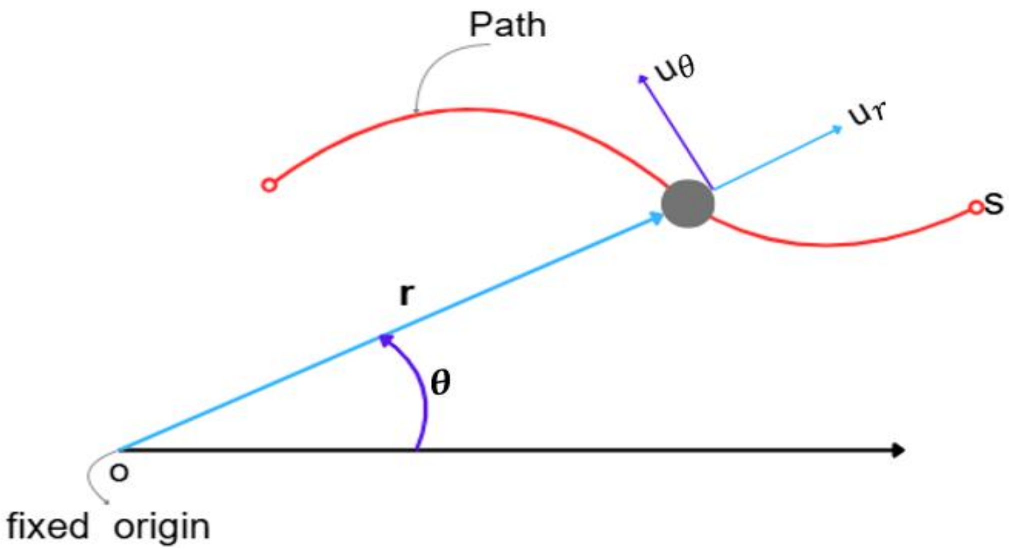


Figure 12. position vector

Velocity

- The velocity v is obtained by taking the time derivative of r. Using a dot to represent the time derivative, we have

