Week 5

Chapter 3
Kinetics of Particles – Newton's Second Law

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Contents

By the end of this lecture, you are able to:

1. Understand the kinetics of particles and its scope
2. Define and explain (Newton's 2nd law) equations of motion
3. Define force and its type
4. Equations of motion for rectilinear motion
5. Equations of motion for Curvilinear motion

Understand the kinetics of particles and its scope

- Kinetics is a branch of dynamics that deals with the relationship between the change in motion of a body and the forces that cause this change [1].
- It basis on is Newton's second law, which states that when an unbalanced force acts on a particle, the

particle will accelerate in the direction of the force with a magnitude that is proportional to the force.[1].

- Recall that we can consider an actual body—including bodies as large as a car, rocket, or airplane—as a particle for the purpose of analyzing its motion, as long as the effect of a rotation of the body about its center of mass can be ignored .
- kinetics of particle is defined as the study of translational motion of rigid bodies with the causing force

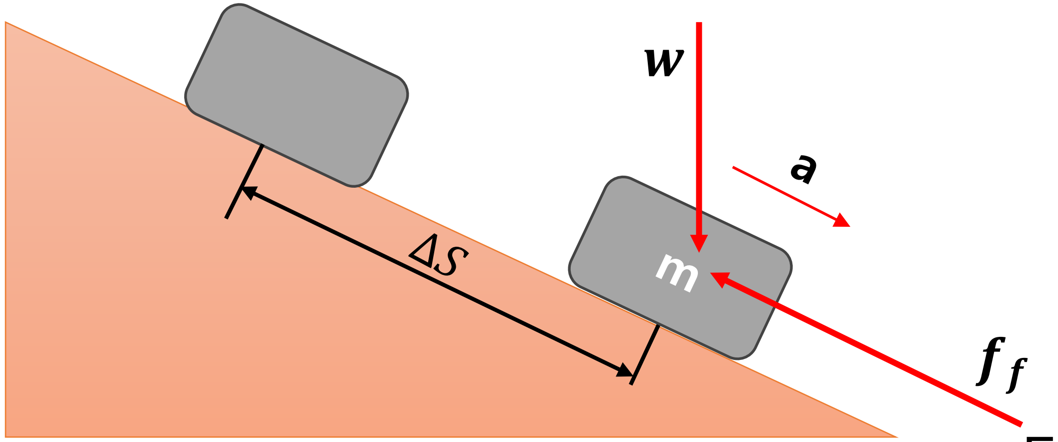


Figure 1. Typical kinetics system

scope Cont'd…

- This chapter requires our combined knowledge of the properties of forces, which we developed in statics, and the kinematics of particle motion just covered in Chapter 2 [1].
- The three general approaches to the solution of kinetics problems are:
- (A) direct application of Newton's second law (called the force- mass-acceleration method),

  (B) use of work and energy principles, and

  (C) solution by impulse and momentum methods
- Each approach has its special characteristics and advantages.

scope Cont'd…

- **Chapter 3 is subdivided into Sections A, B, and C, according to these three methods of solution**

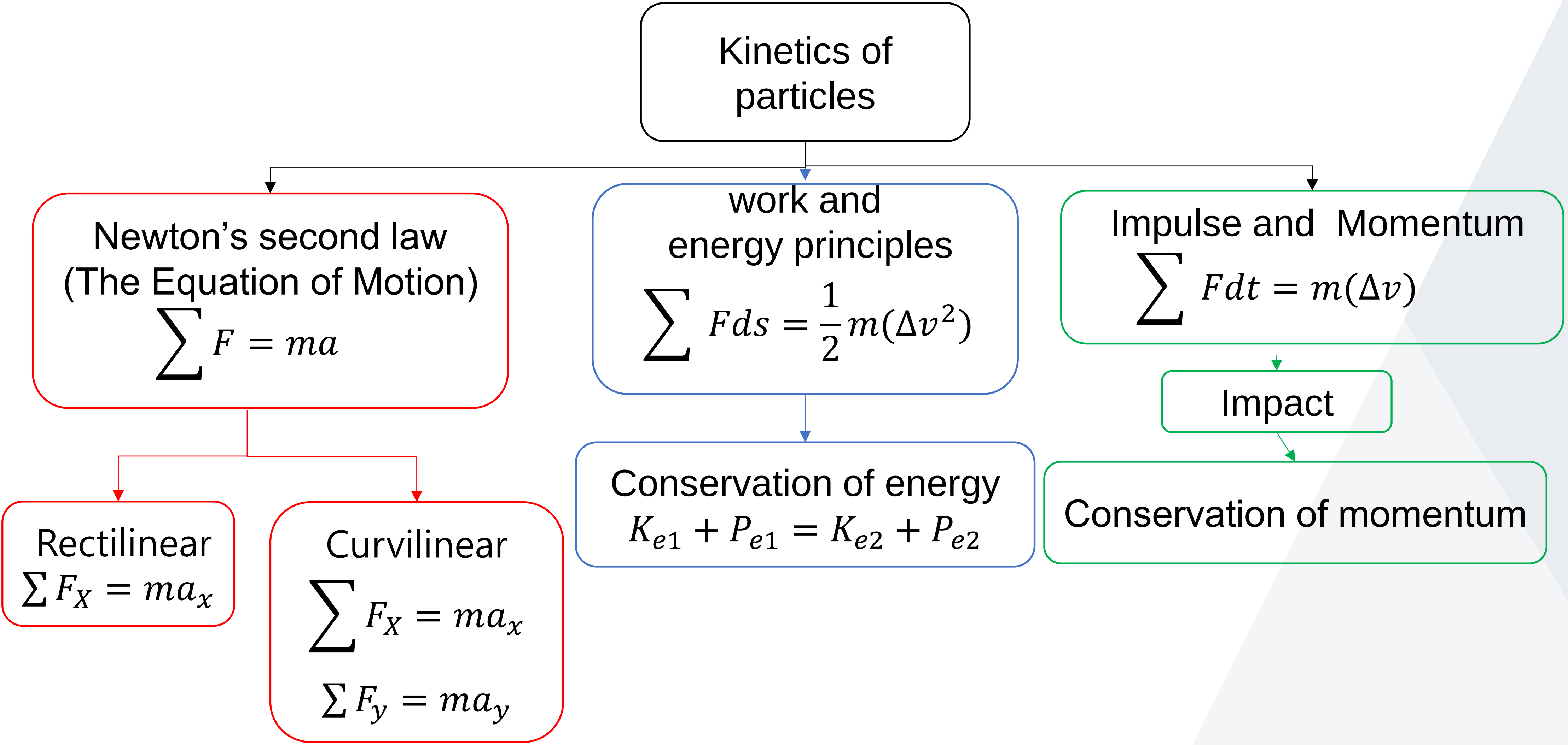


Figure 2. Chapter structure

Forces in Particle Kinetics

- Forces are vectors that have a magnitude and direction. The **Common forces include gravitational force(weight), normal force, friction, Applied Force , tension, and spring force** [1] .

- To study the motion of particles, it is very important to understand how these five types of forces behave. Before looking at each force, we first need to understand three basic ideas: sense, magnitude, and direction

Magnitude of Force

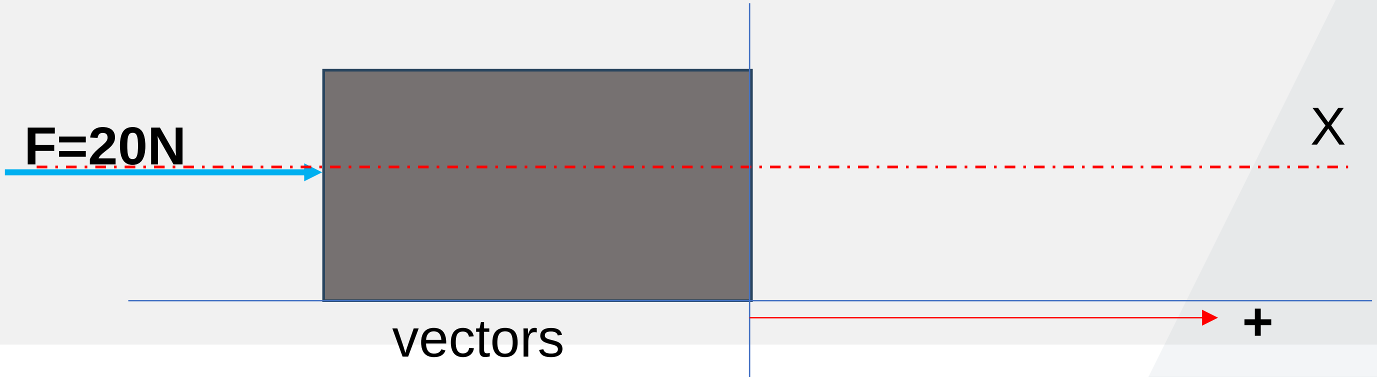
- The size or amount of the force applied. It tells us how strong the force is. With Newton (N) unit . E.g : If you push a box with 20 N of force, the magnitude of the force is 20.

