Week 7

Impulse and Momentum for Particles

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Contents

By the end of this lecture, you are able to:

1. Define and explain impulse momentum
2. Define and explain Linear momentum
3. Define and explain impulse
4. Define Equations for impulse momentum
5. Understand Direct oblique impact and coefficient of restitution
6. Understand conservation of momentum

Understand the impulse momentum

- We now consider a third basic method for the solution of problems dealing with the motion of particles[1].
- This method is based on the principle of impulse and momentum and can be used to solve problems inv olving force, mass, velocity, and time [1].
- It is of particular interest in the solution of problems involving impulsive motion and problems involving im pacts .
- We will start the by first defining the concept of impulse and momentums separately and then will discus s the interconnection between them.
- We will explore how energy lost in an impact is accounted for and the relationship of momentum to collisi ons between two bodies.
- For time- dependent forces, impulse and momentum are often more useful than Newton’s laws.

Momentums

- Momentum and inertia are similar concepts that describe an objects motion, however inertia describes an objects resistance to change in its velocity, and momentum refers to the magnitude and direction of it's motion.
- Any body that is in motion has momentum. A force acting on a body will change its momentum.
- Mathematically, the momentum of a particle is defined as the product of the mass multiplied by the velocity of the motion. Let the variable represent momentum.

$$L = M.v$$

- The vector mv is called the linear momentum, denoted as L. This vector has the same direction as v. The linear momentum vector has units of (kg·m)/s or (slug·ft)/s. mass is constant!
- An object can have a large momentum by having a large mass or a large velocity.

Impulse

- Impulse is the **overall** effect of a force acting over a time interval and It changes momentum.
- It is expressed as the product of impulsive force and change in time.

$$I = F\Delta t$$

- This term is a vector quantity. Since time is a positive scalar, the impulse a cts in the same direction as the force, and its magnitude has units of force-t ime, e.g., N . s or lb · s.
- measures the effect of a force during the time the force acts.
- More precisely in integral form : impulse of a force is defined as:

$$I = \int_{t_1}^{t_2} f\,dt$$

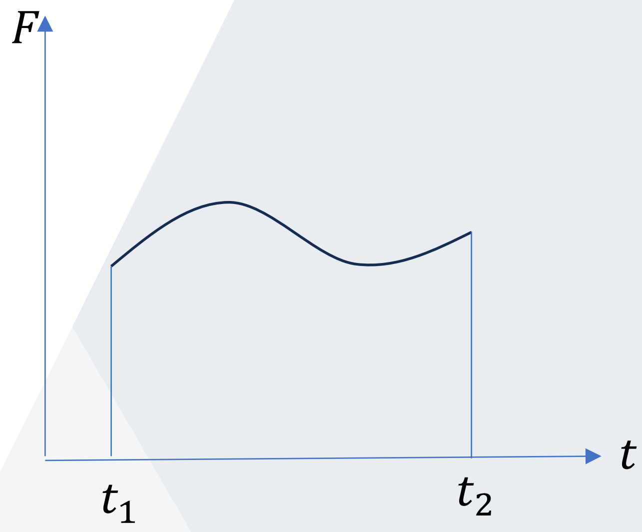


Figure 1. Impulse

- Graphically, it can be represented by the area under the force versus time curve.
- If F is constant, then I = F ($t_2 - t_1$).

N.B. They are typically **much larger** than other forces (like the weight or gravity) acting on the object during extremely **short duration** (Δt)

Impulses in the System

- If the time period over which the motion is studied is very short, some of the external impulses may also be neglected or considered approximately equal to zero.
- The forces causing these negligible impulses are called non-impulsive forces.
- By comparison, forces which are very large and act for a very short period of time produce a significant change in momentum and are called impulsive forces and cannot be neglected in the impulse-momentum analysis.
- Impulsive forces normally occur due to an explosion or the striking of one body against another, whereas non-impulsive forces may include the weight of a body, the force imparted by a slightly deformed spring having a relatively small stiffness, or for that matter, any force that is very small compared to other larger (impulsive) forces.
- To illustrate, consider the effect of striking a tennis ball with a racket as shown in the photo.

Figure 2. impact of tennis ball with racket

During the Impact

- When the racket strikes the tennis ball, the contact time is very short.
- The force of the racket on the ball is impulsive → it causes a large, sudden change in momentum.
- The weight of the ball acts continuously but is very small compared to the impact force.→ It is considered a non-impulsive force and can be neglected during this short impact.

Figure 3. During impact motion of tennis ball

After the Impact

- Once the ball leaves the racket, the time of flight is much longer.
- Now, the impulse due to the ball's weight (and air resistance) becomes significant. These forces cause the change in momentum of the ball during its motion through the air.

Figure 4. After impact motion of tennis ball

- Impulsive forces dominate during short-time collisions. Non-impulsive forces become important over longer time intervals.

Principle of Linear Impulse and Momentum

- **We can use Newton's second law to find how the object's velocity changes(Momentum) as a result of the impulse .**

- From newtons second law we know that:

$$\sum F = ma \qquad \text{Where from kinematics} \quad a = \frac{dv}{dt}$$

$$F = m\frac{dv}{dt}$$

- Rearranging the terms and integrating between the limits $V = v_1 \; at \; t = t_1$ and $V = v_2 \; at \; t = t_2$, We have:

$$\sum \int_{t_1}^{t_2} F dt = \int_{v_1}^{v_2} m dv \qquad \sum \int_{t_1}^{t_2} F dt = m v_2 - m v_1$$

- Moving the initial momentum to the other side of the equation yields

$$m v_1 + \sum \int_{t_1}^{t_2} F dt = m v_2$$

- This equation is referred to as the principle of linear impulse and momentum:

$$mv_1 + \sum \int_{t_1}^{t_2} F dt = mv_2$$

- which states that the initial momentum of the particle at t_1 plus the vector sum of all the impulses applied to the particle during the time interval from t_1 to t_2 is equivalent to the final momentum of the particle at t_{12} .

