Week 8

Mid term

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Contents

In Today section you will Review :

1 Chapter one (week one concept)

2 Chapter two (week two ,three and four concepts)

3 Chapter one (week five, six and seven concepts)

Chapter one

scope of mechanics

- **Mechanics is defined as the science that describes and predicts the *conditions of rest or motion* of *bodies* under *the action of forces* [1].**
- **It consists of the mechanics of solid *bodies,* and mechanics of *fluids*.**

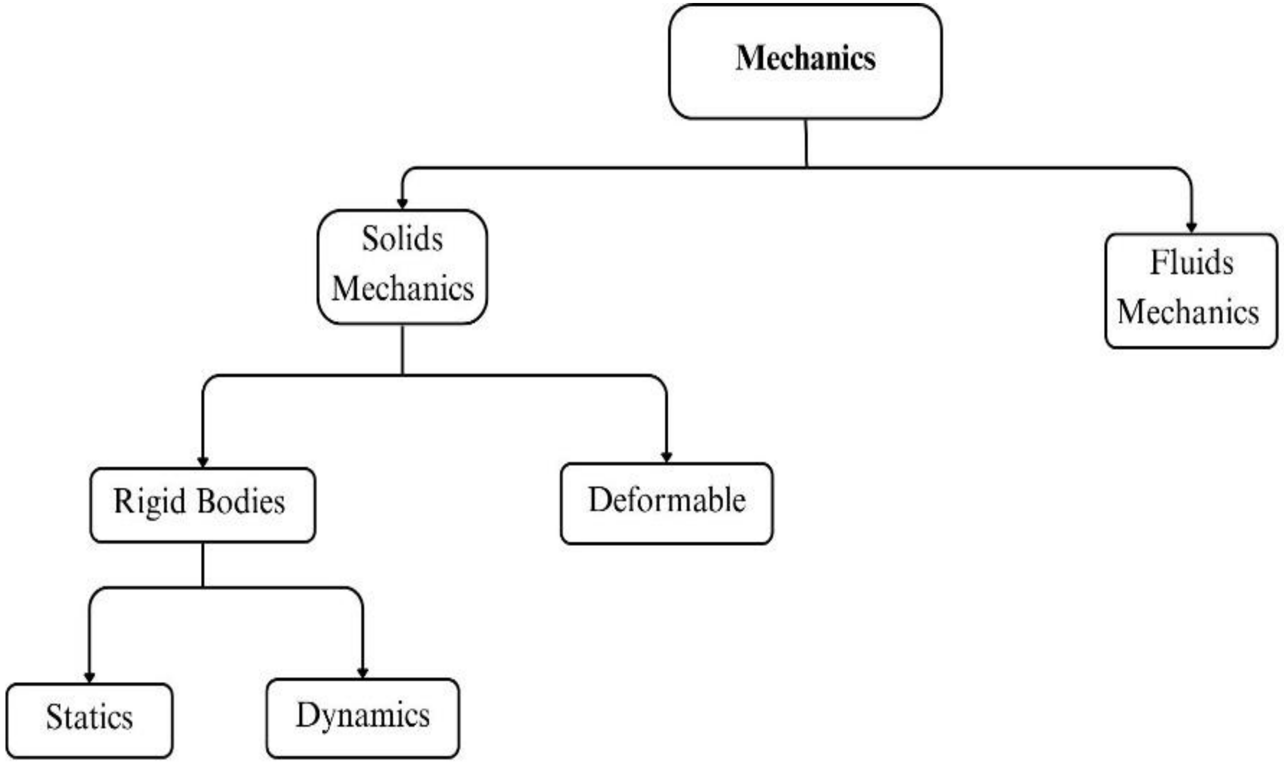


Figure 1. Classifications of engineering mechanics

Rigid Bodies and Particles

- **In dynamics , solid rigid bodies sometimes are treated as a particle and the difference between them is as follows [1] :**

Rigid Bodies Vs Particles

Particles	Rigid Bodies
• is a body of negligible size, having only mass but no shape or dimensions • It is Considered as a point mass having translational motion • Its rotation and internal structure are ignored	• By definition it is body in which the distance between any two points remains constant, even when external forces are applied • Compared to particle, It Has size and shape With Both translational and rotational motion. • it is the main objective of study

- So why we sometimes treat solid rigid bodies as a particle?

The course chapters

- **Based on the above concepts and definitions, This course is divided into five chapters (including this chapter or introduction).**

We begin study by

- treating rigid bodies as particles to study translational motion only (**kinematics of particles**).

Then,

- we connect this translational motion to the forces that cause it (**kinetics of particles).**

After that,

- we remove the particle assumption and study rigid bodies in rotational motion as well as combined translational and rotational motion (general plane motion), which is covered under **kinematics of rigid bodies.**

Finally,

- we relate these rigid body motions to the forces that produce them (**kinetics of rigid bodies**).