Direction of Force

- It is the line along which the force is applied. It tells us where the force is pointing using coordinates. E.g: If you pull a cart towards the east or x , the direction of the force is eastward or x.

Sense of Force

- The orientation of the force along its line of action. It tells us which way along the direction the force is acting (positive or negative along that line).



Forces in Particle Kinetics

Force categories

Applied Force

- Magnitude: Depends on how much force is exerted externally.
- Direction: Along the line in which the external agent applies it.
- Sense: Follows the pulling or pushing action (toward or away)

E.g: F_A is considered as applied force .

Gravitational Force (Weight)

- Magnitude: Equal to $W = m \cdot g$
- Act along the vertical line of action through the object's center of mass.
- Sense: Always downward toward the center of the Earth

E.g: A 10 kg box on the ground
W=10×9.8=98 N downward.

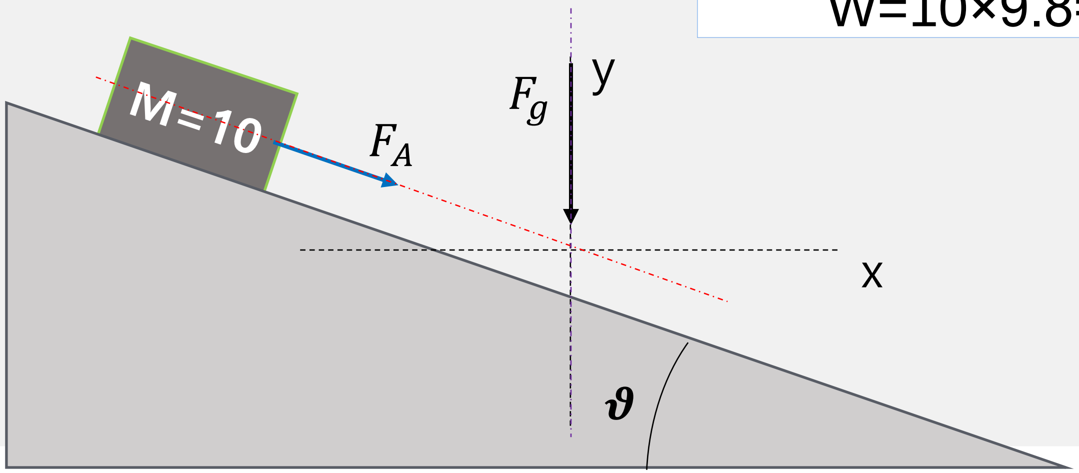


Figure 3. Inclined plane

Forces in Particle Kinetics

Force categories

Normal Force

- Magnitude: For 1D : Obtained using equilibrium condition
  For 2D: Obtained using equation perpendicular to direction of motion condition
- Direction: Perpendicular to the surface in contact
- Sense: Always Reacts

E.g.: For 10 kg box on the inclined floor

$\sum F_z = 0$, $F_N = \frac{F_G}{\cos\theta} = \frac{9.81(10)}{\cos\theta}$

Frictional Force

- Magnitude: $f_f \le \mu F_N$, where μ = coefficient of friction.
  $\mu_s = static\ friction\ coffcent\ used\ at\ rest\ condtion$
  $\mu_k = kenetic\ friction\ coffcent\ used\ at\ rest\ condtion$
- Direction: Along or parallel to the surface of contact or direction of motion.
- Sense: Opposite to direction of motion (attempted motion)

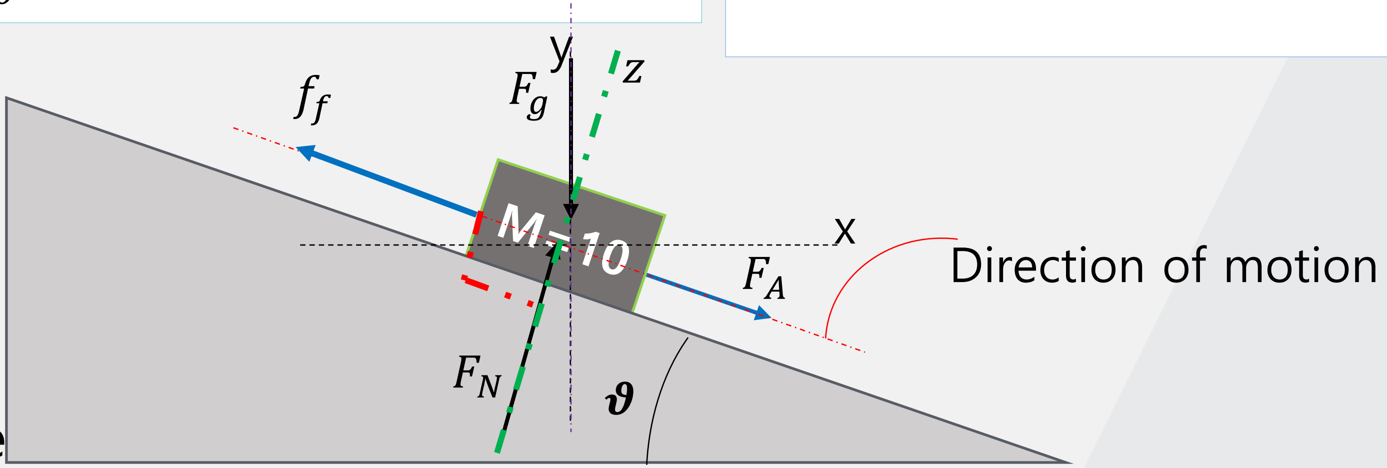


Figure 4. Inclined plane

Forces in Particle Kinetics

Force categories

Tension Force

- Magnitude: Equal throughout the rope/string if massless and frictionless pulley.
- Direction: Along the rope/string.
- Sense: Sense: Away from the object, pulling along the rope.

E.g: Tesion T shown in the diagram

Spring Force

- Magnitude: Fs=kx where k = spring constant, x = displacement

$\Delta x = final\ spring\ length - orginal\ (unstrached) length$

- Direction: Along the axis of the spring..
- Sense: Points towards original unstarched length

E.g: $F_S = k\Delta x = k\,(l_f - l_o)$

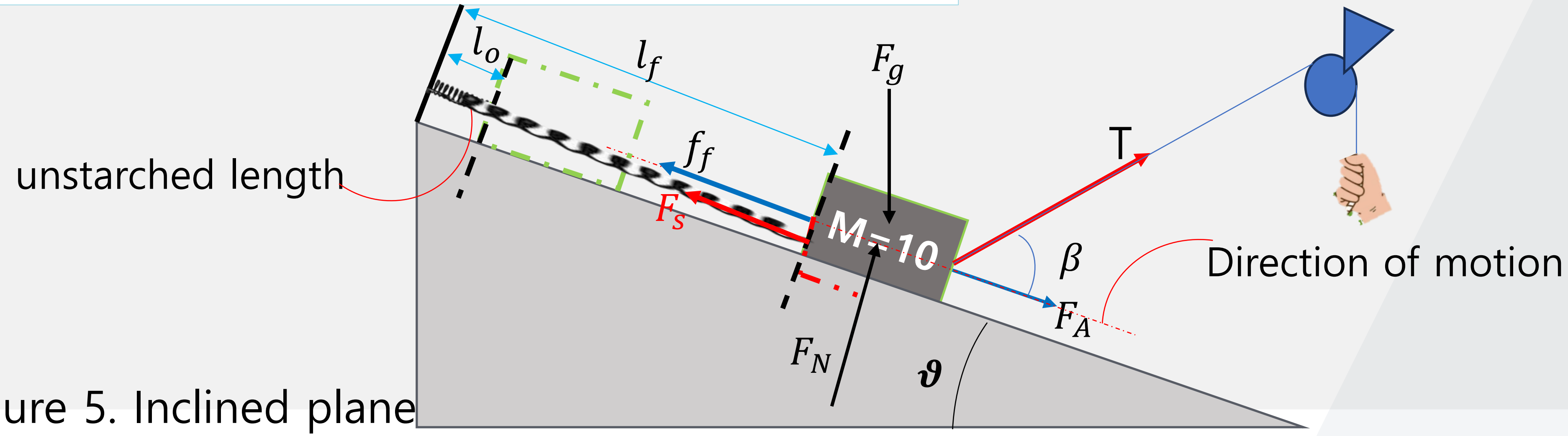
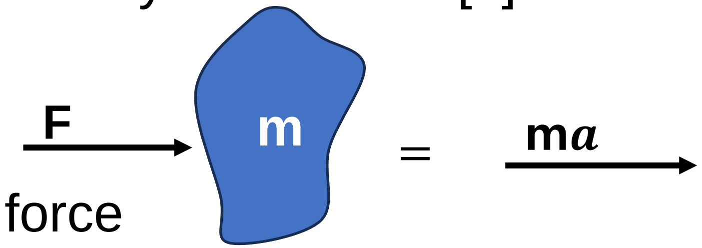


Figure 5. Inclined plane

Newton’s second law

- **If the resultant force acting on a particle is not zero, the particle has an acceleration proportional to the magnitude of the resultant and in the direction of this resultant force** [3].