These three terms are illustrated graphically on the impulse and momentum diagrams shown in Fig. 5.

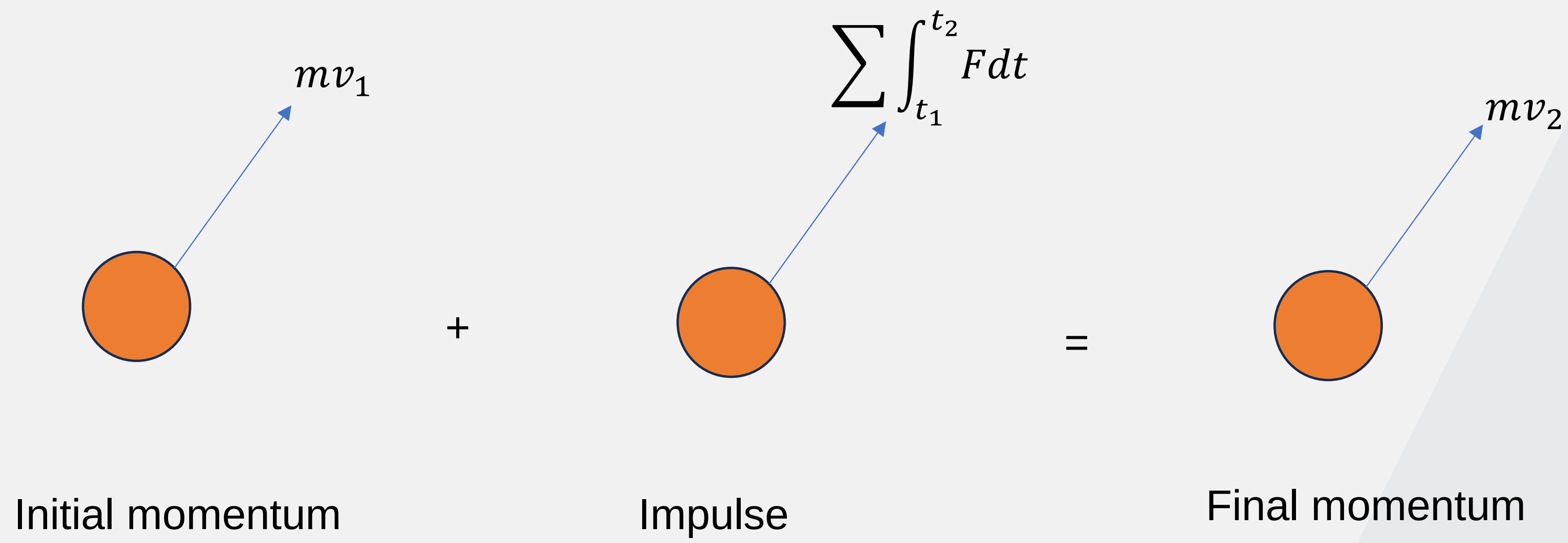


Figure 5. Impulse and momentum diagrams

- If each of the vectors in the impulse momentum equation is resolved into its x, y, z components, we can write the following three scalar equations of linear impulse and momentum.

$X - direction$	$mv_{(x)_1} + \sum \int_{t_1}^{t_2} F_x dt = mv_{x(_2)}$
$\mathrm{y}{-}direction$	$mv_{(y)_1} + \sum \int_{t_1}^{t_2} F_y dt = mv_{y(_2)}$
$z - direction$	$mv_{(z)_1} + \sum \int_{t_1}^{t_2} F_z dt = mv_{z(_2)}$

For system of particles

- When a problem involves two or more particles, we can consider each particle separately and write impulse momentum equation for each particle. We can also add vectorially the momenta of all the particles and the impulses of all the forces involved. We then have

$$\sum mv_1 + \sum \int_{t_1}^{t_2} F dt = \sum mv_2$$

- This equation states that the initial linear momenta of the system plus the impulses of all the external forces acting on the system from t_1 to t_2 is equal to the system's final linear momenta.

Important point

The principle of linear impulse and momentum is used to solve problems involving force, time, and velocity, since these terms are involved in the formulation.

- Establish the x, y, z inertial frame of reference and draw the particle's free-body diagram in order to account for all the forces that produce impulses on the particle.
- establish direction and sense of the particle's initial and final velocities
- In accordance with the established coordinate system, apply the principle of linear impulse and momentum, $mv_1 + \sum \int_{t_1}^{t_2} F dt = mv_2$
- If motion occurs in the x-y plane, the two scalar component equations can be formulated by either resolving the vector components of F from the free-body diagram, or by using the data on the impulse and momentum diagrams.