Chapter 2. kinematics particles

- **Translational motion can be classified as rectilinear and curvilinear**

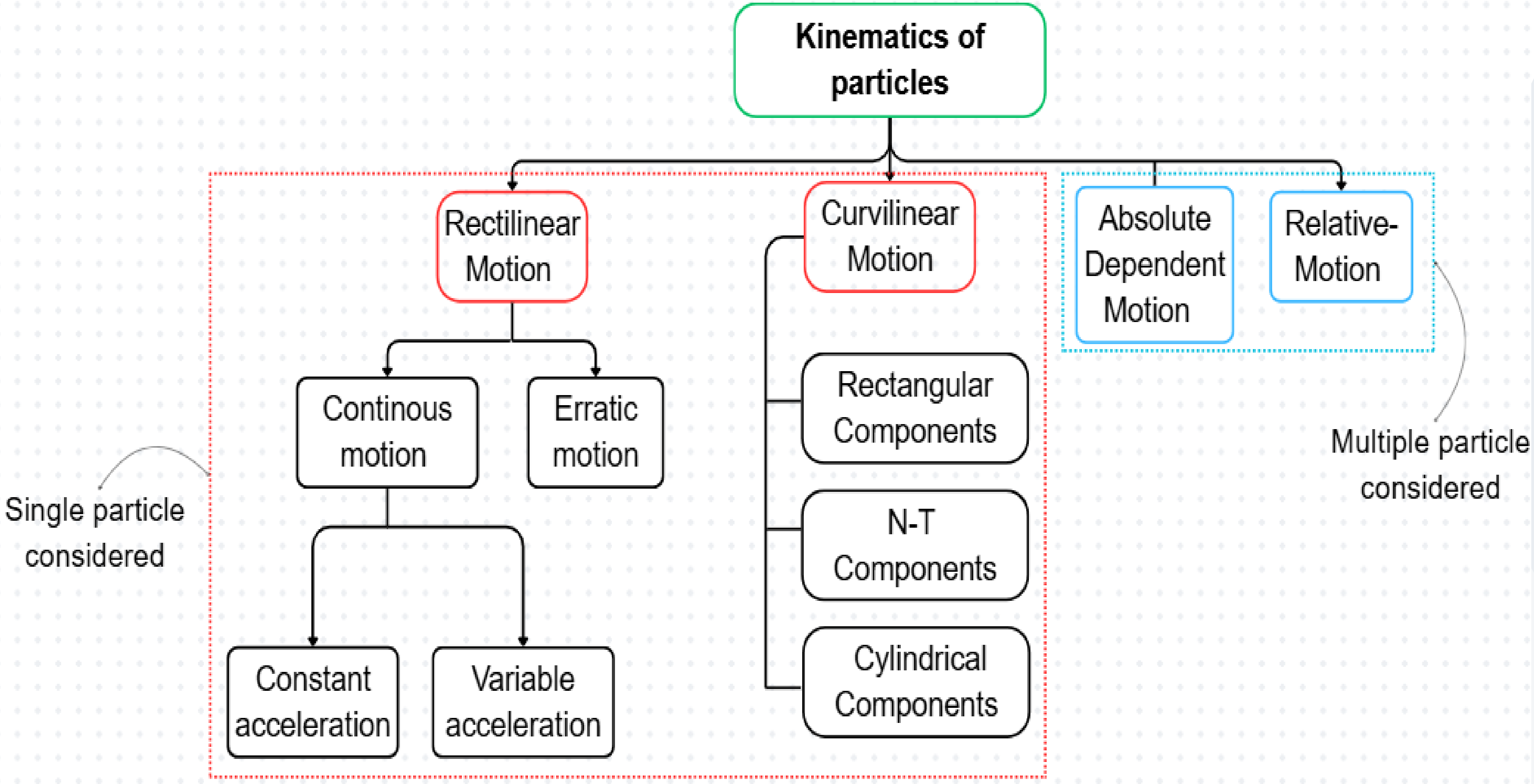


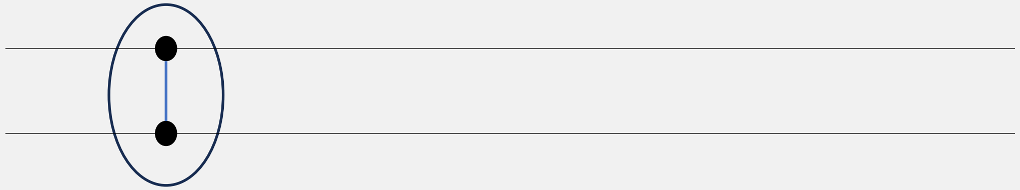
Figure 2. Chapter 2 structure

Rectilinear motion Vs Curvilinear motion

- The key difference between rectilinear and curvilinear motion of particles is summarized below.

Translation motion categories

Rectilinear motion	Curvilinear motion
• All points move in parallel straight lines • scalar form is often enough • 1D or typically x, y, or z- axis is enough to specify the motion at time E.g. Simple lifting mechanisms	• All points move in the parallel curved path direction • Vector approach is crucial • 2D or 3D (rectangular, polar, and cylindrical coordinates are used) E.g. car turning, projectile motion



a. Rectilinear

Figure 3. Rectilinear

b. Curvilinear

Figure 4. Curvilinear

• NB : Today's lecture mainly focuses on the rectilinear motion of a particle.

2.1 ***Rectilinear*** motion

cont'd….

- **The equations are applied differently depending on whether the motion is continuous or erratic.**

Continuous motion	Erratic motion
• The object's motion is smooth and consistent, without sudden stops, starts, or jumps. • can be described by a single, continuous mathematical function for its entire duration. • eg A car moving smoothly on a straight highway at **steady** speed	• The movement is inconsistent, with no regular or predictable pattern, making it difficult to forecast. • different functions are needed for entire duration • it is easier to analyze using graphs • A car in city traffic, stopping suddenly, speeding up, turning, or swerving.

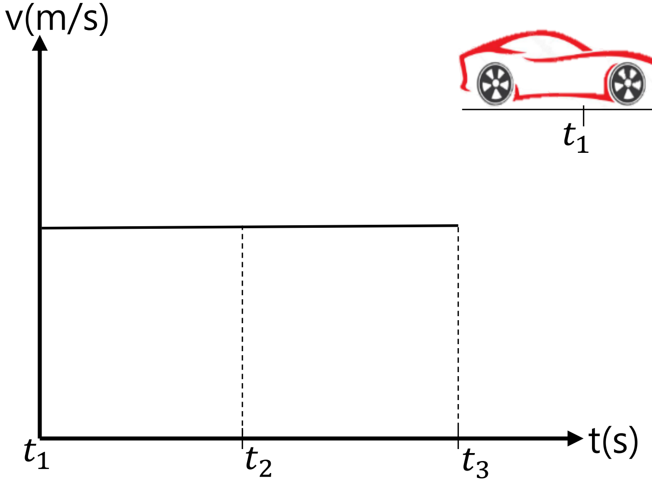


Figure 5. Continuous

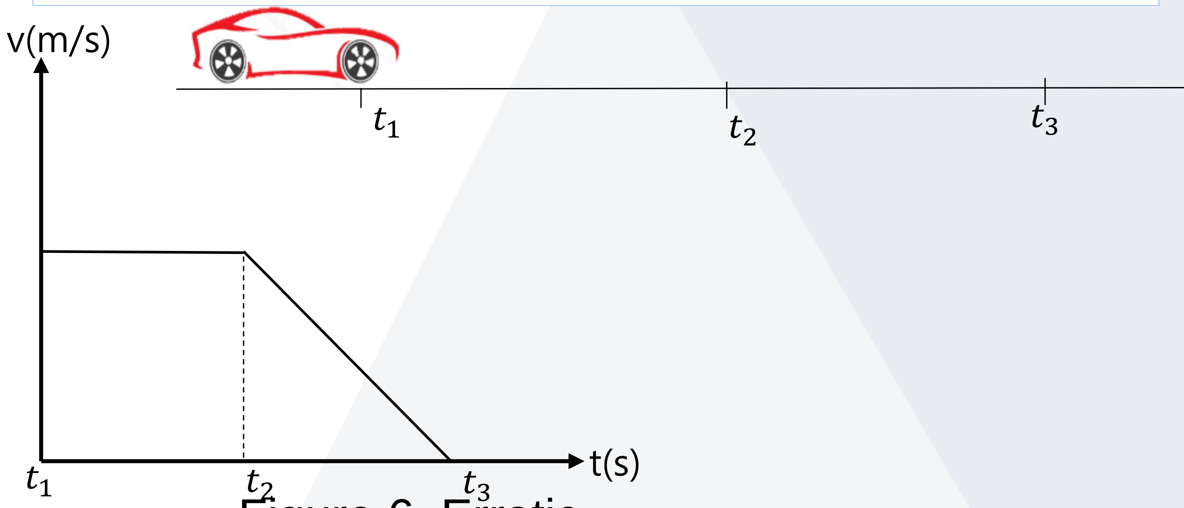


Figure 6. Erratic

Summery of Important in rectilinear motion

Continuous

Variable acceleration

$$\boldsymbol{v} = \frac{\boldsymbol{ds}}{\boldsymbol{dt}}$$

$$a = \frac{\boldsymbol{dv}}{\boldsymbol{dt}}$$

$$a = v\,\frac{dv}{ds}$$ Or $a\,ds = v\,dv$

Constant acceleration

$$V = V_O + \boldsymbol{a_c t}$$

$$S = s_O + v_O t + \frac{1}{2}(a_c t^2)$$

$$V^2 = (V_O)^2 + 2\boldsymbol{a_c}(S - s_O)$$

Erratic motion

Given Graph	Required Graph
S-t	• V-t • a-t
V-t	• s-t • a-t
a-t	• s-t • v-t
a-s	• v-s
V-s	• a-s