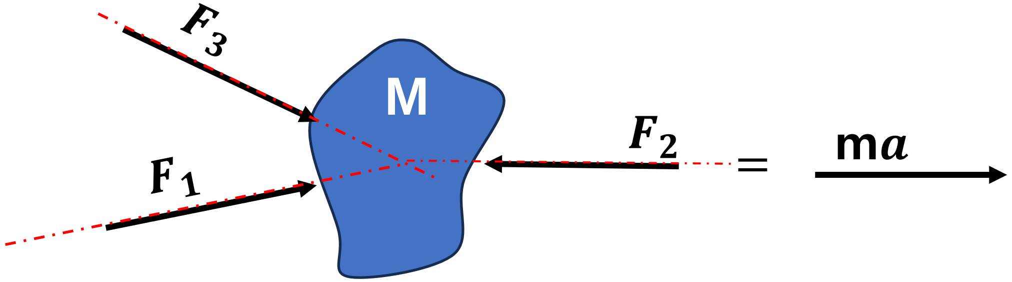
- When a particle P of mass m is acted upon by a force F, the force F and the acceleration a of the particle must therefore satisfy the relation [2].



$$F = ma$$

Figure 6. Single force

- When a particle of mass m is subjected to the action of concurrent forces F1 , F2 , F3 , . . . whose vector sum is ΣF, the above become:



$$\sum F = ma$$

N.B: *forces as concurrent through the mass center.*

Figure 7. Force system

- It is usually called the equation of motion. It gives the instantaneous value of the acceleration corresponding to the instantaneous values of the forces which are acting.

Equation of motion cont'd…….

- In general, The equation of motion states that the unbalanced force on a particle causes it to accelerate.
- We now apply the concept equation of motion to particle motion, starting with rectilinear motion with single and multiple particle and then curvilinear motion.

  - we will analyze the motions of bodies which can be treated as particles.
  - This simplification is possible as long as we are interested only in the motion of the mass center of the body.
  - In this case we may treat the forces as concurrent through the mass center

Figure 6. Continuous vs erratic Motion

1. Rectilinear motions

- If the particle with mass m is moving in straight line and If we choose the line to be x-direction, the acceleration in the y- and z-directions will be zero [3].

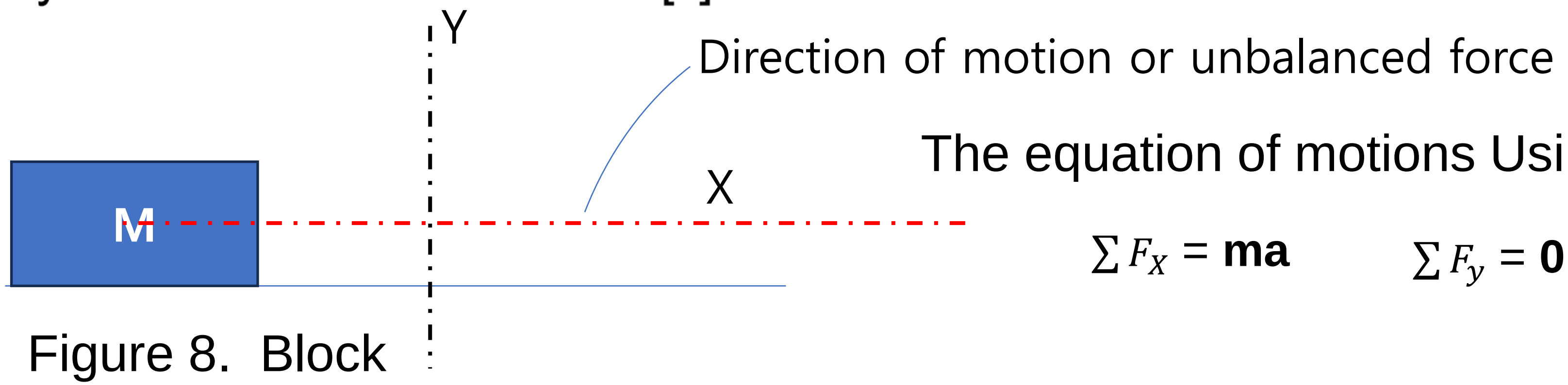


Figure 8. Block

The equation of motions Using Cartesian coordinate

$\sum F_x$ = **ma** $\sum F_y$ = **0** $\sum F_z$ = **0**

Alternatively, if consider normal tangent coordinate and we call the direction of motion tangential direction

$\sum F_t = \mathrm{m}a_t$ $\sum F_n = 0$

General, here first determine the direction in which the motion occurring then. The equation is written as:

The unbalanced force = the **mass** of the particle × its **acceleration** in the **direction** of that unbalanced force.

In all other perpendicular directions, since there is no motion, the conditions of equilibrium must hold.

Important point

Types of problems of dynamics

1) specified kinematic conditions, find forces straightforward application of Newton's law as algebraic equations

2) specified forces, find motion Difficulty depends on the form of force function (t, s, v, a), as the solutions are found by solving a system of differential equations.

Case 1 .The ball has a mass of 30 kg and as speed v = 4 m/s when $\theta = 0°$. Determine the tension in the cord and the when $\theta = 20°$

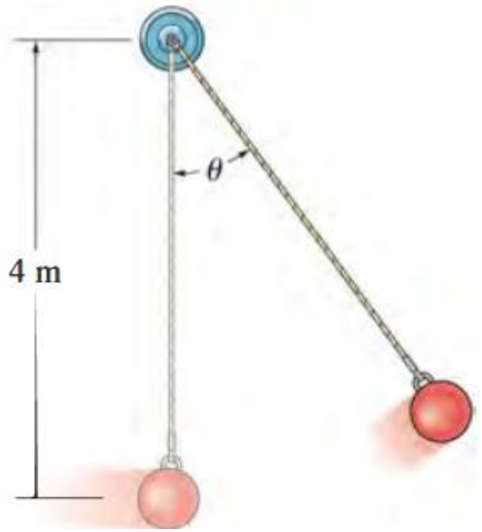


Figure 9. Ball

Case 2. Determine the acceleratio n of the 10kg block when s = 0.4 m. The contact surface is smooth.

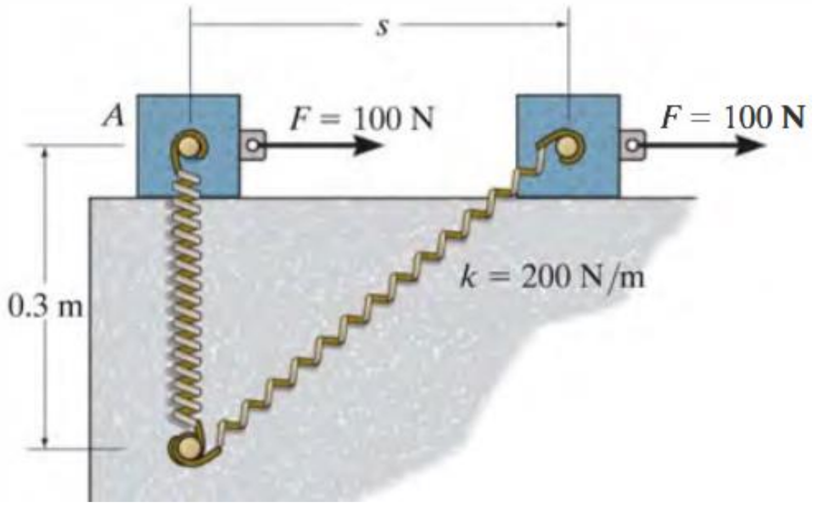


Figure 10. Block with force

Important point

- The third type of kinematics problem deals with systems of particles (like two or more blocks connected by a rope).
- To solve them, we need more than one equation, so we use simultaneous equations. These equations come from two places:
  - Rope relation (kinematics): The motion of one block is linked to the other by the rope. The number of rope relations depends on how many ropes are in the system.
  - Free-body diagram (dynamics): For each particle, we draw forces and apply Newton's second law. Each particle gives one equation of motion.