Impact and conservation of linear momentum

conservation of linear momentum

- When the sum of the external impulses acting on a system of particles is zero, the impulse momentum equation for system of particle reduces to a simplified form, namely,

$$\sum m_i v_{(i)_1} = \sum m_i v_{(i)_2} \qquad i.e \sum \int_{t_1}^{t_2} F dt = 0$$

- consider the collision of two particles of mass $m_A$and m_B. The motion of these particles can be expressed by $mv_{(A)_1}$and $mv_{(B)_1}$ before and after $mv_{(A)_2}$ $and\, mv_{(B)_2}$the collision, respectively, and the above equation can be re written as in the following equation.

$$m_A v_{(A)_1} + m_B v_{(B)_1} = m_A v_{(A)_2} + m_B v_{(B)_2}$$

- The conservation of linear momentum is often applied when particles collide or interact.

- To apply it, analyze the free-body diagram (FBD) of the entire system to identify:
- Internal forces → always cancel out in equal and opposite pairs.
- External forces → may create external impulses and thereby determine in what direction(s) linear momentum is conserved

Impact

- Impact occurs when two bodies collide with each other during a very short period of time, causing relatively large (impulsive) forces to be exerted between the bodies.
- The striking of a hammer on a nail, or a golf club on a ball, are common examples of impact loadings.
- In general, there are two types of impact namely Central impact and oblique impact.
- The type of impacts are determined by observing the to imaginary lines namely line of impact, and plane of contact,. which is perpendicular line of impact.

The line of impact is a line that passes through the centers of mass of two colliding bodies, or is the common normal at the point of contact.

The plane of impact is the plane that is tangent to the surfaces of the bodies at the point of contact and is perpendicular to the line of impact

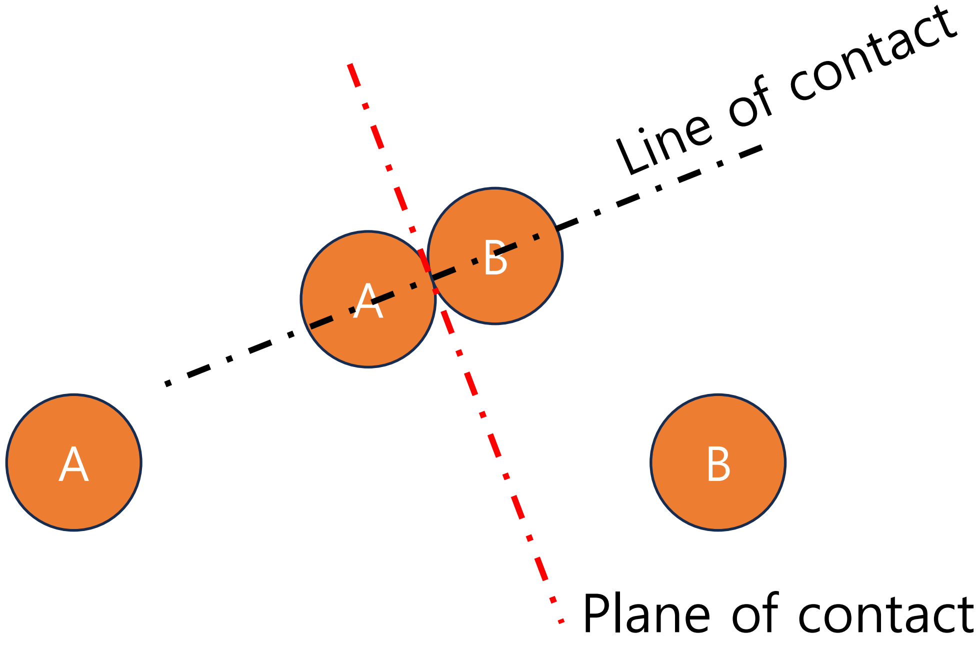


Figure 6. Line of contact and plane of contact

Types of Impact

- Central impact occurs when the direction of motion of the mass centers of the two colliding particles is along a line passing through the mass centers of the particles.

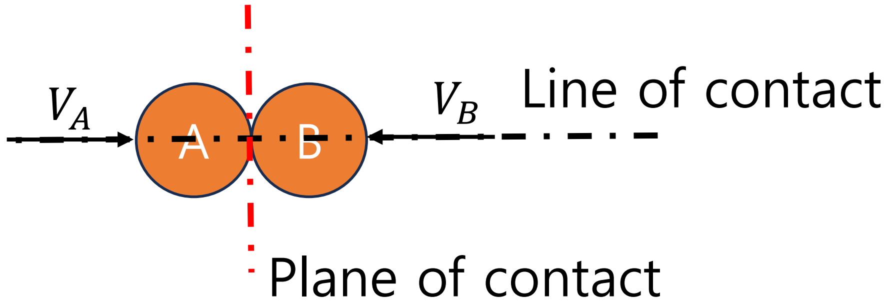


Figure 7. central impact

- When the motion of one or both of the particles make an angle with the line of impact, Fig. 8, the impact is said to be oblique impact.

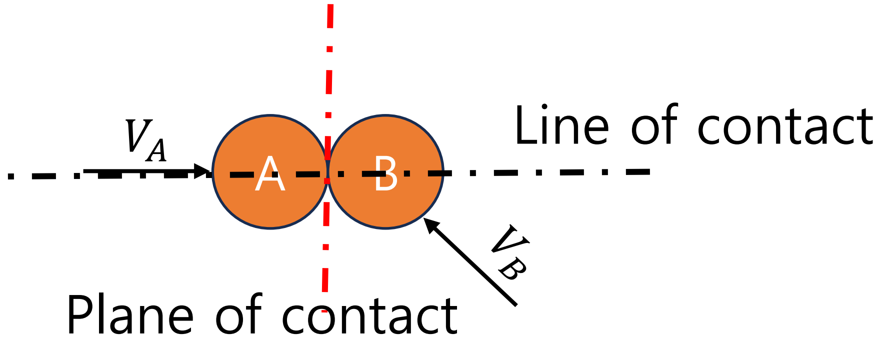


Figure 8. oblique impact

Impact analysis

- To illustrate the method for analyzing the mechanics of impact, consider the case involving the central impact of the two particles A and B shown in Fig. 9.[3].