$$V = \frac{d}{dt}(r u_r) = \dot{r}\, u_r + r\, \dot{u}_r$$

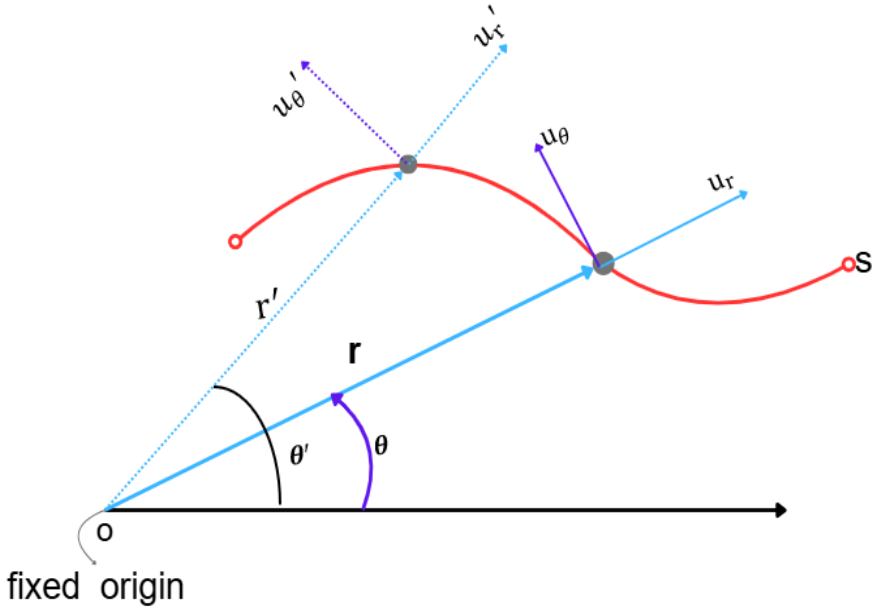


Figure. 13 change in unit vector

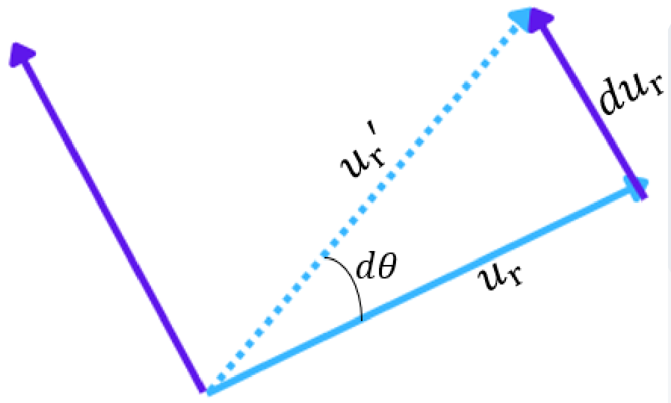


Fig. 14 particle direction change

Consequently, $\frac{du_t}{dt} = \dot{u}_r = \frac{d\theta}{dt} u_\theta$

- Substituting into the above equation, the velocity can be written in component form as:

$$V=\dot{r}\,u_r + r\dot{\theta}u_\theta \qquad \text{where} \qquad v_r=\dot{r} \qquad v_\theta=r\dot{\theta}$$

  - *radial component* (v_r) is a measure of the rate of increase or decrease in the length of the radial coordinate i.e., $\dot{r}$
  - *transverse component* (v_θ) can be interpreted as the rate of motion along the circumference of a circle having a radius r

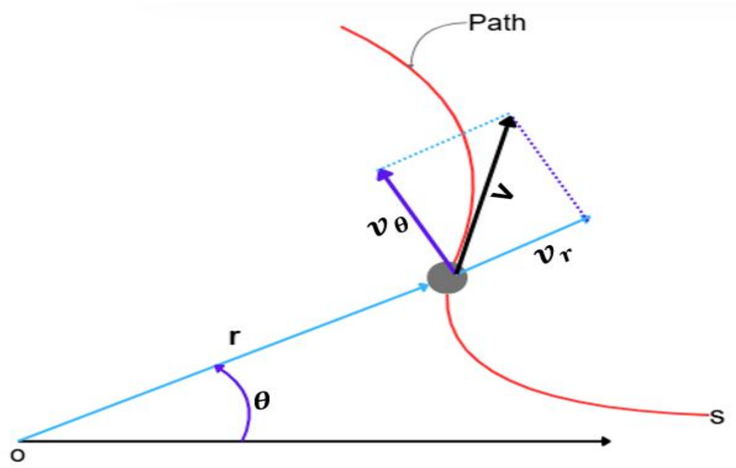


Figure. 15 Velocity vector

- Since v_r and v_θ are mutually perpendicular, the magnitude of velocity or speed is simply the positive value of:

$$\mathbf{v}=\sqrt{(\boldsymbol{v_r})^2+(\boldsymbol{v_\theta})^2}$$

- The direction of v is, of course, tangent to the path, Fig. 15

- The term $\dot{\theta}=d\theta/dt$, is called the ***angular velocity***, since it indicates the time rate of change of the angle θ. Common units used for this measurement are rad/s.

Acceleration

Taking the time derivatives of velocity equation, we obtain the particle's instantaneous acceleration,

$$a = \frac{dV}{dt} = \frac{d}{dt}(\dot{r}\,u_r + r\dot{\theta}u_\theta) = (\ddot{r}\,u_r + \dot{r}\,\dot{u}_r + \dot{r}\,\dot{\theta}u_\theta + r\,\ddot{\theta}\,u_\theta + r\dot{\theta}\,\dot{u}_\theta)$$