2.2 Understand curvilinear motion

- **Involves a particle moving along a curved path, resulting in continuous changes in direction and magnitude[1].**
- Here, both the **magnitude** and **direction** of velocity and acceleration vary, necessitating a vectoral analysis to fully describe the motion**.**

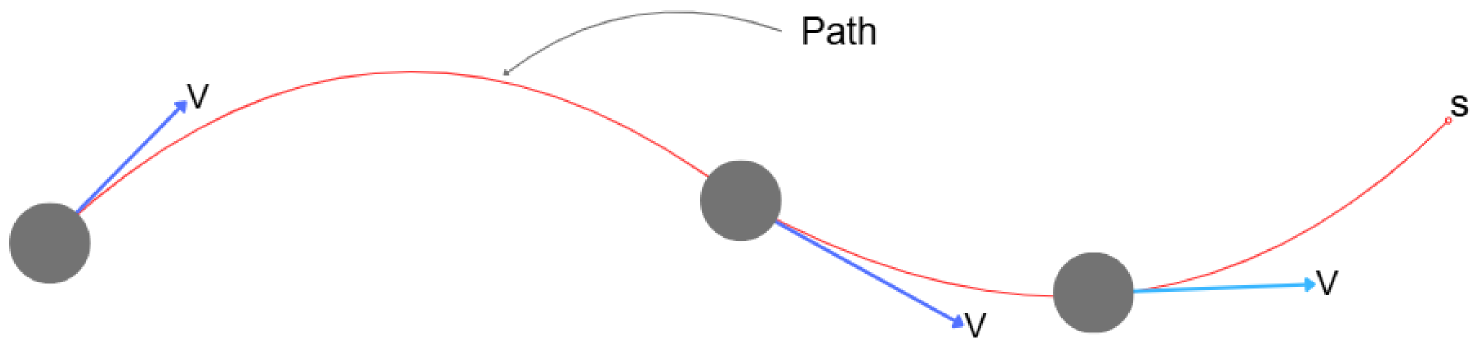
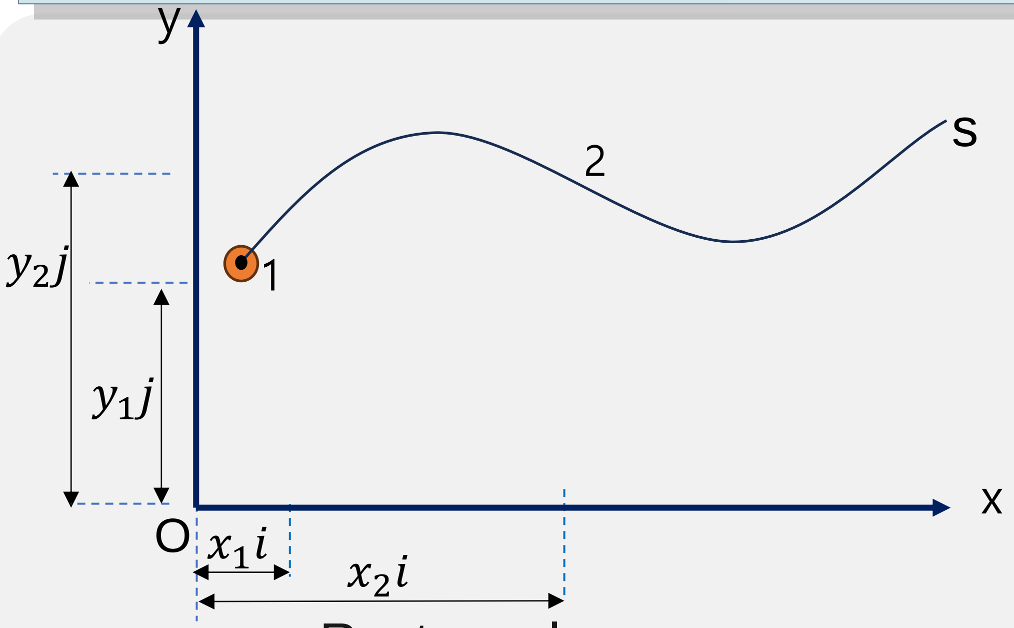


Figure 7. curvilinear motion

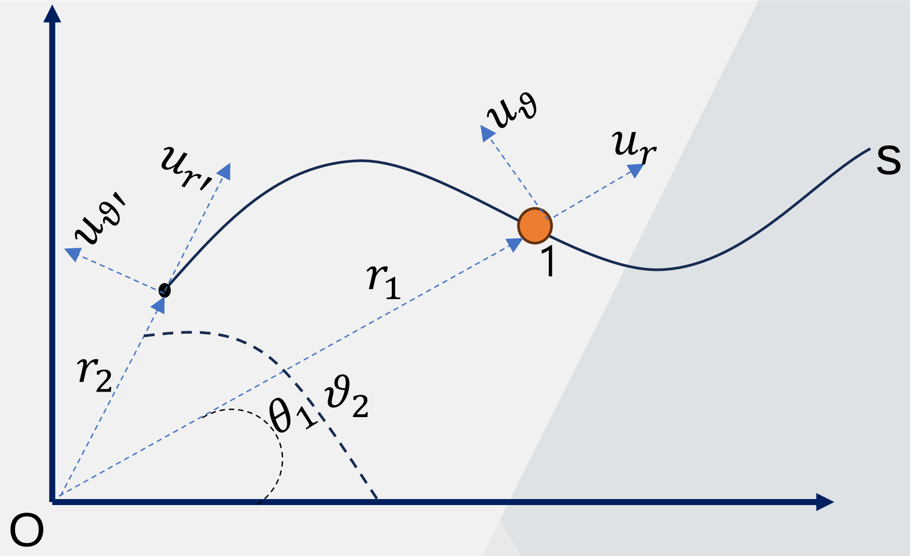
Unlike rectilinear motion where the direction is constant, in curvilinear motion, the ***velocity's direction is constantly changing*** as the ball follows the curve.

NB: To fully describe this type of motion, we need to use a different method than the one we used for rectilinear motion

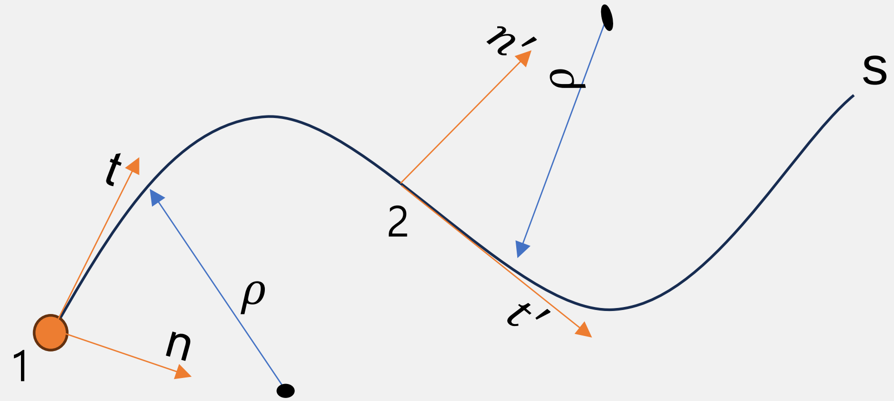
Coordinate systems



Rectangular



Polar Coordinates



Normal and Tangential Coordinates

Figure 8. Coordinate systems

Summery on the Important Equations

Rectangular

Position $\boldsymbol{r} = x\boldsymbol{i} + y\boldsymbol{j} + z\boldsymbol{k}$

Velocity $V = V_x\boldsymbol{i} + V_y j + V_z k$ $\quad v = \sqrt{(v_x)^2 + (v_y)^2 + (v_z)^2}$

