Week 9

Kinematics of Rigid Bodies- Types of Motion

Lecturer : Daniel Hambissa Datti

Contents

By the end of this lecture, you are able to:

1. Understand the Kinematics of Rigid Bodies and its scope
2. Define different Types of Motion of rigid bodies
3. Define Translational motion
4. Define rotational motion
5. General plane motion

Understand the Kinematics of Rigid Bodies and its scope

- Kinematics is often described as the “geometry of motion. The principles relate the displacement, velocity , acceleration, and time of a body’s motion, without reference to the cause of the motion [1].
- Recall that , A rigid body is an object that does not deform under the influence of forces. The distance bet ween any two points in the body remains constant.

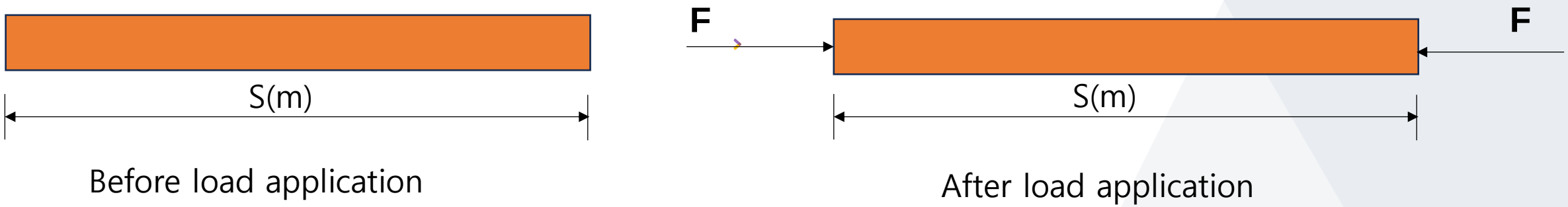


Figure 1. Rigid bodies

- Unlike particles that exhibit only translational motion, rigid bodies experience a combination of translation al and rotational motions, causing each point in the body to move differently see figure 2.
- In on previous studies of particle kinematics, we developed the relationships governing the displacement, velocity, and acceleration of points as they moved along straight or curved paths [1].
- In rigid-body kinematics we use these same relationships but must also account for the rotational motion of the body. Thus rigid-body kinematics involves both linear and angular displacements, velocities, and ac celerations

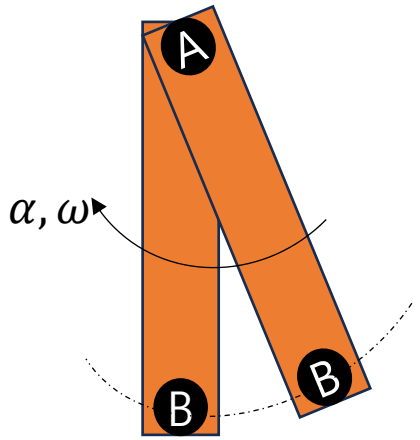


Figure 2. Rigid bodies motion

scope Cont'd…

- We will investigate the relations between the time, the positions, the velocities, and the accelerations of the various particles forming a rigid body.

This study is important for the design of gears, cams, and mechanisms used for many mechanical operatio ns.

- Once the kinematics is thoroughly understood, then we can apply the equations of motion, which relate the forces on the body to the body's motion.

- Thus, this chapter is covered considering different types of motions of rigid bodies, namely Kinematics of rigid body study – during translational motion , rotational Motion and general plane motion.