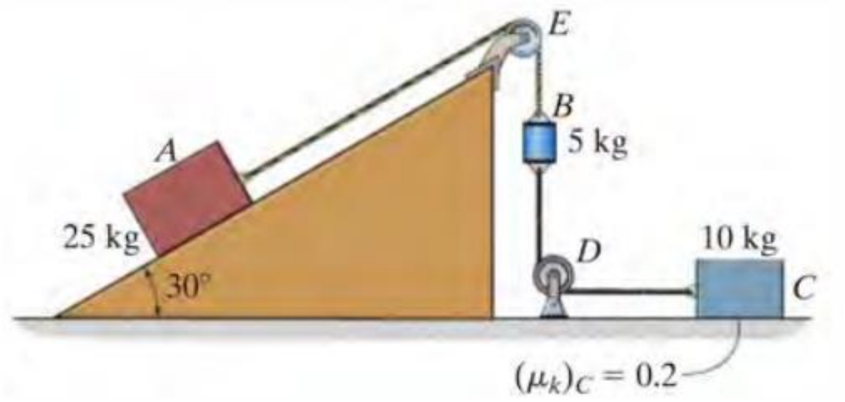


Figure 11. dependent Motion

1. Curvilinear motions

- As discussed earlier, the equations of motion of a particle relate mass, acceleration and the force which causes the motion.
- When a particle moves along a curved path, the equation of motion for the particle may be written in 2 or 3 dimensions in indifferent form depending on the coordinate types under consideration [4]. .
- The acceleration of a particle is a vector which can be resolved into two perpendicular components and also we will have unbalanced force resolved at least in 2 components along the chosen coordinate directions.

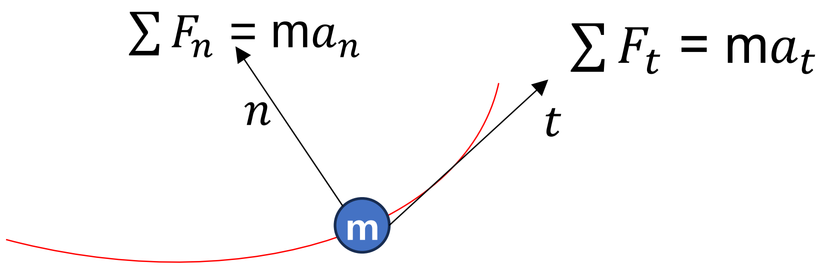


Figure 12. Normal-tangent

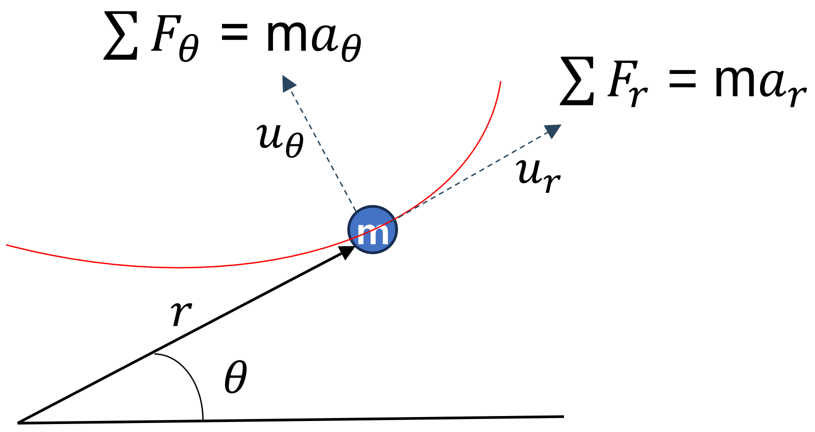


Figure 13. Polar

- we look at the forces that cause the particle to change both the speed and the direction of motion.

Equation of motion – Normal tangent

- When a particle moves along a curved path which is known, the equation of motion for the particle may be written in the tangential, normal, and binormal directions [4].
- We resolve the forces and the acceleration of the particle into components along the tangent to the path (in the direction of motion) and the normal (toward the inside of the path).

$$\sum F_t = ma_t \quad \sum F_n = ma_n \quad \sum F_{bn} = 0$$

- Recall that $a_t = \frac{dv}{dt}$ represents the time rate of change in the magnitude of velocity and acts in the direction of motion
- Likewise, ($a_n = \frac{v^2}{\rho}$) represents the time rate of change in the velocity's direction ,and always acts toward the path's center of curvature
- N.B. there is no motion of the particle in the binormal direction, since the particle is constrained to move along the path

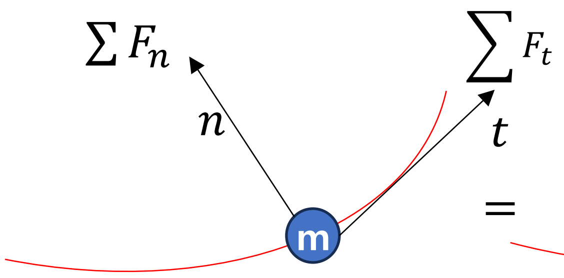


Figure 14. FBD

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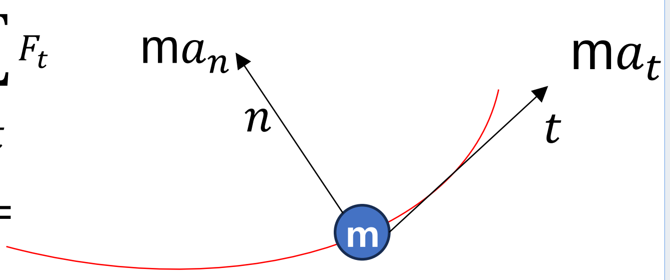

Figure 15. kinetic diagram

Important point

- Force Resolution in normal tangent Coordinates
- Sometimes the direction forces such us applied force and gravitational force may not align with normal or tangent directions [3]. .
- If If y= $f(x)$ is known , the angle θ between these force and tangent directions is obtained using the given equation.

$$\tan(\theta)=\frac{dy}{dx}$$

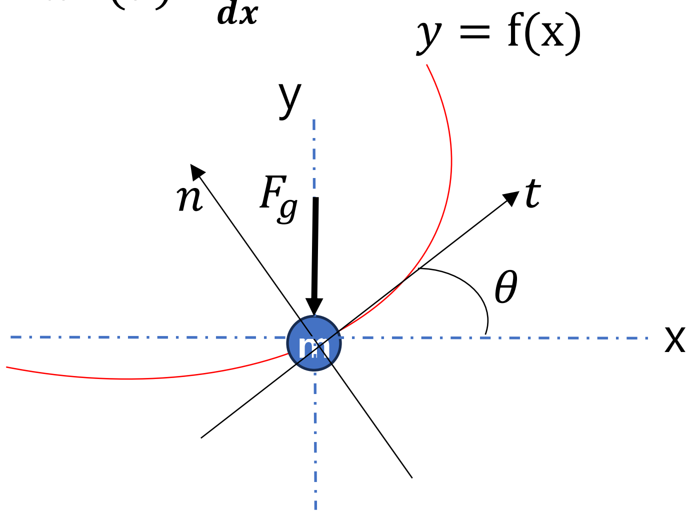

Figure 16. Force Resolution

Equation of motion- polar coordinate

- Consider a particle P, that moves in a plane under the action of several forces. When all the forces acting on a particle are resolved into cylindrical with coordinates r and θ, i.e., along the unit-vector directions u_θ *and* a_r Fig. 13-16, the equation of motion can be expressed as:

$$\sum F_\theta = ma_\theta \qquad \sum F_r = ma_r \qquad \sum F_{bn} = 0$$

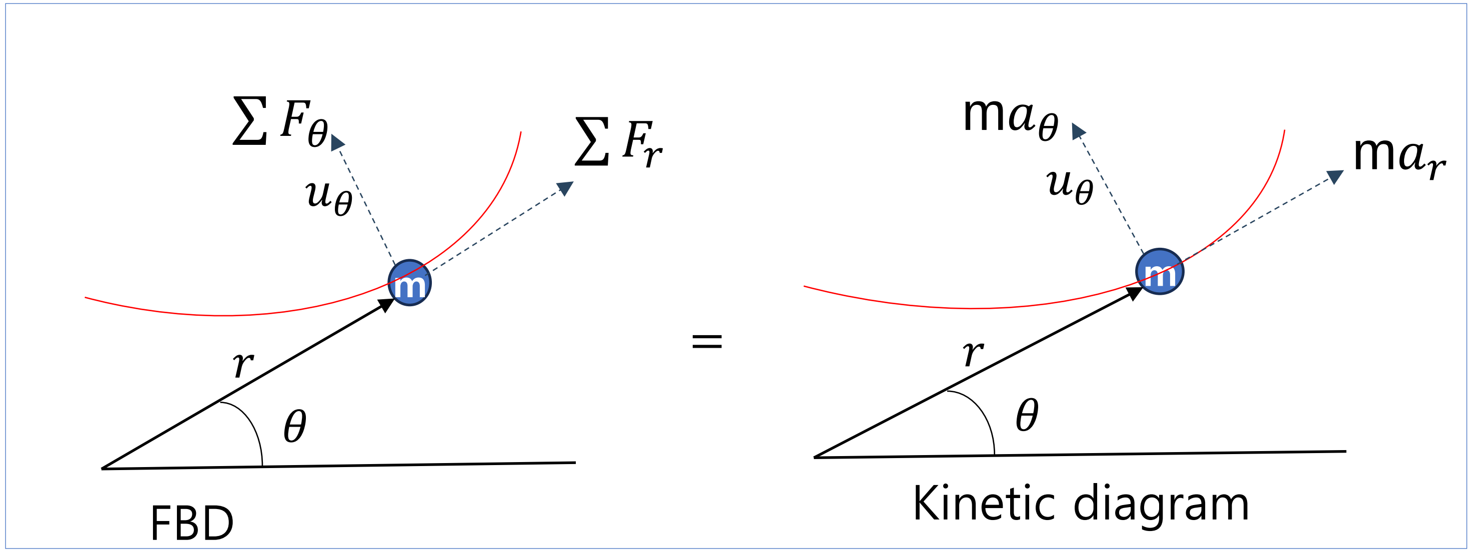

Figure 17. Polar coordinate

Substituting $a_\theta = r\ddot{\theta} - 2\dot{r}\dot{\theta}$ and $a_r = \ddot{r} - r\dot{\theta}^2$ we have:

$$\sum F_r = m\,(\ddot{r} - r\dot{\theta}^2)$$

$$\sum F_\theta = m(r\ddot{\theta} - 2\dot{r}\dot{\theta})$$

Important point

Force Resolution in Polar Coordinates

- Sometimes acceleration is unknown, but forces are given and some of These forces such us friction force and normal force may not align with radial or transverse directions [3]. .
- To solve we need to Find the angle between tangential and radial directions that help us to Resolve forces into radial and transverse components and Use these components to calculate the required acceleration.

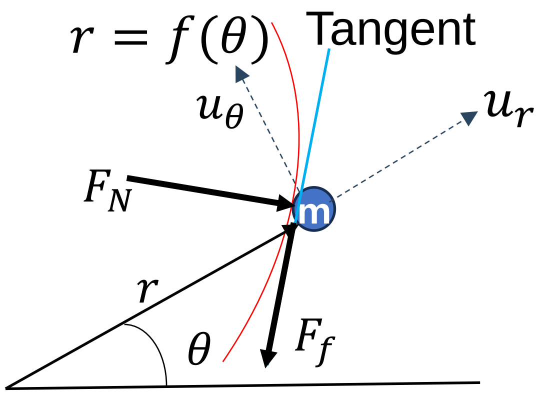

Figure 18. Force Resolution

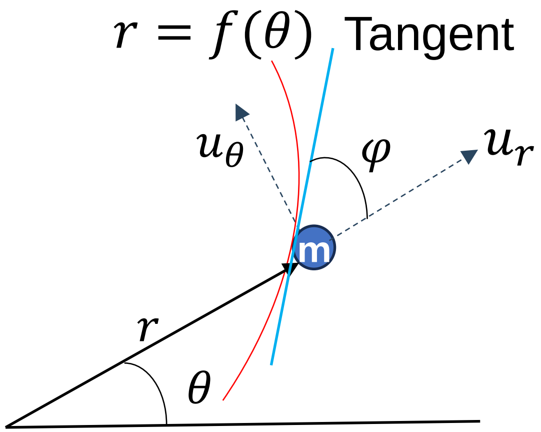

Figure 19. Angle Resolution

- If $r = f(\theta)$ is known, The angle is obtained using the given equation.

$$\tan(\varphi) = \frac{r}{dr/d\theta}$$

Procedure for problem solving

- Two of the most important tools you will use in solving dynamics problems, particularly those involving Newton's second law, are the Freebody diagram and the kinetic diagram [3].
- These diagrams will help you to model dynamic systems and apply appropriate equations of motion.
- A Free Body Diagram is a graphical representation that shows all the external forces acting, coordinates or axis, dimensions on a particle or body, isolated from its surroundings.
- A Kinetic Diagram (also called an Inertia Diagram) shows the inertial (ma) effect of the particle's motion as a vector, typically drawn alongside the FBD to apply Newton's second law.

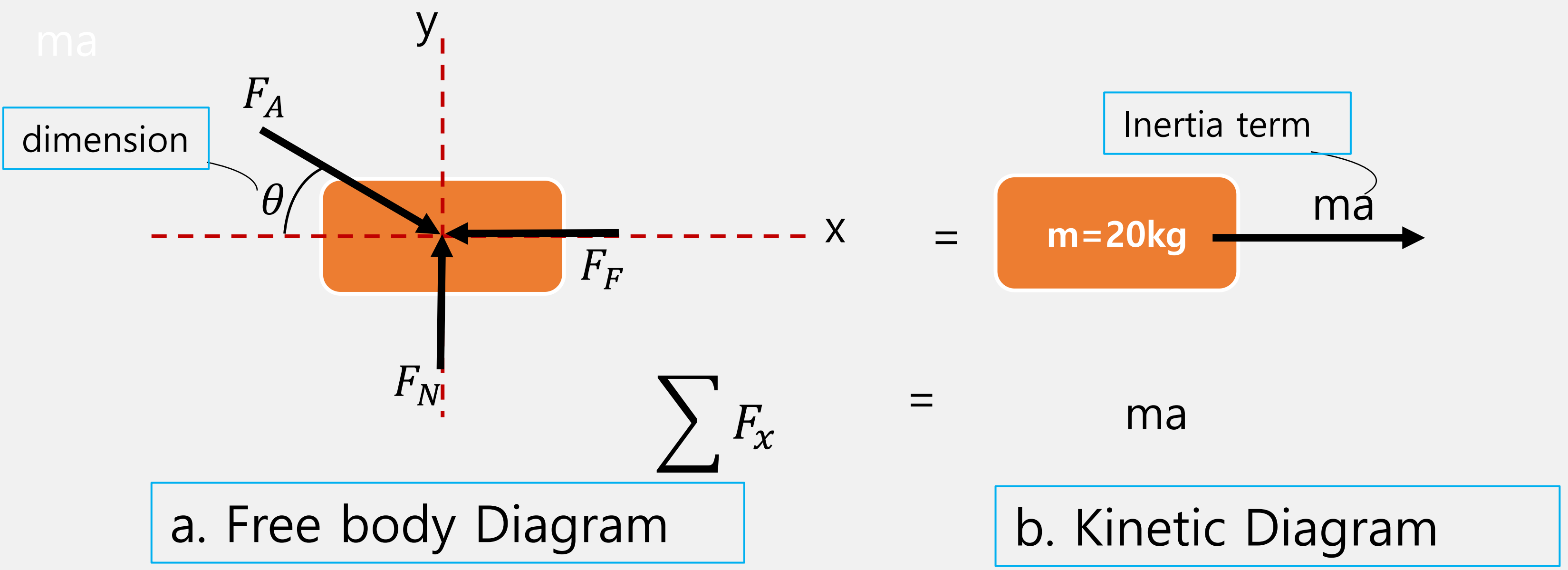

Figure 20. FBD and kinetic diagram

Procedure for problem solving

Steps for free-body diagram

- **Body:** Define your system by isolating the body (or bodies) of interest. If a problem has multiple bodies (such as constrained motion), you may have to draw multiple free-body diagrams and kinetic diagrams.
- **Axes** : Draw an appropriate coordinate system (e.g., Cartesian, normal and tangential, or radial and transverse).
- **Support Forces:** Replace supports or constraints with appropriate forces (e.g., two perpendicular forces for a pin, normal forces, friction forces).
- **Applied Forces and Body Forces:** Draw any applied forces and body forces (also sometimes called field forces) on your diagram (e.g., weight, magnetic forces, a known pulling force).
- **Dimensions:** Add any angles or distances that are important for solving the problem.