Acceleration a= $a_x i + a_y j + a_z k$ $\quad \boldsymbol{a} = \sqrt{(a_x)^2 + (a_y)^2 + (a_z)^2}$

Motion of a Projectile

Horizontal Motion $(a_x = 0)$

$X = X_O + (v_o)_x t$

$V_x = constant$

Vertical Motion $(a_y) = -g$

$V_y = (v_o)_y - \boldsymbol{gt}$

$Y = y_O + (v_o)_y t - \frac{1}{2} g t^2)$

$V_y{}^2 = (V_O)^2 - 2\boldsymbol{g}\,(Y - y_O)$

Cylindrical (Polar-coordinate)

Velocity

$V = \dot{r}\, u_r + r\dot{\theta} u_\theta$ where $v_r = \dot{r}$

$\mathbf{v} = \sqrt{(v_r)^2 + (v_\theta)^2}$ $\quad v_\theta = r\dot{\theta}$

Acceleration

$a = a_r u_r + a_\theta\, u_\theta$ where $a_\theta = 2\dot{r}\dot{\theta} - r\,\ddot{\theta}$

$\mathbf{a} = \sqrt{(a_r)^2 + (a_\theta)^2}$ $\quad a_r = \ddot{r} - r\dot{\theta}^2$

r is radial coordinate $\quad \theta$ is *transverse* coordinate

Normal-tangent

Velocity V= $v\, \boldsymbol{u_t}$

Acceleration $\boldsymbol{a} = \boldsymbol{a_t}\, \boldsymbol{u_t} + \boldsymbol{a_n} \boldsymbol{u_n}$

$$\boldsymbol{a_n} = \frac{\boldsymbol{v}^2}{\boldsymbol{\rho}} \qquad \rho = \frac{[1 + (\frac{dy}{dx})^2]^{\frac{3}{2}}}{\frac{d^2y}{dx^2}}$$

$$\boldsymbol{a} = \sqrt{(\boldsymbol{a_n})^2 + (\boldsymbol{a_t})^2}$$

If $\boldsymbol{a_t}$ is varies,

$\boldsymbol{a_t} = \frac{dv}{dt}$

$\boldsymbol{a_t}\, ds = v\, dv$

If $\boldsymbol{a_t}$ is constant,

$V = V_O + \boldsymbol{a_t t}$

$S = s_O + v_O t + \frac{1}{2}(a_t t^2)$

$V^2 = (V_O)^2 + 2\boldsymbol{a_t}\,(S - s_O)$

2.3 Understand Motion of multiple Particles

- As shown in Figure 10, the motion of the two planes is independent as they move freely. In contrast, in Figure 11, the movement of block A affects the motion of block B because they are connected by a rope [1].

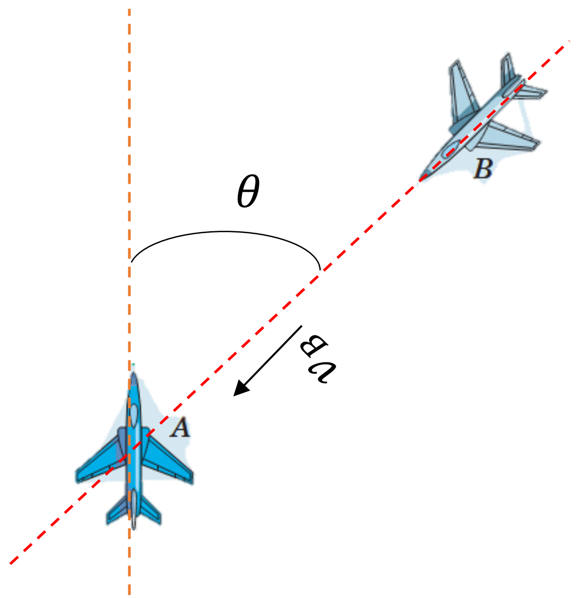


Figure 9. Independent motion

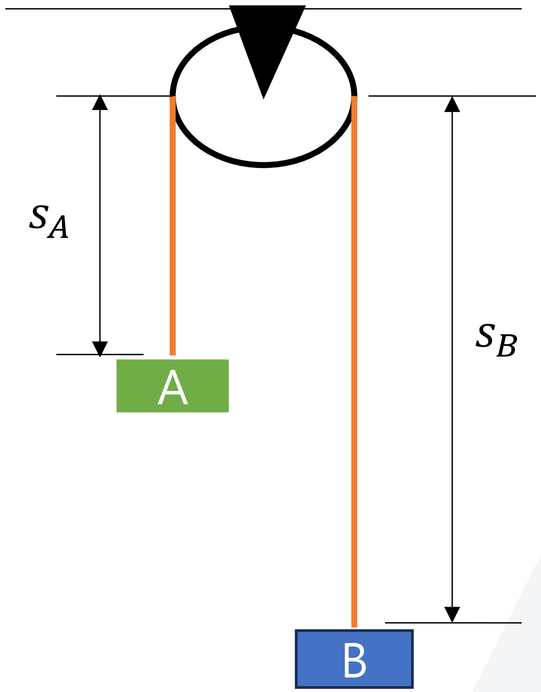


Figure 10. Dependent (constrained) motion

- Constraint equations and relative motion principles are key tools for solving problems involving interconnected particles.

Chapter 3. Understand the kinetics of particles

- kinetics of particle is defined as the study of translational motion of rigid bodies with the causing force

The three general approaches to the solution of kinetics problems are:

(A) direct application of Newton's second law (called the force- mass-acceleration method),

(B) use of work and energy principles, and

(C) solution by impulse and momentum methods

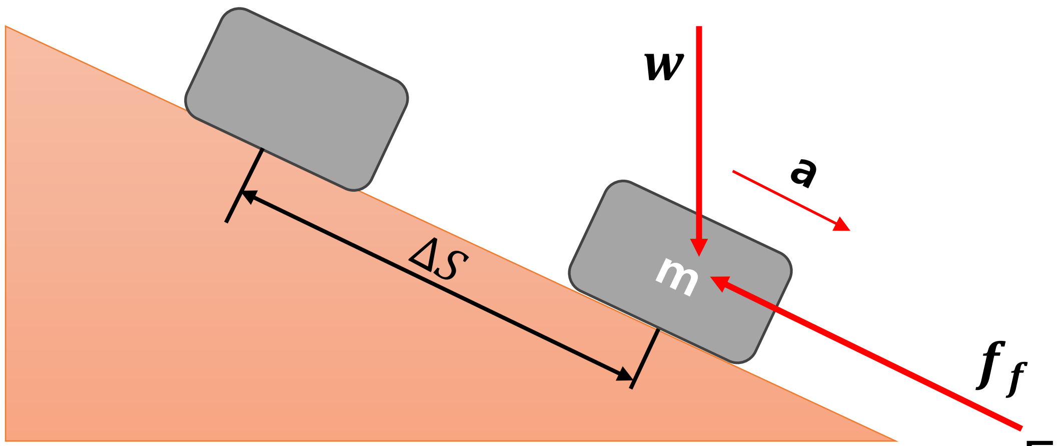


Figure 11. Typical kinetics system

- **Chapter 3 is subdivided into Sections A, B, and C, according to these three methods of solution**