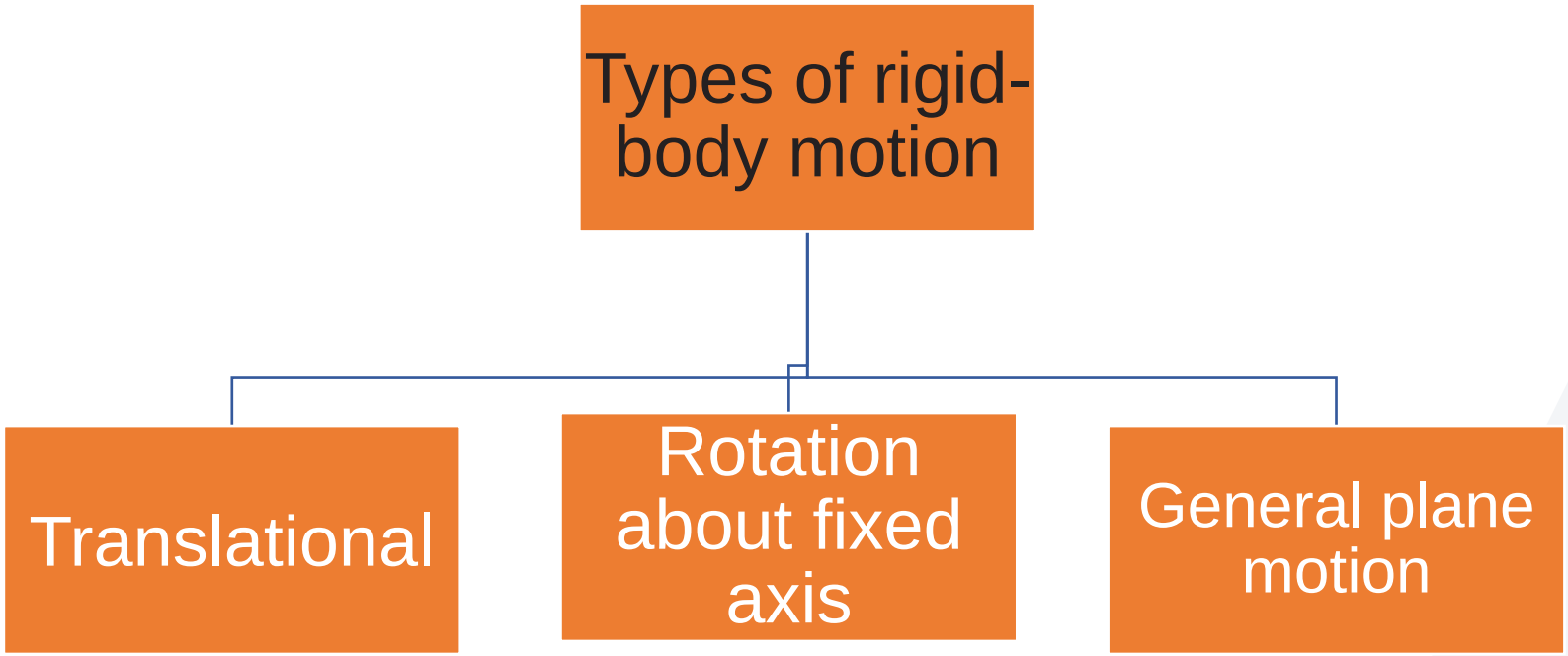


Figure 3. Types Rigid body motions

Each type depends on how all points in the body move relative to one another.

Translation motion

- All points in the body move in parallel paths.
- Lines connecting any two points remain parallel during motion.
- No rotation occurs.

a. Rectilinear

b. Curvilinear

Figure 4. Translational motion of rigid bodies

- NB : Today's lecture mainly focuses on the rectilinear motion of a particle.

Velocity and Acceleration in Translational Motion

- Consider a rigid body which is subjected to either rectilinear or curvilinear translation in the x-y plane