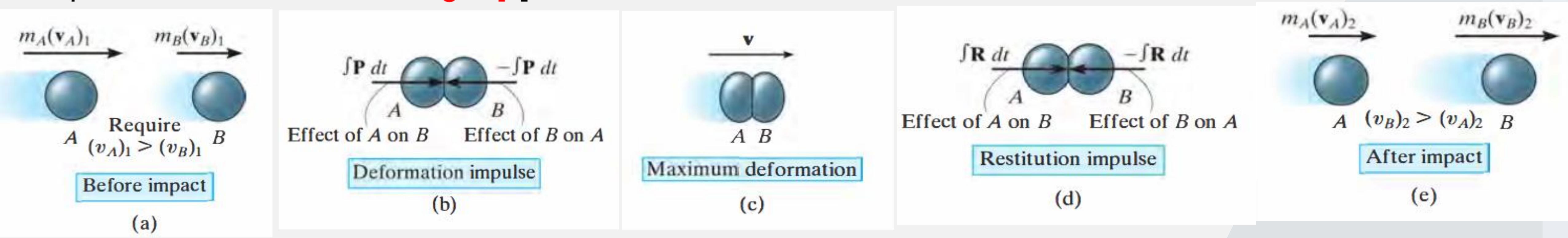


Figure 9. impact process

- The particles have the initial momenta shown in Fig. 9a. Provided $V_{A_1} > V_{B_1}$, collision will eventually occur.
- Once the particles start to collide hey exert equal and opposite impulse forces ($\int pdt$) on each other. Fig. 9b
- Then particles reach maximum deformation. They move together with equal velocity. Fig. 9c
- Afterward, the particles may either permanently deform or return to their original shape ,Fig. 9d.
- After the restitution impulse, the particles separate and move with new velocities according to the law of momentum. Fig. 9e

Mathematical expression

- In a system of colliding particles, momentum is conserved because the internal impulses during deformation and restitution cancel each other out.

$$m_A v_{(A)_1} + m_B v_{(B)_1} = m_A v_{(A)_2} + m_B v_{(B)_2}$$

- In most problems the initial velocities of the particles will be known, and it will be necessary to determine their final velocities V_{A_2} and V_{B_2} .

- To determine the final velocities, in addition to the momentum equation, we need another equation known as the **coefficient of restitution (*e*)**, which is given by:

$$e = \frac{v_{(B)_2} - v_{(A)_2}}{v_{(A)_1} - v_{(B)_1}}$$

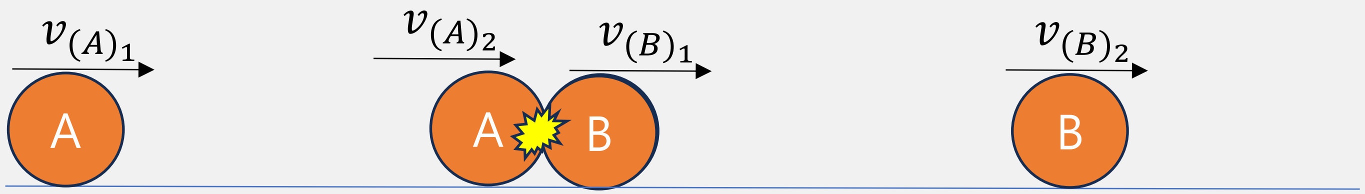
where:

- $v_{(B)_2} - v_{(A)_2}$ = Relative velocity after separation
- $v_{(A)_1} - v_{(B)_1}$ = Relative velocity before collision

Coefficient of Restitution (e)

- The coefficient of restitution (e) is defined as the ratio of the relative velocity of separation just after impact to the relative velocity just before impact.
- In general, the value of e ranges between 0 and 1 and one should be aware of the physical meaning of these two limits.
- Elastic Impact (e = 1). This means no kinetic energy is lost during the collision ,all the energy before impact is conserved after impact.

$$v_{(A)_1} - v_{(B)_1} = v_{(B)_2} - v_{(A)_2}$$

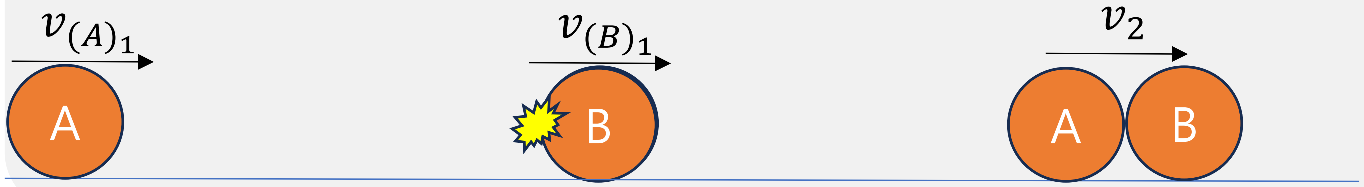


Before impact After impact

Figure 10. Elastic impact

- Plastic (Inelastic) Impact (e = 0) . After the collision, both bodies stick together and move with a common velocity. In this case, maximum energy loss occurs during the collision.