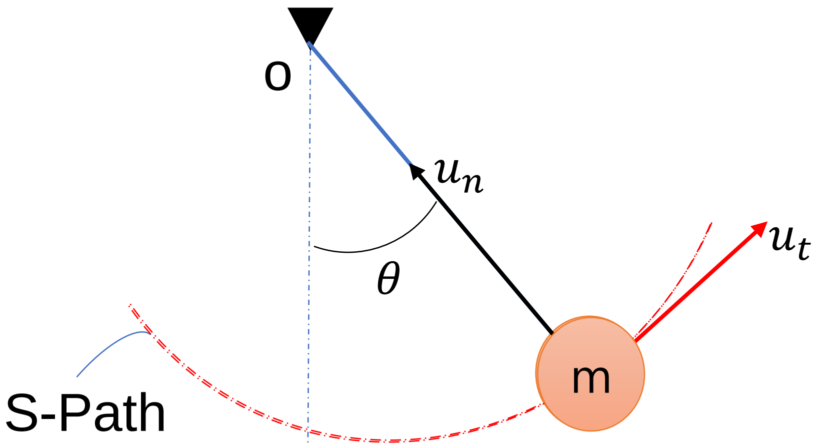

Figure 21. system

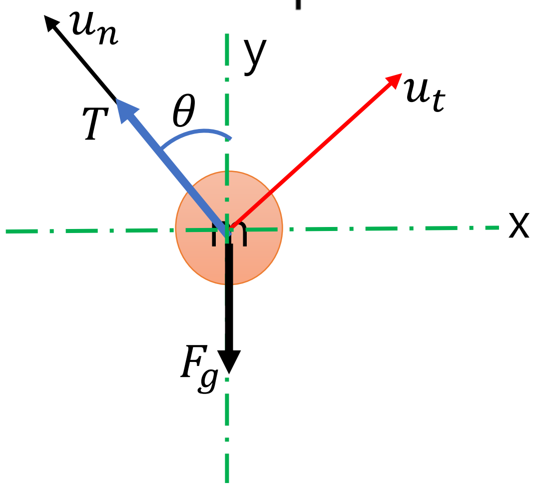

Figure 22. free body

Procedure for problem solving

steps cont'd….

Steps for kinetic diagram

- Body: This is the same body as in the free-body diagram; place this beside the free-body diagram.
- Inertial terms: Draw the ma term to be consistent with the coordinate system. Generally, draw this term in different components (e.g., ma_x and ma_y or ma_n and ma_t). If they are unknown quantities, it is best to draw them in the positive directions as defined by your coordinates.

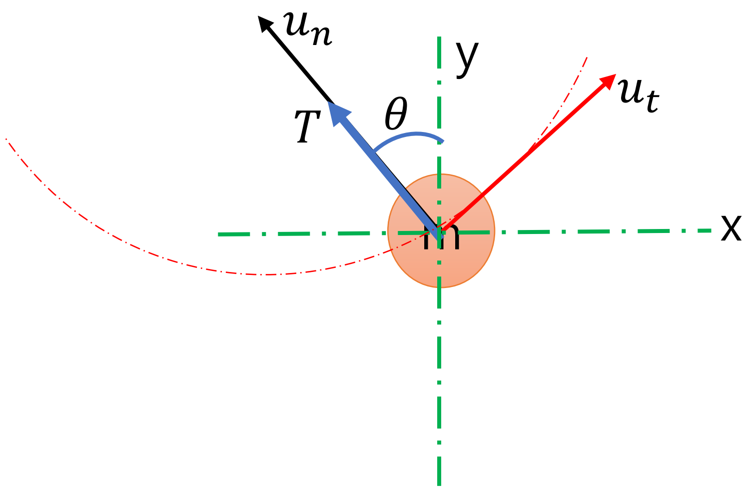

Figure 23. free body

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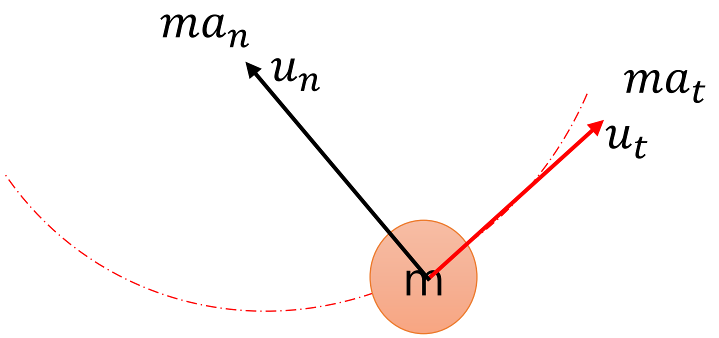


Figure 24. Kinetic diagram

Equation of motion for the conditions shown

$$\sum F_t = ma_t$$

$$\sum F_n = ma_n$$

Summary on equation of motion

- Kinetics is the study of the relation between forces and the acceleration they cause.
- This relation is based on Newton's second law of motion, expressed mathematically as

$$\sum F = ma$$

- Before applying the equation of motion, it is important to first draw the particle's FBD and kinetic diagram.

Rectilinear motion

Equation of motion only applied in 1D and in other direction are in equilibrium

$$\sum F_x = ma_x \qquad \sum F_y = 0 \qquad \sum F_z = 0$$

Curvilinear motion

Polar Co-ordinate

$\sum F_\theta = ma_\theta$ $\quad a_r = (\ddot{r} - r\dot{\theta}^2)$

$\sum F_r = ma_r$ $\quad a_\theta = (r\ddot{\theta} - 2\dot{r}\dot{\theta})$

$\sum F_{bn} = 0$

Normal-tangent Co-ordinate

$\sum F_t = ma_t$ $\quad a_t = \frac{dv}{dt}$

$\sum F_n = ma_n$ $\quad (a_n = \frac{v^2}{\rho})$

$\sum F_{bn} = 0$

Problem 1

The spring has a stiffness k = 200 N/m and is unstretched when the 25-kg block is at A. Determine the acceleration of the block when s = 0.4 m. The contact surface between the block and the plane is smooth [3]. .