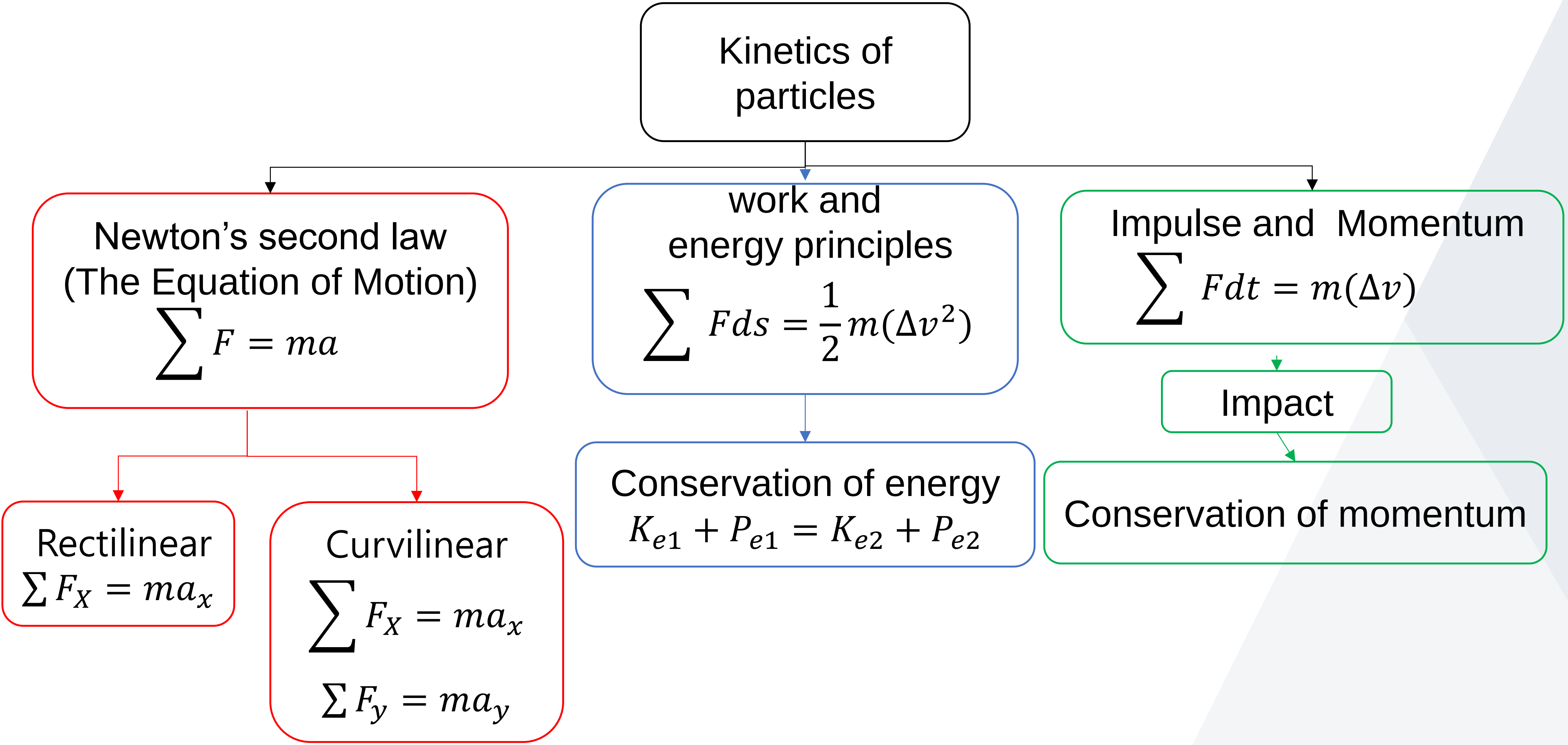


Figure 12. Chapter 3 structure

3.1 Summery on equation of motion

- Kinetics is the study of the relation between forces and the acceleration they cause.
- This relation is based on Newton's second law of motion, expressed mathematically as

$$\sum F = ma$$

- Before applying the equation of motion, it is important to first draw the particle's FBD and kinetic diagram.

Rectilinear motion

Equation of motion only applied in 1D and in other direction are in equilibrium

$$\sum F_x = ma_x \qquad \sum F_y = 0 \qquad \sum F_z = 0$$

Curvilinear motion

Polar Co-ordinate

$\sum F_\theta = ma_\theta$ $\quad a_r = (\ddot{r} - r\dot{\theta}^2)$

$\sum F_r = ma_r$ $\quad a_\theta = (r\ddot{\theta} - 2\dot{r}\dot{\theta})$

$\sum F_{bn} = 0$

Normal-tangent Co-ordinate

$\sum F_t = ma_t$ $\quad a_t = \frac{dv}{dt}$

$\sum F_n = ma_n$ $\quad (a_n = \frac{v^2}{\rho})$

$\sum F_{bn} = 0$

3.2 Summery on work energy

Work –energy equation

- useful for solving problems that involve force, velocity, and displacement
- Equation $T_1 + \sum U_{1-2} = T_2$
- Applied in all cases.

Conservation of energy

- A conservative force does work that is independent of its path. Two examples are the weight of a particle and the spring force.
- The work done by a conservative force depends upon its position relative to a datum.
- When this work is referenced from a datum, it is called potential energy..
- Equation $T_1 + V_1 = T_2 + V_2$
- Applied in certain cases.

3.3 Principle of Linear Impulse and Momentum

$$mv_1 + \sum \int_{t_1}^{t_2} F dt = mv_2$$

- which states that the initial momentum of the particle at t_1 plus the vector sum of all the impulses applied to the particle during the time interval from t_1 to t_2 is equivalent to the final momentum of the particle at t_{12} .

These three terms are illustrated graphically on the impulse and momentum diagrams shown in Fig. 5.