$$v_{(B)_2} = v_{(A)_2} = V_2$$



Before impact After impact Figure 11.plastic impact process

Central impact

Important point

- For Two particles undergoing a **direct central impact,** If the Coefficient of restitution (e), Mass of each particle and Initial velocities before impact following are known, then goal is to find their final velocities just after collision.
- Then, the magnitudes of final velocities can be obtained, by simultaneously solving using two main equations: Conservation of Momentum Coefficient of Restitution Equation.

$$m_A v_{(A)_1} + m_B v_{(B)_1} = m_A v_{(A)_2} + m_B v_{(B)_2}$$

$$e = \frac{v_{(B)_2} - v_{(A)_2}}{v_{(A)_1} - v_{(B)_1}}$$

- If the y axis is established within the plane of contact and the x axis along the line of impact, the impulsive forces of deformation and restitution act only in the x direction.

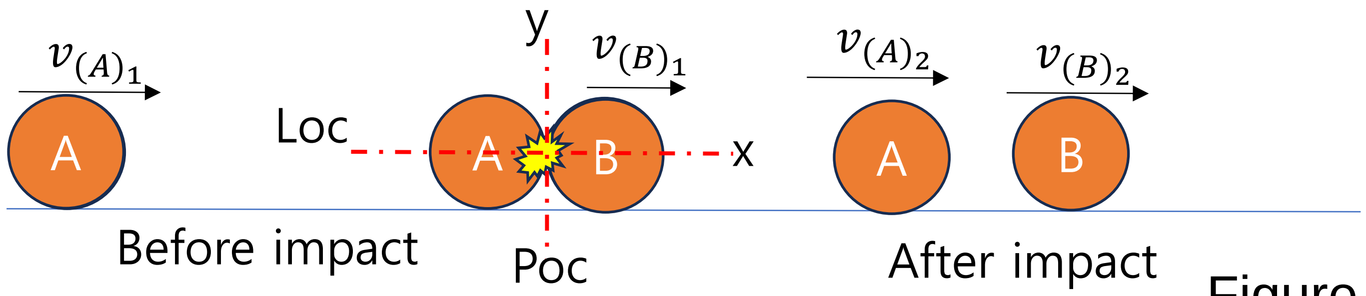


Figure 12. Central impact

- Both conservation of energy and The coefficient of restitution equations applied along line of impact or x axis.

Oblique impact

- When oblique impact occurs between two smooth particles, the particles move away from each other with velocities having unknown directions as well as unknown magnitudes.
- Provided the initial velocities are known, then four unknowns are present in the problem.

- As shown in Fig.13 , these unknowns may be represented either as $V_{A_2}, V_{B_2}, \theta_2$ and α_2 or as the x and y components of the final velocities.

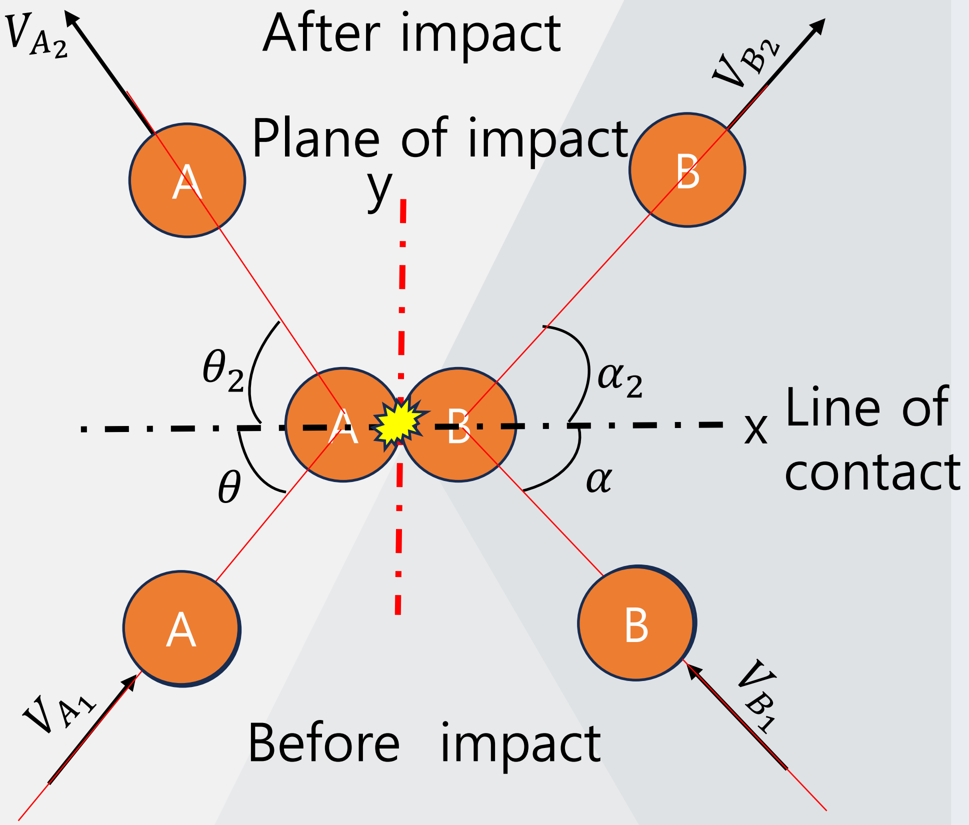


Figure 13.oblique impact

- The motion can be analyzed by resolving the velocities along the line of impact (x-axis) and along the plane of contact (y-axis).