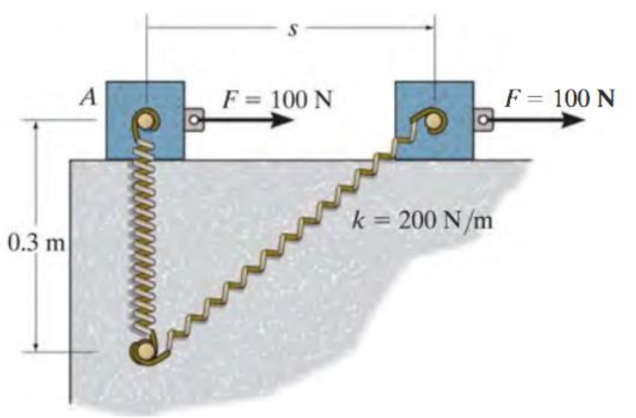
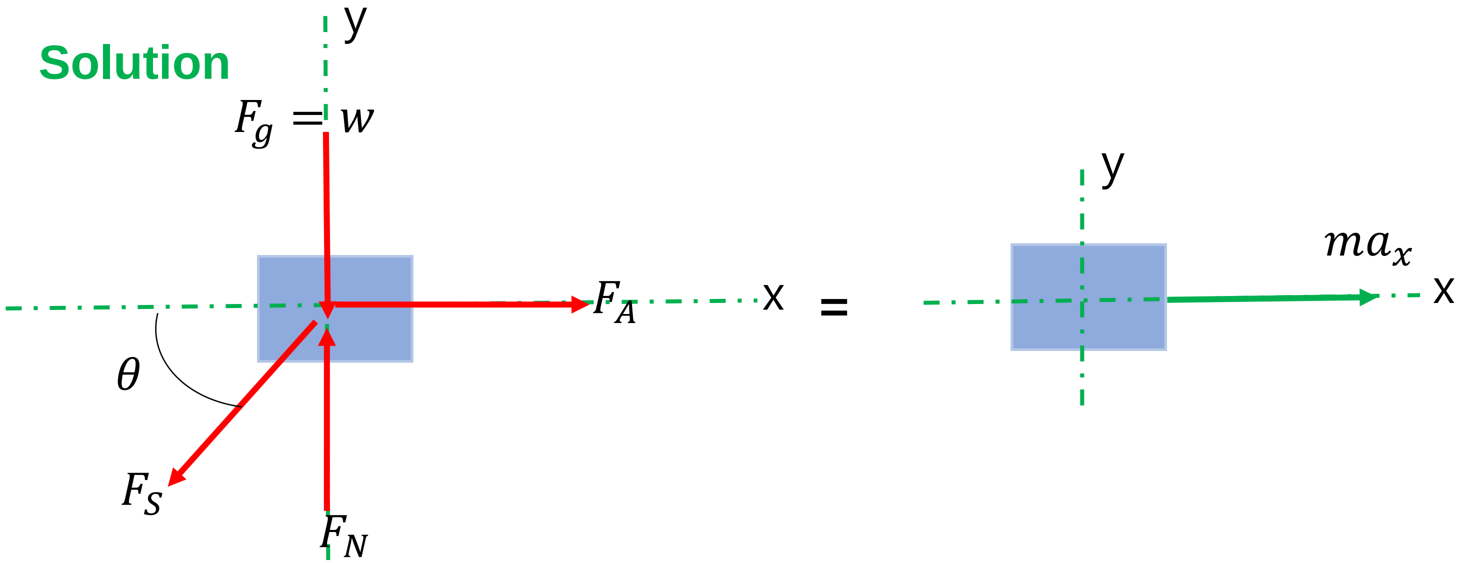


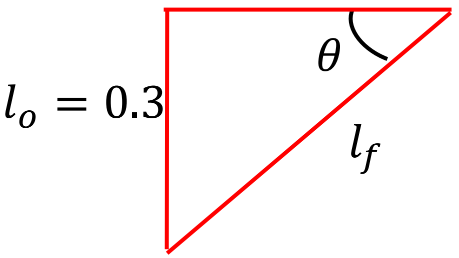
Figure 25. Problem 1

Solution



FBD

KD



From geometry:

$l_f = 0.5$

$\theta = 36^0$

Given

$$K = 200 N/m$$

$$m = 25kg$$

$$s = -0.4m$$

$$f_f = 0 (smooth\ surface)$$

$$F_A = 100N$$

Required

$\boldsymbol{a} = ?$

$$\sum F_X = ma_x$$

$$F_A - F_s cos\theta = ma_x$$

Force magnitude

$$F_s = k\Delta s = K(l_f - l_o), F_s = 200(0.5 - 0.2) = 40N$$

$$a_x = 2.7\ {}^{m}/_{s^2}$$

ans

Problem 2

The 1.5-kg cylinder C travels along the path described by $r = (0.6 \sin\theta)$m. If arm OA rotates counterclockwise with a constant angular velocity of $\dot{\theta}$ = 3 rad/s, determine the force exerted by the smooth slot in arm OA on the cylinder at the instant θ= 60°. The spring has a stiffness of 100 N/m and is unstretched when θ = 30° . The cylinder is in contact with only one edge of the slotted arm. Neglect the size of the cylinder. Motion occurs in the vertical plane [1].

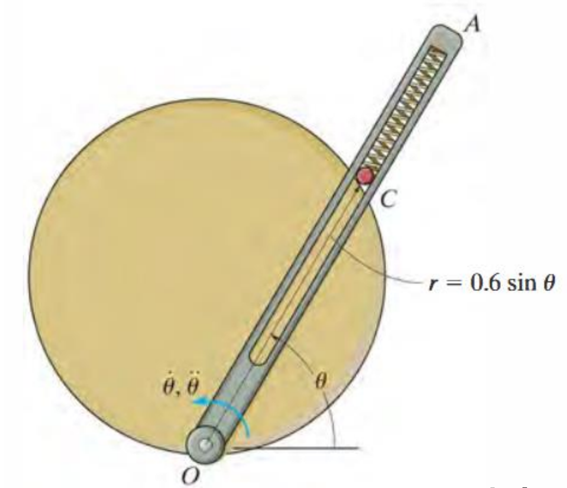
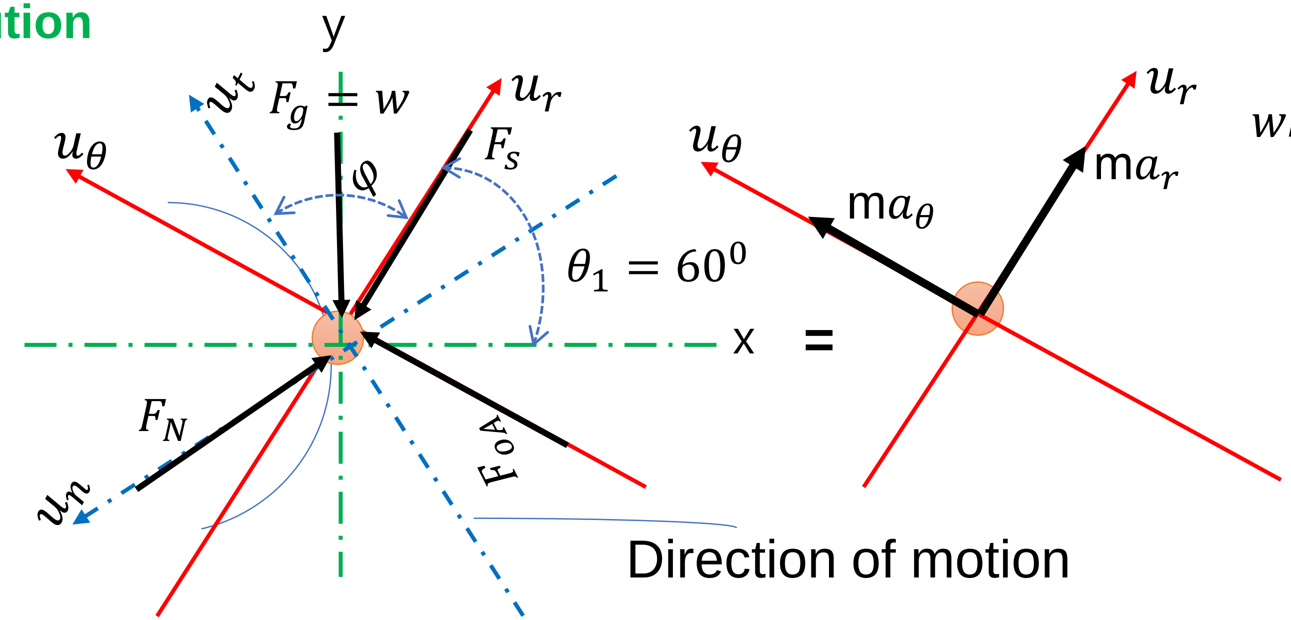


Figure 26. Problem 2

Solution



$$where: tan(\varphi) = r \Big/ \frac{dr}{d\theta}$$

$$tan(\varphi) = \frac{0.6 \sin 60}{0.6 \cos 60}$$

$$(\varphi) = 60^0$$

Given

- $M=1.5kg$
- $r(\theta) = (0.6 \sin\theta)$m
- $\theta_0 = 30^0 \ and \ \theta_1 = 60^0$
- $\dot{\theta}$ = 3 rad/s
- $\ddot{\theta} = 0 \ (Constant)$
- $k = 100 \ N/m$