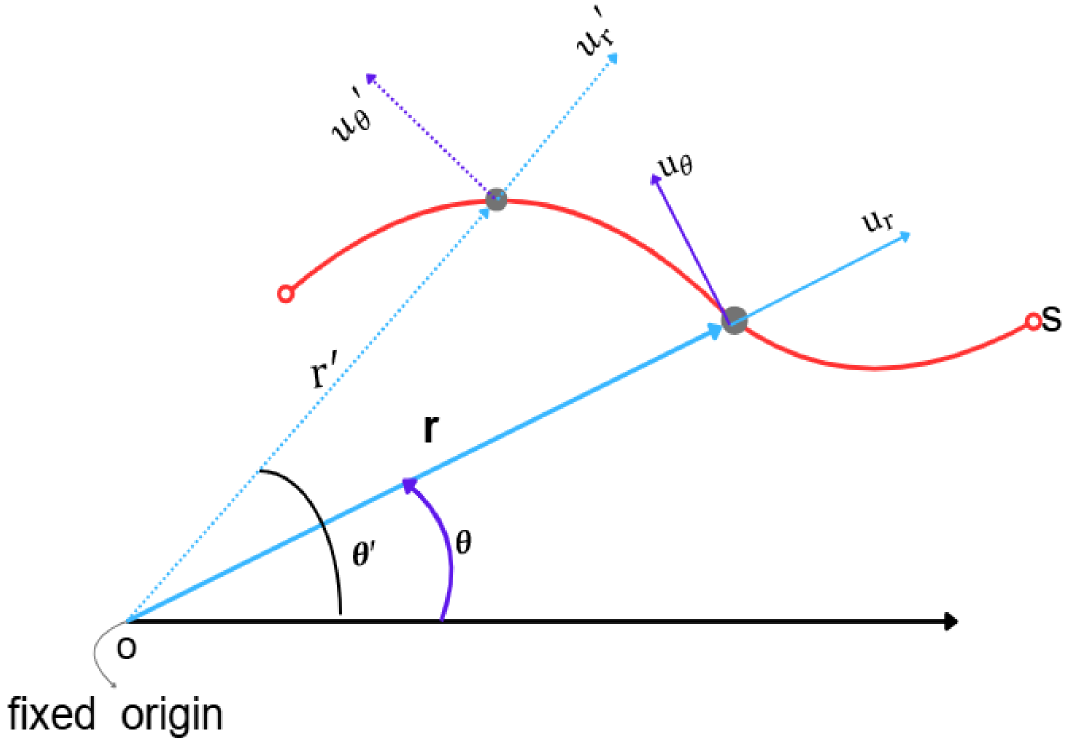


Figure 16. Particle motion.

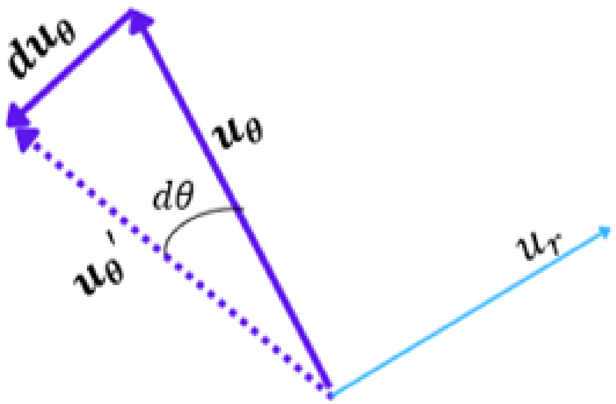


Figure. 17 unit vector

$$\frac{du_r}{dt} = \dot{u}_\theta = -\dot{\theta}u_r$$

cont'd…

- Substituting into the above equation, the acceleration can be written in component form as: **a** $=(\ddot{r}\, u_r - r\dot{\theta}^2 u_r + \dot{r}\dot{\theta} u_\theta + \dot{r}\,\dot{\theta} u_\theta + r\,\ddot{\theta}\, u_\theta)$ or

$$a = a_r u_r + a_\theta\, u_\theta$$

Where: $a_r = \ddot{r} - r\dot{\theta}^2$ $\quad a_\theta = 2\dot{r}\dot{\theta} - r\,\ddot{\theta}$

- Since a_r and a_θ are mutually perpendicular, are always perpendicular, the magnitude of acceleration is simply the positive value of:

$$\mathbf{a} = \sqrt{(\boldsymbol{a_r})^2 + (\boldsymbol{a_\theta})^2}$$

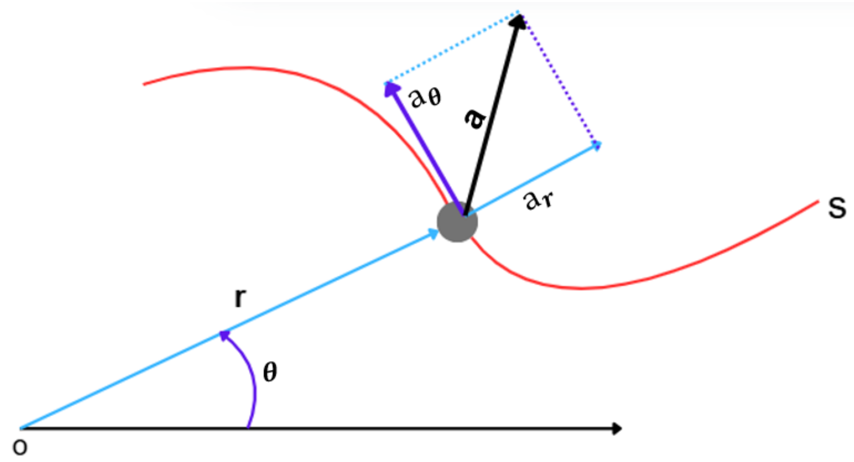


Figure. 18 Acceleration vector

- The direction is determined from the vector addition of its two components. In general, a will not be tangent to the path.

- The term $\ddot{\theta} = \frac{d^2\theta}{dt^2}$, is called the angular acceleration since it measures the change made in the angular velocity during an instant of time. Units for this measurement are $\frac{rad}{s^2}$.

Important Points

- Polar coordinate measure ***change in and direction magnitude in the r and θ direction***
- The velocity will be described as : $V = v_r\, u_r + v_\theta u_\theta$ $v_r = \dot{r}$ $v_\theta = r\dot{\theta}$
- The acceleration will be vector. $a = a_r u_r + a_\theta\, u_\theta$ $a_r = \ddot{r} - r\dot{\theta}^2$ $a_\theta = 2\dot{r}\dot{\theta} - r\,\ddot{\theta}$
- Two types of problems generally occur:

1. If the coordinates are specified as time parametric equations, $\mathbf{r} = \mathbf{r}(\boldsymbol{t})$ *and* $\theta = \theta(\boldsymbol{t})$ then the time derivatives can be found directly.
2. If the time-parametric equations are not given, then the path $r = \boldsymbol{f}(\theta)$ must be known. Using the chain rule of calculus we can then find the relation between $\dot{r}$ *and* $\dot{\theta}$, and between $\ddot{\theta}$ and $\ddot{r}$.

- Once the r and θ components of **v** and **a** have been determined, the magnitudes of these vectors are found from the Pythagorean theorem

Summery on the Important Equations

Rectangular

Position $\boldsymbol{r} = x\boldsymbol{i} + y\boldsymbol{j} + z\boldsymbol{k}$

Velocity $V = V_x\boldsymbol{i} + V_y j + V_z k$ $\quad v = \sqrt{(v_x)^2 + (v_y)^2 + (v_z)^2}$

Acceleration a= $a_x i + a_y j + a_z k$ $\quad \boldsymbol{a} = \sqrt{(a_x)^2 + (a_y)^2 + (a_z)^2}$

Motion of a Projectile

Horizontal Motion $(a_x = 0)$

$X = X_O + (v_o)_x t$

$V_x = constant$

Vertical Motion $(a_y) = -g$

$V_y = (v_o)_y - \boldsymbol{g}\boldsymbol{t}$

$Y = y_O + (v_o)_y t - \frac{1}{2} g t^2)$

$V_y{}^2 = (V_O)^2 - 2\boldsymbol{g}\,(Y - y_O)$

Cylindrical (Polar-coordinate)

Velocity

$V = \dot{r}\, u_r + r\dot{\theta} u_\theta$ where $v_r = \dot{r}$

$\mathbf{v} = \sqrt{(v_r)^2 + (v_\theta)^2}$ $\quad v_\theta = r\dot{\theta}$

Acceleration

$a = a_r u_r + a_\theta\, u_\theta$ where $a_\theta = 2\dot{r}\dot{\theta} - r\,\ddot{\theta}$

$\mathbf{a} = \sqrt{(a_r)^2 + (a_\theta)^2}$ $\quad a_r = \ddot{r} - r\dot{\theta}^2$

r is radial coordinate $\quad\theta$ is *transverse* coordinate

Normal-tangent

Velocity V= $v\, \boldsymbol{u_t}$

Acceleration $\boldsymbol{a} = \boldsymbol{a_t}\, \boldsymbol{u_t} + \boldsymbol{a_n} \boldsymbol{u_n}$

$$\boldsymbol{a_n} = \frac{\boldsymbol{v^2}}{\boldsymbol{\rho}} \qquad \rho = \frac{[1 + (\frac{dy}{dx})^2]^{\frac{3}{2}}}{\frac{d^2y}{dx^2}}$$