Along line of impact

- The momentum of the system is conserved always

$$m_A v_{(A_1)_x} + m_B\, v_{(B_1)_x} = m_A v_{(A_2)_x} + m_B v_{(B_2)_x}$$

- The coefficient of restitution (e) relates the relative velocities of the two bodies along the line of impact before and after collision:

$$e = \frac{v_{(B_2)_x} - v_{(A_2)_x}}{v_{(A_1)_x} - v_{(B_1)_x}}$$

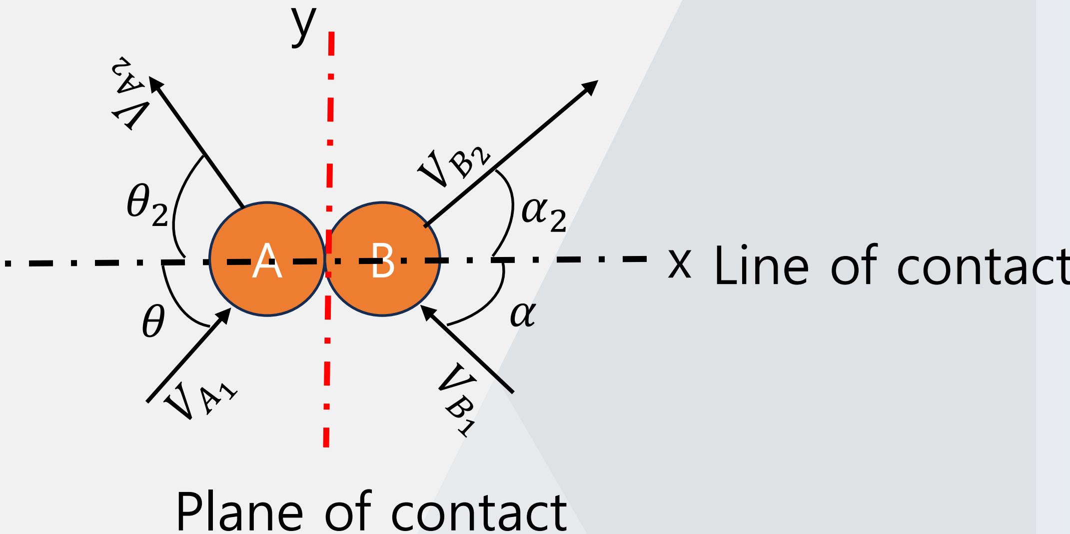


Figure 14. oblique impact

Along plane impact or contact

- No impulse acts along the plane of contact (y-axis).Therefore, the components of velocity along y remain unchanged:

$$v_{(A_1)y} = v_{(A_2)y} \qquad v_{(B_1)y} = v_{(B_2)y}$$

Magnitudes of final velocity and directions

Magnitudes

$$For\ particle\ A, V_{A_2} = \sqrt{(v_{(A_2)_x})^2 + (v_{(A_2)_y})^2}$$

$$For\ particle\ B, V_{B_2} = \sqrt{(v_{(B_2)_x})^2 + (v_{(B_2)_y})^2}$$

directions

$$tan\theta_2 = \frac{(v_{(A_2)_y})}{(v_{(A_2)_x})}$$

$$tan\alpha_2 = \frac{(v_{(B_2)_y})}{(v_{(B_2)_x})}$$

Oblique cont'd….

Important point

- If the y axis is established within the plane of contact and the x axis along the line of impact, the impulsive forces of deformation and restitution act only in the x direction.
- Momentum of the system is conserved along the line of impact
- The coefficient of restitution, e can also applied in line of impact
- Perpendicular to the line of impact or plane of contact , since no impulse acts on particles in this direction. As a result the initial velocity before impact is equal to the final velocity (after impact)
- Maximum we have four unknown and they can be obtained By resolving the velocity or momentum vectors into components along the x and y axes.
- Can be solved simultaneously and the magnitudes are obtained using Pythagorean equation

Summery on Impulse momentum

- This is a vector equation that can be resolved into rectangular components and used to solve problems that involve force, velocity, and time.
- Expressed mathematically as:

$$mv_1 + \sum \int_{t_1}^{t_2} F dt = mv_2$$

- where: mv is referred to us momentum and $\sum \int_{t_1}^{t_2} F dt$ is the sum of impulse forces over a given time interval

System of particle

$$\sum mv_1 + \sum \int_{t_1}^{t_2} F dt = \sum mv_2$$

- The conservation of momentum

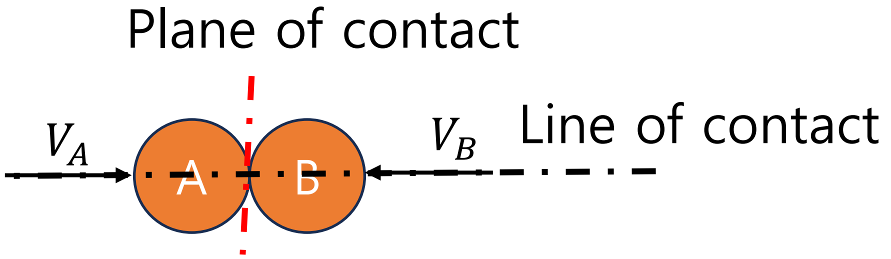
$$m_A v_{(A)_1} + m_B v_{(B)_1} = m_A v_{(A)_2} + m_B v_{(B)_2}$$