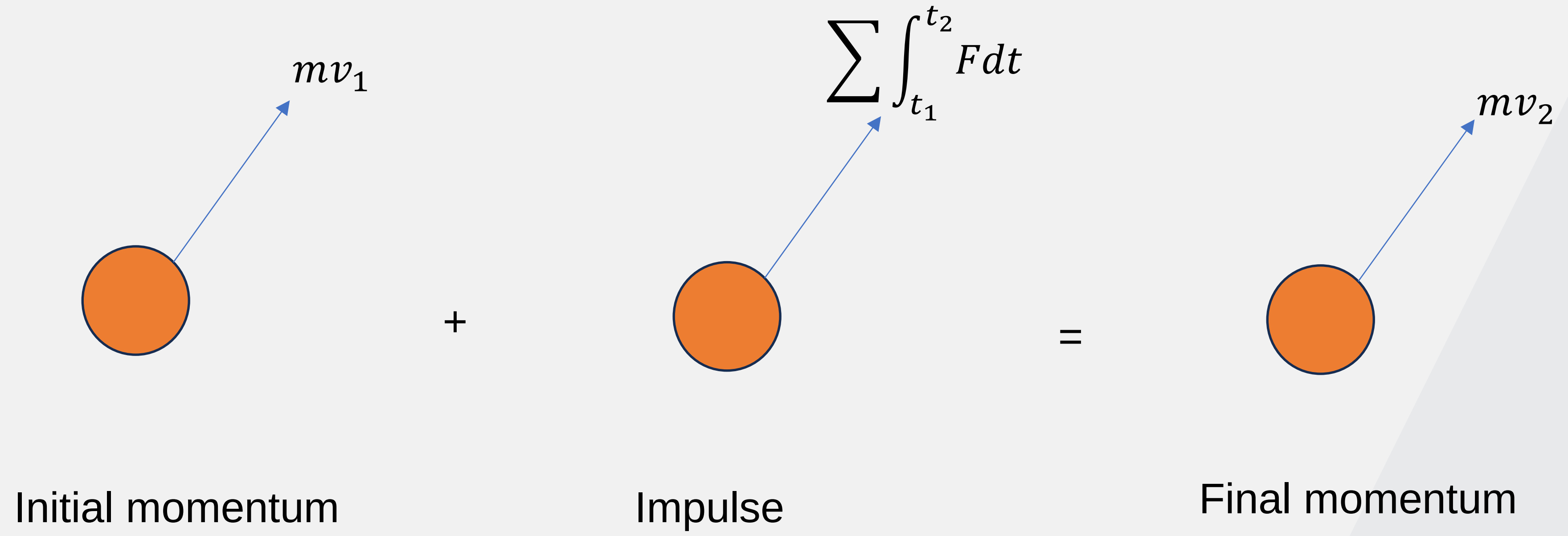


Figure 13. Impulse and momentum diagrams

- If each of the vectors in the impulse momentum equation is resolved into its x, y, z components, we can write the following three scalar equations of linear impulse and momentum.

$$mv_{(x)_1} + \sum \int_{t_1}^{t_2} F_x dt = mv_{x(2)}$$

$$mv_{(y)_1} + \sum \int_{t_1}^{t_2} F_y dt = mv_{y(2)}$$

$$mv_{(z)_1} + \sum \int_{t_1}^{t_2} F_z dt = mv_{z(2)}$$

For system of particles

- When a problem involves two or more particles, we can consider each particle separately and write impulse momentum equation for each particle. We can also add vectorially the momenta of all the particles and the impulses of all the forces involved. We then have

$$\sum mv_1 + \sum \int_{t_1}^{t_2} F dt = \sum mv_2$$

- This equation states that the initial linear momenta of the system plus the impulses of all the external forces acting on the system from t_1 to t_2 is equal to the system's final linear momenta.

Impact analysis

- Central impact occurs when the direction of motion of the mass centers of the two colliding particles is along a line passing through the mass centers of the particles.

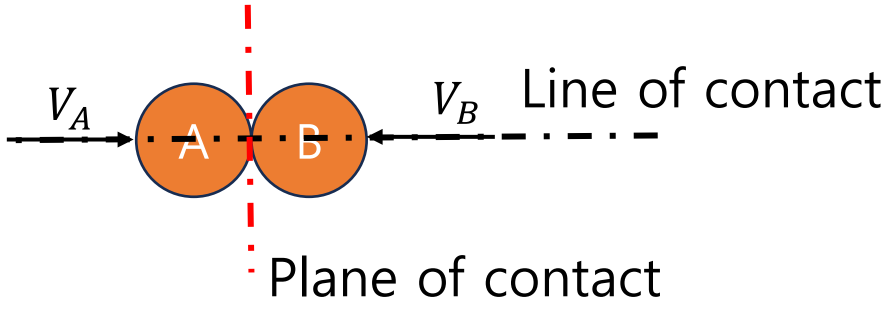


Figure 14. central impact

- When the motion of one or both of the particles make an angle with the line of impact, Fig.16, the impact is said to be oblique impact.

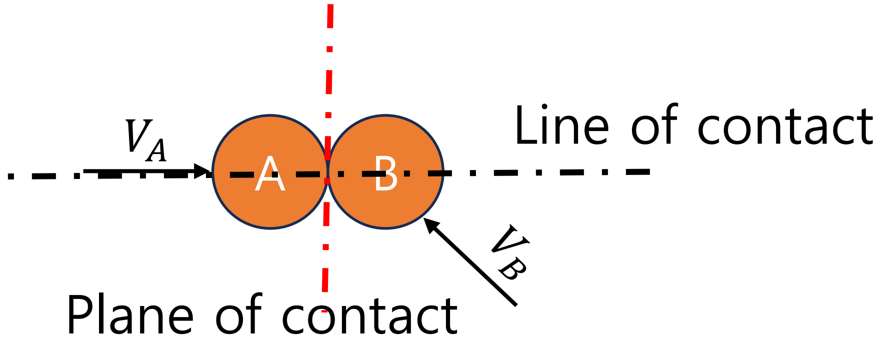


Figure 15. oblique impact

Central impact

- For Two particles undergoing a **direct central impact,** If the Coefficient of restitution (e), Mass of each particle and Initial velocities before impact following are known, then goal is to find their final velocities just after collision.
- Then, the magnitudes of final velocities can be obtained, by simultaneously solving using two main equations: Conservation of Momentum Coefficient of Restitution Equation.

$$m_A v_{(A)_1} + m_B v_{(B)_1} = m_A v_{(A)_2} + m_B v_{(B)_2}$$

$$e = \frac{v_{(B)_2} - v_{(A)_2}}{v_{(A)_1} - v_{(B)_1}}$$

- If the y axis is established within the plane of contact and the x axis along the line of impact, the impulsive forces of deformation and restitution act only in the x direction.

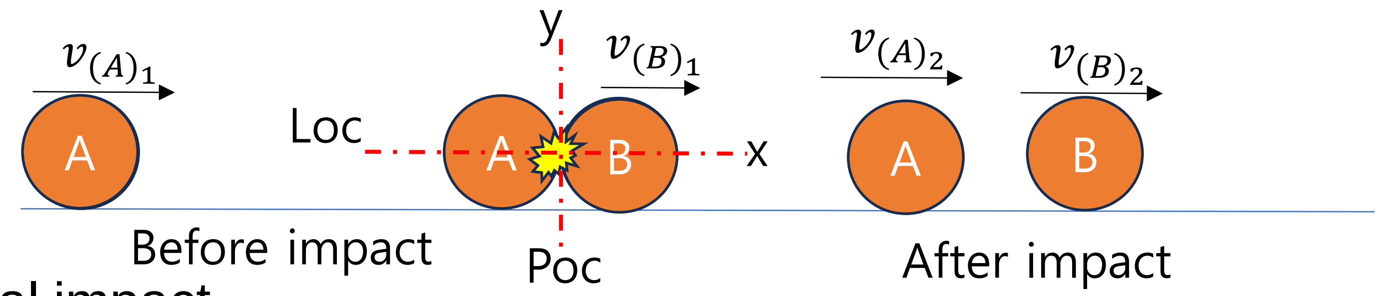


Figure 16. Central impact

- Both conservation of energy and The coefficient of restitution equations applied along line of impact or x axis.