$$\boldsymbol{a} = \sqrt{(\boldsymbol{a_n})^2 + (\boldsymbol{a_t})^2}$$

If $\boldsymbol{a_t}$ is varies, $\quad \boldsymbol{a_t} = \frac{dv}{dt}$

$\boldsymbol{a_t}\, ds = v\, dv$

If $\boldsymbol{a_t}$ is constant, $\quad V = V_O + \boldsymbol{a_t t}$

$S = s_O + v_O t + \frac{1}{2}(a_t t^2)$

$V^2 = (V_O)^2 + 2\boldsymbol{a_t}\,(S - s_O)$

Problem 1

The velocity of a particle is given by v = { $16t^2$i + 4 t^3 j + (5t + 2)k} m/s, where t is in seconds. If the particle is at the origin when t = 0, determine the magnitude of the particle's acceleration when t = 2 s. Also, what is the x, y, z coordinate position of the particle at this instant [2]?

Given

v = $16t^2$i + 4 t^3 j + (5t + 2)k

t = 2 s

Required

a) lal=?

$b)\ x_{i,}\ y_{j,}\ and\ z_{k,}\ =?$

Acceleration: obtained by applying differentiating velocity vector.

$$a = \frac{dv}{dt} = 32ti + 12t^2\text{j} + 5\text{k}$$

When t=2s, a=$64i + 48$j + 5k

lal=$\sqrt{64^2 + 48^2 + 5^2} = 80.2\ {}^{m}/{}_{s^2}$

Position: obtained by integrating the velocity function,

$$\int_0^s ds = \int_0^t vdt$$

$$\int_0^s ds = \int_0^t (16t^2\text{i} + 4\ t^3\ \text{j} + (5\text{t} + 2)\text{k})dt$$

$$s = \frac{16}{3}\ t^3 i + \ t^4 j + \left(\frac{5t^2}{2} + 2t\right)k$$

When t=2s, the coordinate of the particle is:

s= (42.7i+ 16j+ 14k) m

Problem 2

cont'd…

The box slides down the slope described by the equation y = $(0.05x^2)$ m, where x is in meters. If the box has x components of velocity and acceleration of v_x= -3 m/s and a_x = -1.5 $^m/_{s^2}$ at x = 5 m, determine the y components of the velocity and the acceleration of the box at this instant [3].

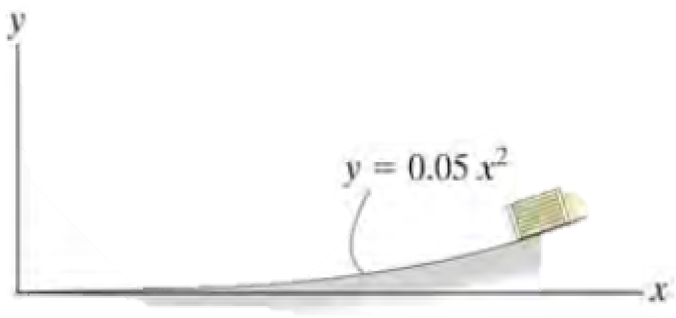


Figure 17. box

Given

y = $(0.05x^2)$ m

v_x= -3 m/s

a_x = -1.5 $^m/_{s^2}$

x = 5 m

Required

$a)\ v_y$= ?

$b)\ a_x$=?

Velocity: by taking the first time derivative of the path's equation using the ch ain rule.

y = $(0.05x^2)$

$\dot{y} = 0.1x\dot{x}$ =or $V_y = 0.1xv_x$

At x=5m and $v_x = -3m/s$ $\boldsymbol{V_y = 1.5m/s}$

Acceleration: obtained by taking the second time derivative of the path's equ ation using the chain rule.

$\ddot{y} = 0.1[\dot{x}\ \dot{x}\ +\text{x}\ \ddot{x}]$

Or

$a_y = 0.1[{v_x}^2 + xa_x]$

At x=5m , $v_x = -3\ m/s$ and a_x = -1.5 $^m/_{s^2}$

$\boldsymbol{a_y = 0.15\ ^m/_{s^2}}$

Problem 3

If the car decelerates uniformly along the curved road from 25 m/s at A to 15 m/s at C, determine the acceleration of the car at B [1].