- The Coefficient of Restitution

$$e = \frac{v_{(B)_2} - v_{(A)_2}}{v_{(A)_1} - v_{(B)_1}}$$

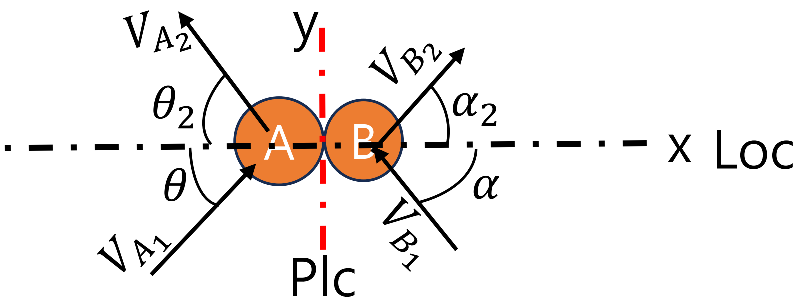
Impact

Central impact



- The magnitudes of after impact velocities can be obtained, by simultaneously solving using two main equations.

Oblique impact



- The impulsive forces of deformation and restitution act only in the LOC and gives as components of final velocity in that direction.
- In PLC initial velocity before impact is equal to the final velocity .

Problem 1

If cylinder A is given an initial downward speed of 2 m/s, determine the speed of each cylinder when t = 3 s. Neglect the mass of the pulleys [1].

Solution

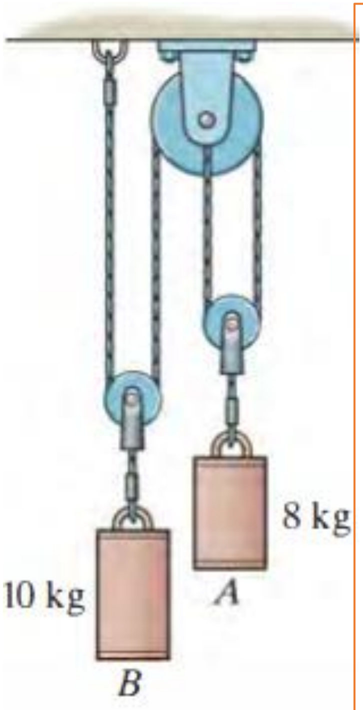


Figure 15. P1

Given

- $V_{(A)1} = 2\ m/s$
- $t = 3sec$

Required

$V_{Af} = ?$

$V_{Bf} = ?$

- Free-Body Diagram:

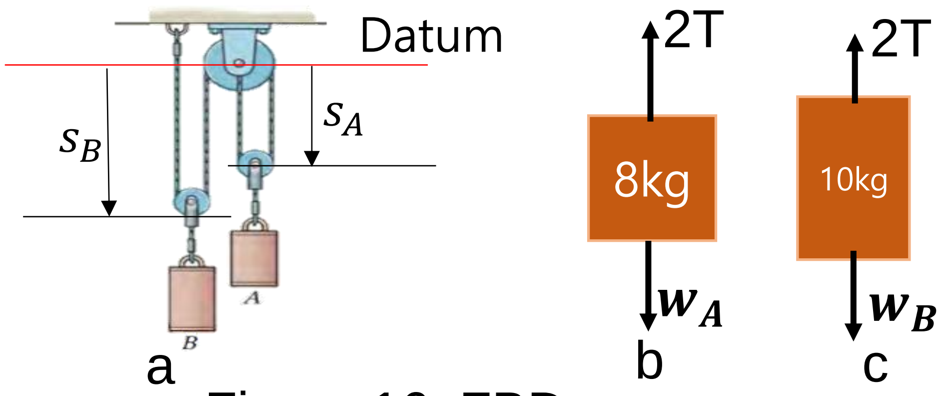


Figure 16. FBD

- Kinematics: Expressing the length of the cable in terms of s_B and s_A by referring to: $2s_B + 2s_A = l$

,and Taking the time derivative $V_B + V_A = 0$ $\therefore V_{(B)1} = -V_{(A)1} = 2m/s \uparrow$

- Principle of Impulse and Momentum:

By referring to Fig. *b*

$$m_A v_{(A)1} + \sum \int_{t_1}^{t_2} F_{(A)y} dt = m_A v_{(A)2}$$

$$8v_{(A)1} + 2T(3) - w_A(3) = 8v_{(A)2} ..(eq1)$$

By referring Fig. *c*

$$m_B v_{(B)1} + \sum \int_{t_1}^{t_2} F_{(B)y} dt = m_B v_{(B)2}$$

$$8v_{(A)1} + 2T(3) - w_A(3) = 8v_{(A)2} ..eq(2)$$

where: $V_{(A)2} = -V_{(B)2}$

Solving Eqs 1 and 2, $V_{(A)2} = \frac{1.27m}{s}\ and\ T = 43.6N$

Problem 2

Two coins A and B have the initial velocities shown just before they collide at point O. If they have weights of $W_A = 13.2 \times 10^{-3}\,lb$ and $W_B = 6.6 \times 10^{-3}\,lb$ the surface upon which they slide is smooth, determine their speeds just after impact. The coefficient of restitution is e = 0.6 [1].