Required

$F_{OA} = ?$

$$\sum F_r = ma_r$$

$$\sum F_\theta = ma_\theta$$

Where from kinematics

$$a_r = (\ddot{r} - r\dot{\theta}^2)$$

$$a_\theta = (r\ddot{\theta} - 2\dot{r}\dot{\theta})$$

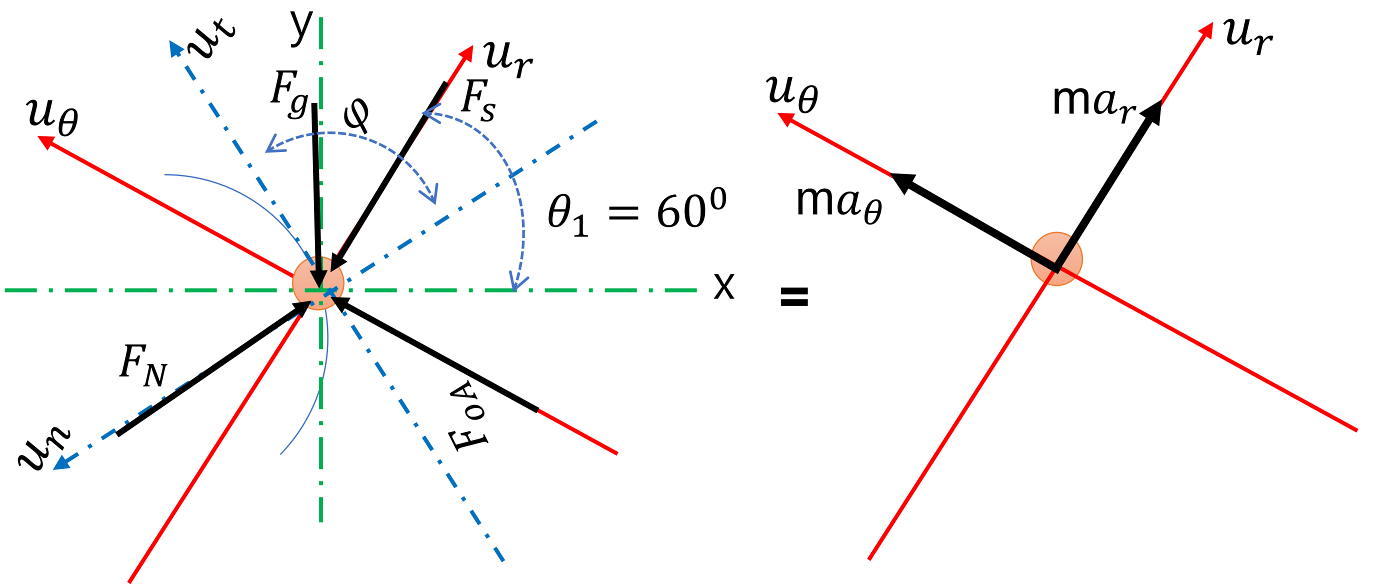
Solution cont'd…….

Given

- $M=1.5kg$
- $r = (0.6 \sin\theta)$m
- $\theta_0 = 30^0$ and $\theta_1 = 60^0$
- $\dot{\theta} = 3$ rad/s
- $\ddot{\theta} = 0$ $(Constant)$
- $k = 100\ N/m$

Required

$\boldsymbol{F_{OA}} = ?$



$where: tan(\varphi) = {}^{r}\!/\frac{dr}{d\theta}$

$$tan(\varphi) = \frac{(0.6 \sin 60}{0.6 \cos 60}$$

$$(\varphi) = 60^0$$

r= $0.6(sin\theta) = 0.519\ m$ $\quad \dot{r} = 0.6(cos\theta)\dot{\theta} = 0.9\ m/s$ $\quad \ddot{r} = 0.6((-sin\theta)\dot{\theta}^2 + (\cos\theta)\ddot{\theta}) = (-4.65\ m/s^2)$

$\sum F_r$ =m$(\ddot{r} - r\dot{\theta}^2)$ $\quad -F_s - F_g \sin\theta_1 + F_N \cos(\varphi) = m(\ddot{r} - r\dot{\theta}^2) \ldots \ldots eq\ 1$

$\sum F_\theta = m(r\ddot{\theta} - 2\dot{r}\dot{\theta})$ $\quad F_{oA} - F_g \cos\theta_1 - F_N \sin(\varphi) = m((r\ddot{\theta} - 2\dot{r}\dot{\theta}) \ldots eq\ 2$

Where: $F_s = k\Delta x = 100(r_{@60^0} - r_{@30^0}) = 21.9N$, and $F_g = mg$ =1.5(9.81)=14.7 N

With two equations(eq 1 and 2) and two unknowns ($F_{oA} and\ F_N$), the system of equations can be solved.

Activity

3. The 5-lb collar slides on the smooth rod, so that when it is at A it has a speed of 10 ft/s. If the spring to which it is attached has an unstretched length of 3 ft and a stiffness of k = 10 lb/ft, determine the normal force on the collar and the acceleration of the collar at this instant [1] .

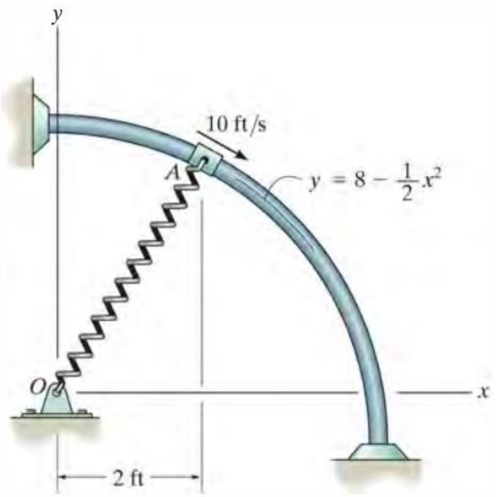


Figure 27. Problem 3

4. Determine the acceleration of the system and the tension in each cable. The inclined plane is smooth, and the coefficient of kinetic friction between the horizontal surface and block C is $\mu_k = 0.2$ [1].

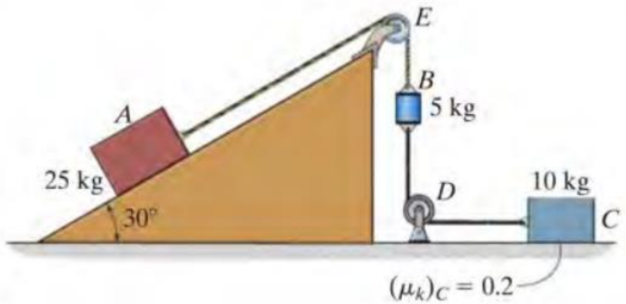


Figure 28. Problem 4

Activity

5. The motor winds in the cable with a constant acceleration, such that the 20-kg crate moves a distance s = 6 m in 3 s, starting from rest. Determine the tension developed in the cable. The coefficient of kinetic friction between the crate and the plane is 0.3 [1]. .

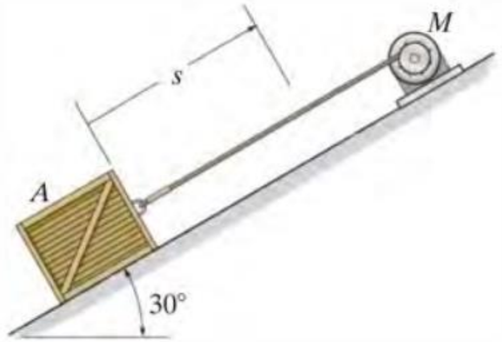


Figure 29. Problem 5

6. The 1 0-lb block A travels to the right at V_A= 2 ft/s at the instant shown. If the coefficient of kinetic friction is 0.2 between the surface and A, determine the velocity of A when it has moved 4 ft. Block B has a weight of 20 lb [2]. .

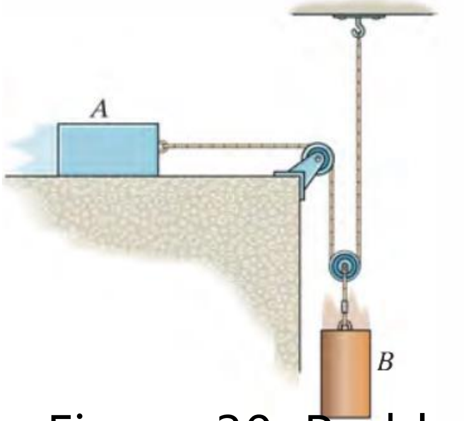


Figure 30. Problem 6

Summary

In This Lecture We Covered:

1. Introduction to Kinetics of Particles→ Scope and definition of kinetics
2. Equation of Motion of Particles→ Force and its types
3. Equation of Motion for Rectilinear Motion
4. Equation of Motion for curvilinear motion graphs – Normal tangent and polar coordinate
5. Steps to Solve kinematics of particle Problems→ description
6. Solved Problems

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