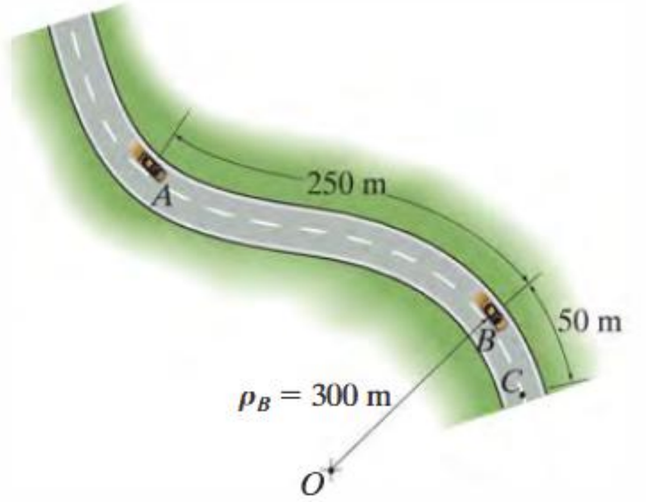


Figure 18. Car

Given

v_A =25 m/s

$v_c = 15$ m/s

$s_{AB} = 250$

$ms_{Ac} = 300m$

$\rho = 300$m

$a_t = constant$

Required

I a_B I=?

The magnitude of the acceleration at point be is calculated as:

I a_B I=$\sqrt{a_t{}^2 + a_n{}^2}$

Tangential acceleration

$v_c{}^2 = v_a{}^2 + 2a_t(s_{Ac})$

$a_t = -0.666\, {}^{m}/_{s^2}$

Then the velocity at point b will be:

$v_B{}^2 = v_a{}^2 + 2a_t(s_{Ab})$

$v_B{}^2 = 295\, m/s$

Normal acceleration

$a_n = \frac{v_B{}^2}{\rho_{Bc}} = 0.966\, {}^{m}/_{s^2}$

I a_B I=$\sqrt{a_t{}^2 + a_n{}^2}$=1.64 ${}^{m}/_{s^2}$

Activity 1

1)At the bottom of a loop in the vertical plane an airplane has a horizontal velocity of 315 mi/h and is speeding up at a rate of 10 $^{ft}/_{s^2}$. The radius of curvature of the loop is 1 mi. The plane is being tracked by radar at O.What are the recorded values of $\dot{r}, \ddot{r}$, $\dot{\theta}$ and $\ddot{\theta}$, for this instant [4]?

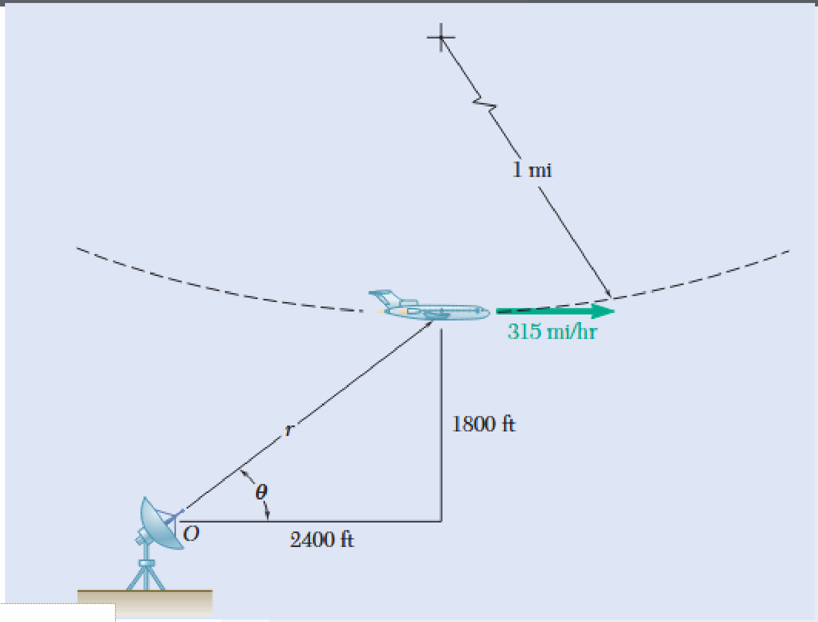


Figure 19. Radar

2) Skateboard rider leaves the ramp at A with an initial velocity v_A at a 30° angle. If he strikes the ground at B, determine v_A and the time of flight [1].