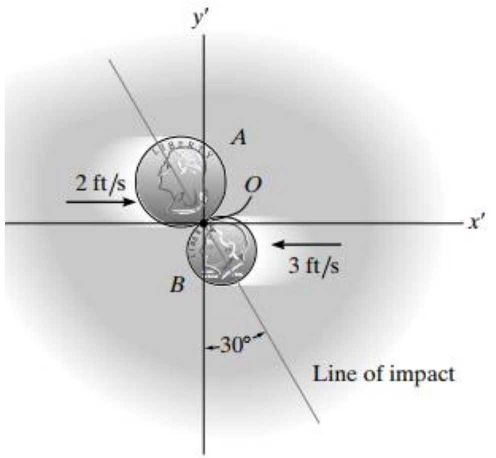


Figure 17. Problem 2

Given

$m_A = \frac{W_A}{g} = 0.4 \times 10^{-3}$

$m_B = \frac{W_B}{g} = 0.2 \times 10^{-3}$

$v_{A_1} = 2ft/s$

$v_{B_1} = -3ft/s$

Required

$v_{A_2} = ?$ $v_{B_2} = ?$

Solution

Along Line of impact(m)

Conservation of momentum

$$m_A v_{(A_1)m} + m_B v_{(B_1)m} = m_A v_{(A_2)m} + m_B v_{(B_2)m}$$

$$0.4 \times 10^{-3} (2 \sin 30) - 0.2 \times 10^{-3} (3 \sin 30) = 0.4 \times 10^{-3}\ v_{(A_2)m} + 0.2 \times 10^{-3}\ v_{(B_2)m}$$

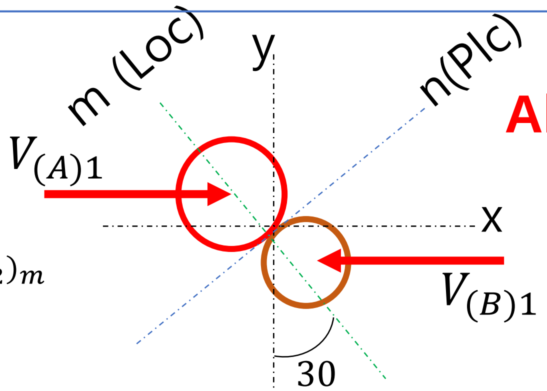


Figure 18. Fbd

Along plane of impact(m)

$$m_A v_{(A_1)n} = m_A v_{(A_2)n}$$

$$v_{(A_2)n} = 2\cos 30 = 1.73ft/s$$

$$m_B v_{(B_1)n} = m_B v_{(B_2)n}$$

$$v_{(B_2)n} = 3\cos 30 = 2.6ft/s$$

The coefficient of restitution (e) $e = \frac{v_{(B_2)m} - v_{(A_2)m}}{v_{(A_1)m} - v_{(B_1)m}}$

$$0.65 = \frac{v_{(B_2)m} - v_{(A_2)m}}{2\sin 30 - (-3\sin 30)}$$

Solving: $v_{(A_2)m} = \left(-\frac{0.375ft}{s}\right), v_{(B_2)m} = 1.25ft/s$

Magnitudes

$$v_{A_2} = \sqrt{(v_{(A_2)m})^2 + (v_{(A_2)n})^2} = 1.77ft/s \qquad v_{B_2} = \sqrt{(v_{(B_2)m})^2 + (v_{(B_2)n})^2} = 2.88ft/s$$

Activity

5. The motor winds in the cable with a constant acceleration, such that the 20-kg crate moves a distance s = 6 m in 3 s, starting from rest. Determine the tension developed in the cable. The coefficient of kinetic friction between the crate and the plane is 0.3 [1]. .

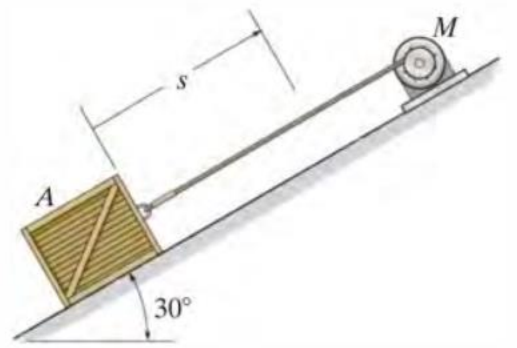


Figure 29. Problem 5

6. The 1 0-lb block A travels to the right at V_A= 2 ft/s at the instant shown. If the coefficient of kinetic friction is 0.2 between the surface and A, determine the velocity of A when it has moved 4 ft. Block B has a weight of 20 lb [2]. .

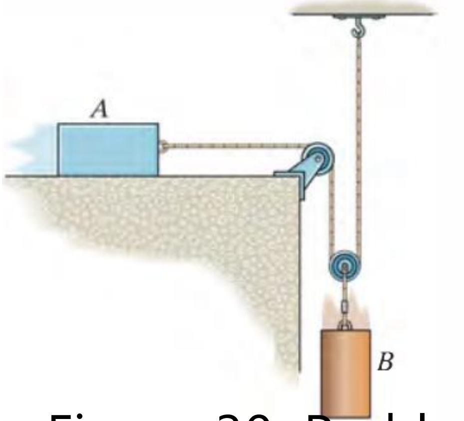


Figure 30. Problem 6

Summary

In This Lecture We Covered:

1. Introduction to Kinetics of Particles→ Scope and definition of kinetics
2. Equation of Motion of Particles→ Force and its types
3. Equation of Motion for Rectilinear Motion
4. Equation of Motion for curvilinear motion graphs – Normal tangent and polar coordinate
5. Steps to Solve kinematics of particle Problems→ description
6. Solved Problems

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