Oblique impact

- The motion can be analyzed by resolving the velocities along the line of impact (x-axis) and along the plane of contact (y-axis).

Along line of impact

- The momentum of the system is conserved always

$$m_A v_{(A_1)_x} + m_B\, v_{(B_1)_x} = m_A v_{(A_2)_x} + m_B v_{(B_2)_x}$$

- The coefficient of restitution (e) relates the relative velocities of the two bodies along the line of impact before and after collision:

$$e = \frac{v_{(B_2)_x} - v_{(A_2)_x}}{v_{(A_1)_x} - v_{(B_1)_x}}$$

Along plane impact or contact

- No impulse acts along the plane of contact (y-axis).Therefore, the components of velocity along y remain unchanged:

$$v_{(A_1)_y} = v_{(A_2)_y} \qquad v_{(B_1)_y} = v_{(B_2)_y}$$

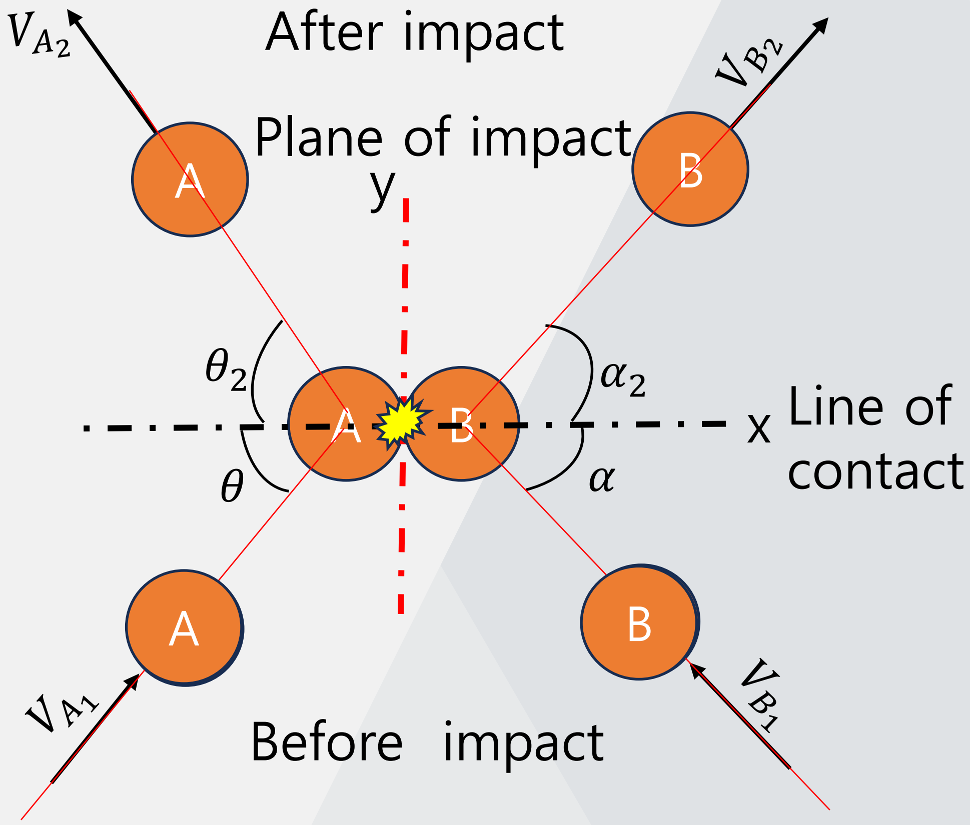


Figure 17.oblique impact

Magnitudes of final velocity and directions

Magnitudes

$$For\ particle\ A, V_{A_2} = \sqrt{(v_{(A_2)_x})^2 + (v_{(A_2)_y})^2}$$

$$For\ particle\ B, V_{B_2} = \sqrt{(v_{(B_2)_x})^2 + (v_{(B_2)_y})^2}$$

directions

$$\theta_2 = \frac{(v_{(A_2)_y})}{(v_{(A_2)_x})}$$

$$\alpha_2 = \frac{(v_{(B_2)_y})}{(v_{(B_2)_x})}$$

Summery on chapter three

- The You now have at your disposal three different methods for the solution of kinetics problems:
  - The direct application of Newton’s second law
  - The method of work and energy :
  - The method of impulse and momentum
- To derive maximum benefit from these three methods, you should be able to choose the method best suited for the solution of a given problem.
- You also should be prepared to solve problems that require you to use multiple principles.

Choosing the Right Method

- Work & Energy Method – faster and simpler in many cases. Has limitations – cannot determine acceleration or normal force.
- $\Sigma F = ma$ is required when acceleration or normal force is unknown.
- For non-impulsive motion, both $\Sigma F = ma$ and Work–Energy give quick solutions.
- For impact or collision problems, only Impulse–Momentum is practical, because:

✓ $\Sigma F = ma$ becomes too complex.

✓ Work–Energy cannot be used (energy loss during impact).

- Some problems combine methods:

✓Impact phase → use Impulse–Momentum & relative velocity.

✓Before/After impact → use Work–Energy.

✓Normal force calculation → use $\Sigma F = ma$.

Choosing the Right Method Example

System setup:

•Pendulum A (mass m_A, length l) released from rest at position A_1.
•Pendulum B (mass m_B, same length l) initially at rest.
•After impact (coefficient of restitution e), pendulum B swings through an angle θ.