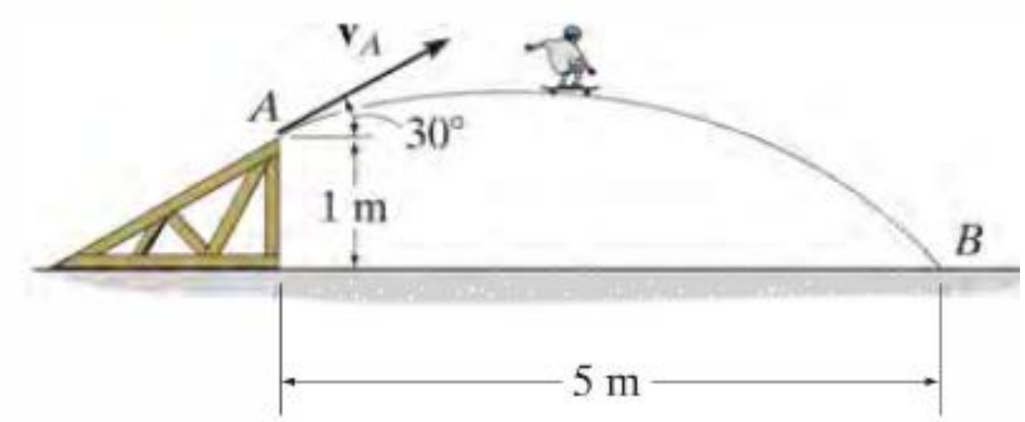


Figure 20. skateboard rider

3) The platform is rotating about the vertical axis such that at any instant its angular position is $\theta = (4t^{3/2})$ rad, where t is in seconds. A ball rolls outward along the radial groove so that its position is r = $(0.1t^3)$ where t is in seconds. Determine the magnitudes of the velocity and acceleration of the ball when t = 1.5 S [1].

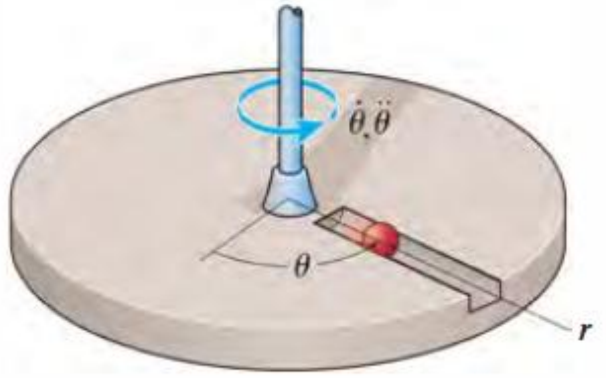


Figure 21. platform

4) When x = 10 ft, the crate has a speed of 20 ft/s which is increasing at $6\ ^{ft}/_{s^2}$. Determine the direction of the crate's velocity and the magnitude of the crate's acceleration at this instant [1].

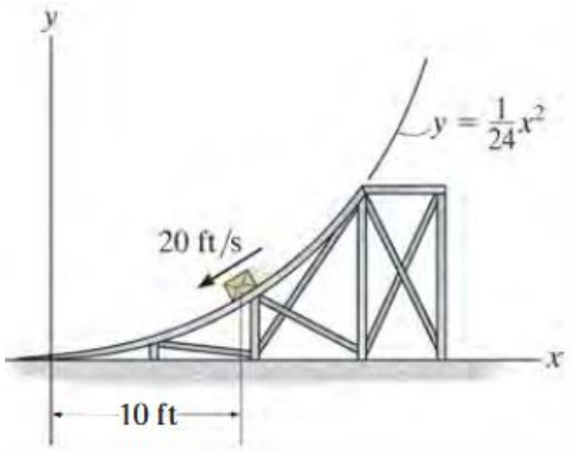


Figure 22. crate

Summary

In Today Lecture We studied:

1. Scope of Curvilinear Motion
2. Rectangular Components
3. Normal and Tangential Components
4. Understand Cylindrical Components
5. The difference between the 3 - coordinates
6. Solving problems and Understand how to use the equations

References

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[3] Engineering Mechanics - Dynamics, Meriam J.L.,John Wiley & Sons, 9th ed., 2020.,

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