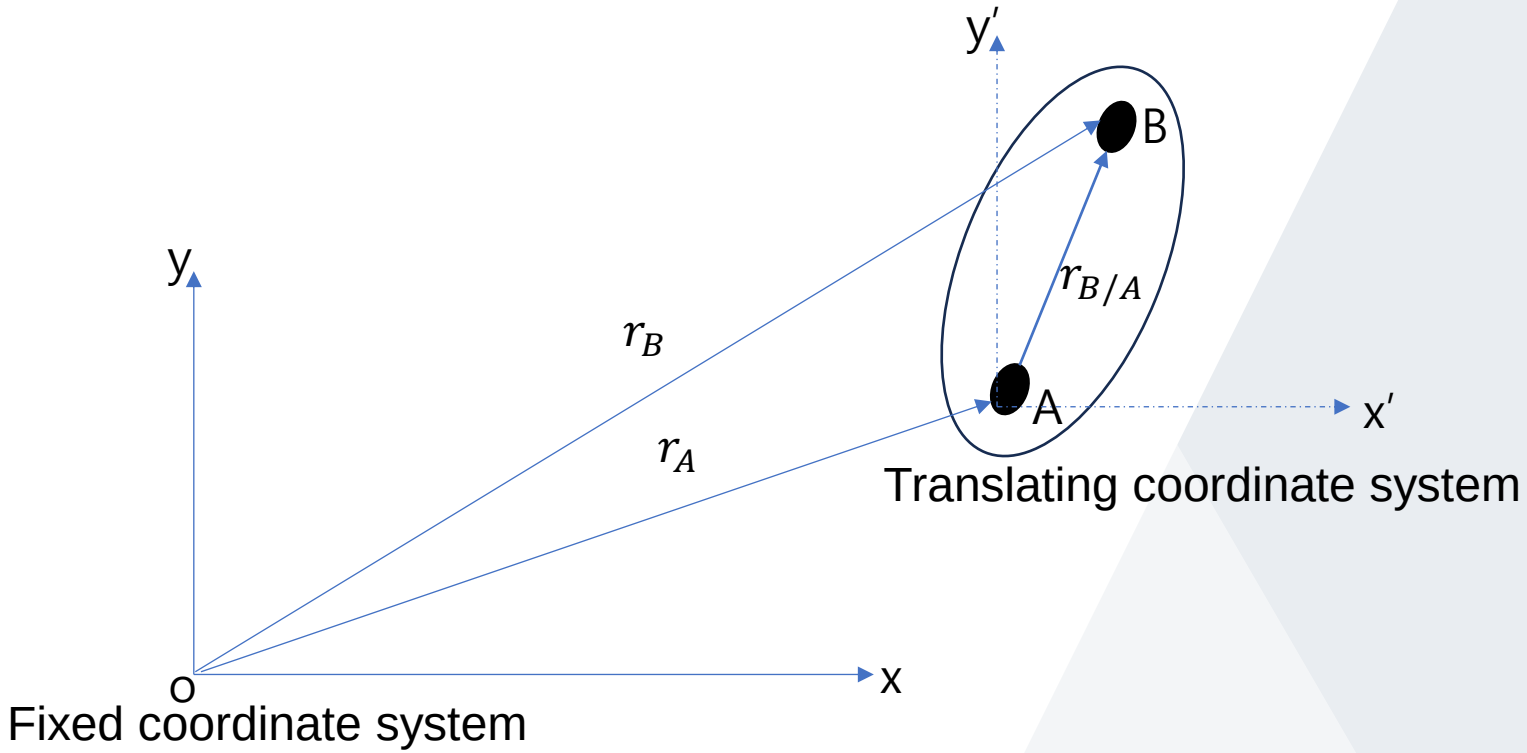


Figure 5. Translational motion Expression

Velocity and Acceleration in Translational Motion

- Position: $r_B = r_A + r_{B/A}$
- Velocity ($\frac{dr_B}{dt}$): $V_A = V_B$
- Acceleration : $a_A = a_B$