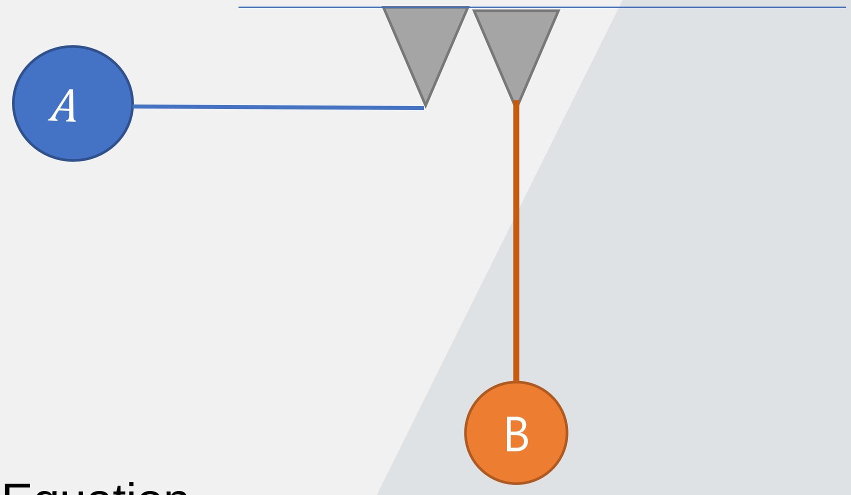
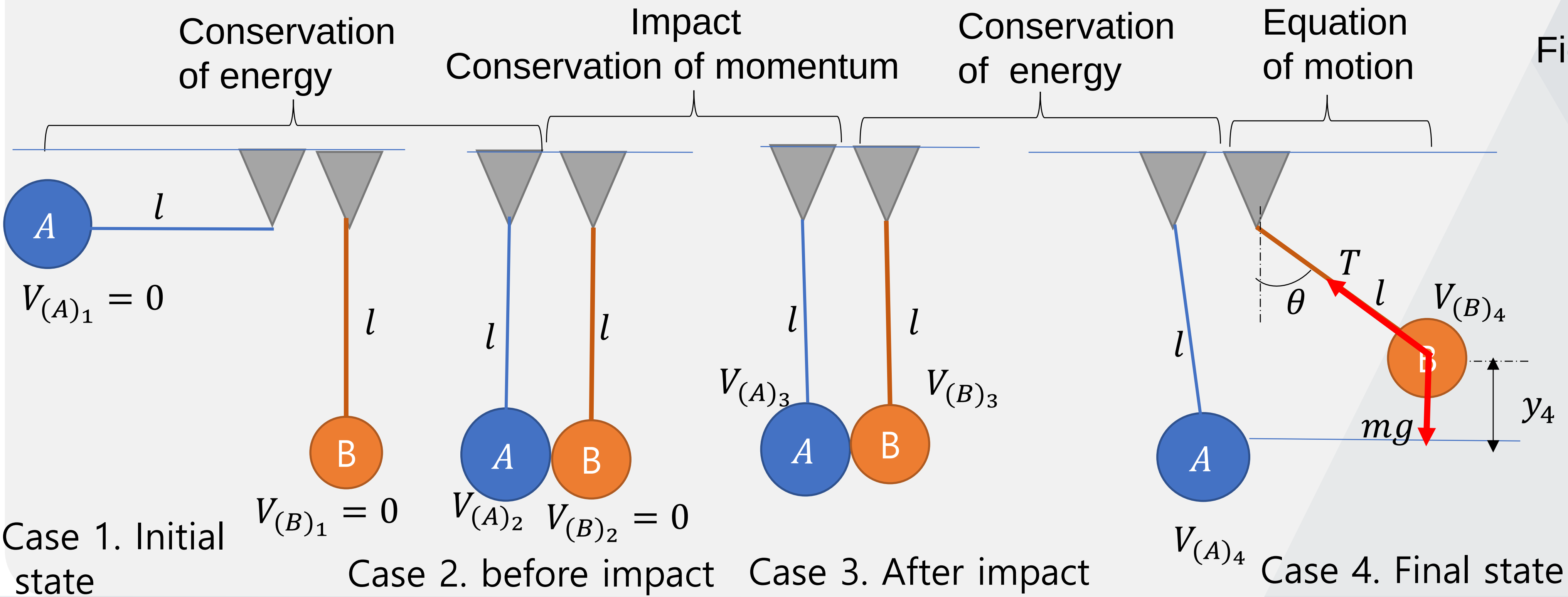


Figure 18. Pendulum



Activity 1

- The 15-lb suitcase A is released from rest at C. After it slides down the smooth ramp, it strikes the 10lb suitcase B, which is originally at rest. If the coefficient of restitution between the suitcases is e = 0.3 and the coefficient of kinetic friction between the floor DE and each suitcase is $\mu_k = 0.4$, determine (a) the velocity of A just before impact, (b) the velocities of A and B just after impact, and (c) the distance B slides before coming to rest.

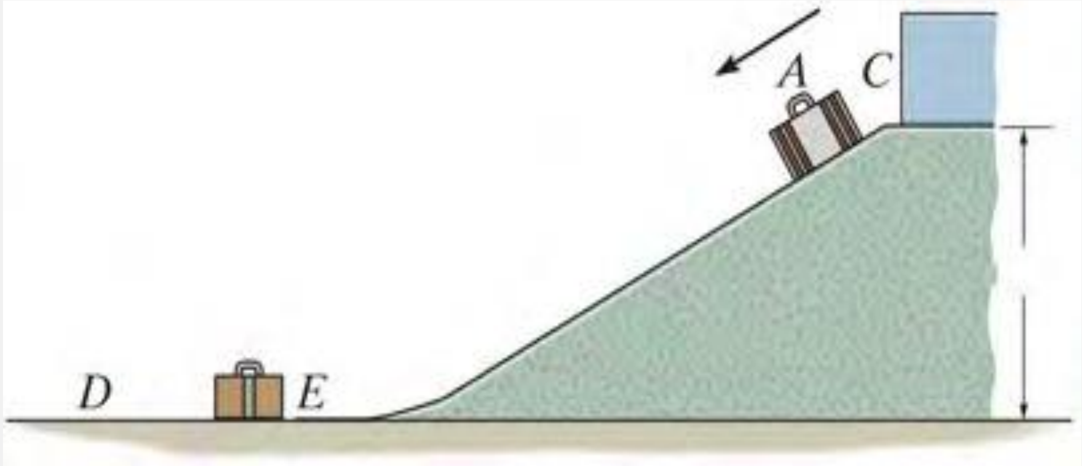


Figure 19. Activity 1

Activity 2

- A 2-kg block A is pushed up against a spring compressing it a distance x. The block is then released from rest and slides down the 20° incline until it strikes a 1-kg sphere B which is suspended from a 1-m inextensible rope. The spring constant k 5 800 N/m, the coefficient of friction between A and the ground is 0.2, the distance A slides from the unstretched length of the spring d 5 1.5 m, and the coefficient of restitution between A and B is 0.8. Knowing the tension in the rope is 20 N when α = 30°, determine the initial compression x of the spring

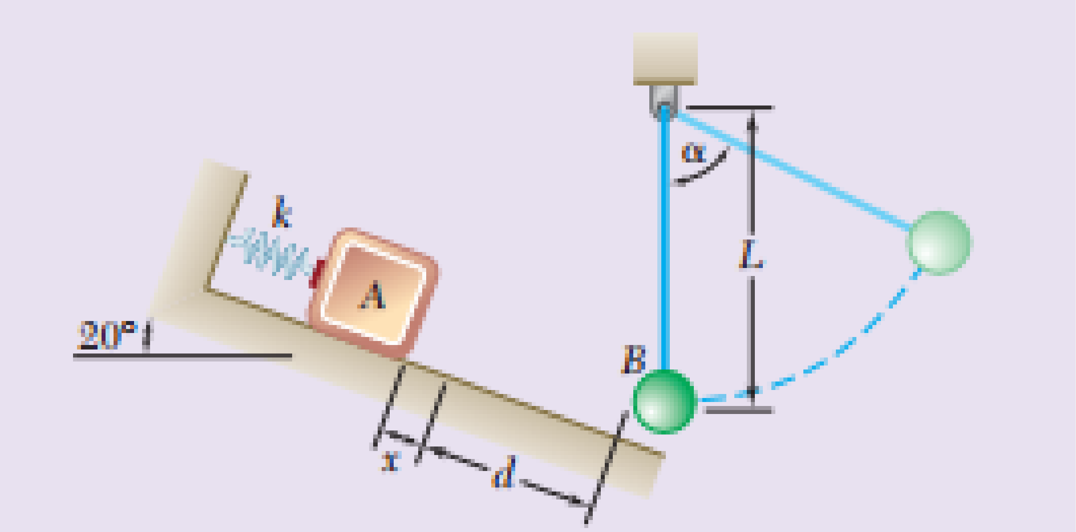


Figure 20. Activity 2

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