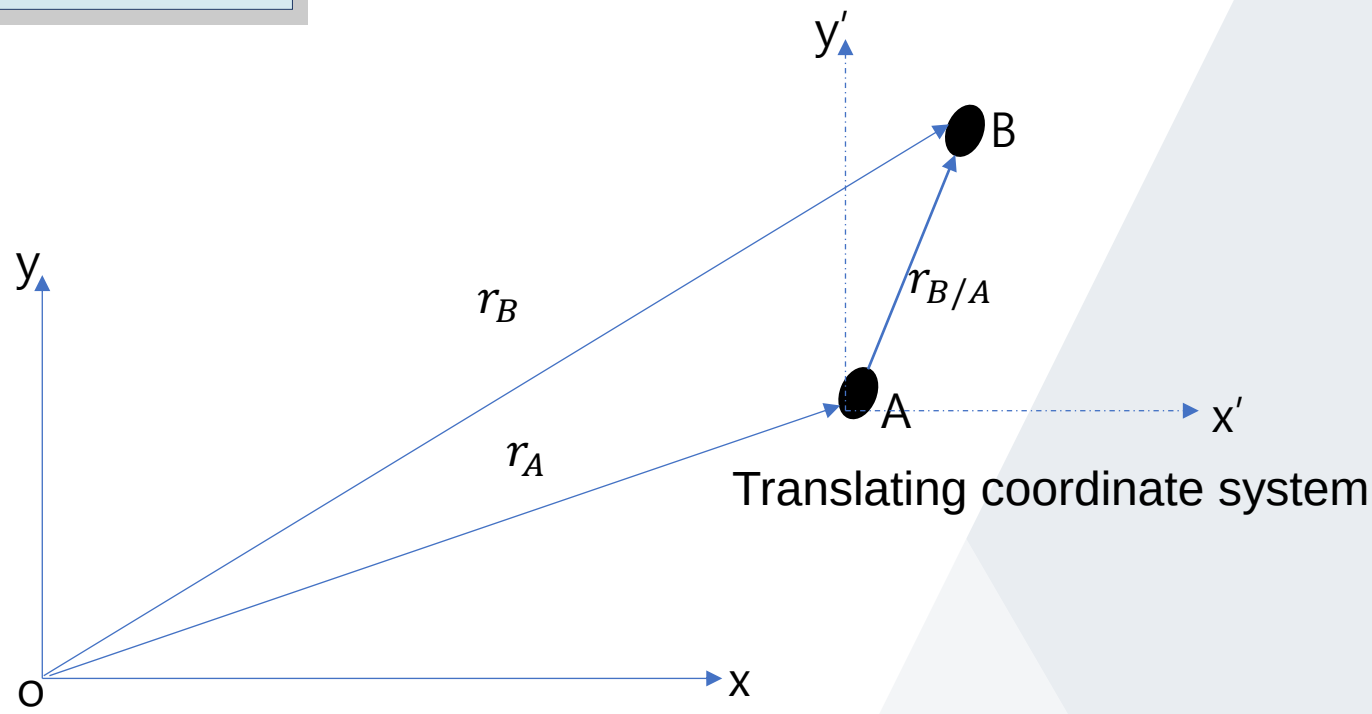


Figure 6. vector Expression

Here $V_A, V_B, a_A \ and \ a_b$ denote absolute velocities and absolute velocities since these vectors are measured with respect to the fixed x, y axes.

The term $\frac{d(r_{B/A}}{dt}$ = 0, since the magnitude of $r_{B/A}$ is constant by definition of a rigid body, and because the body is translating the direction Of $r_{B/A}$ is also constant.

Translation cont'd….

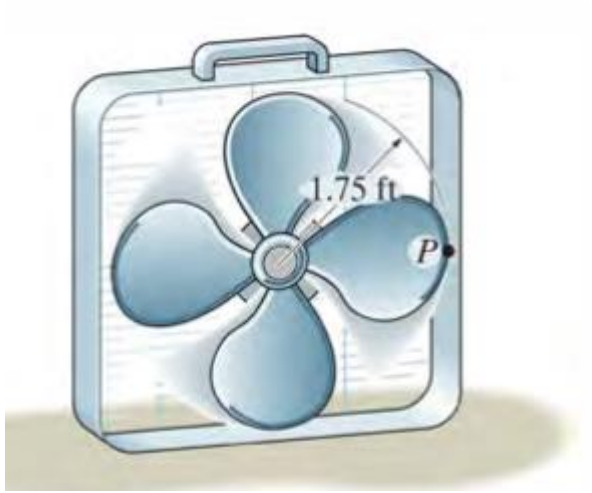
- Thus, when a rigid body is in translation, all the points of the body have the same velocity and the same acceleration at any given instant .
- In the case of rectilinear translation, all particles of the body move along parallel straight lines, and their velocity and acceleration keep the same direction during the entire motion
- In the case of curvilinear translation, the velocity and acceleration change in direction as well as in magnitude at every instant.
- Thus, our earlier study of the motion of a point (particle) in Chapter 2 enables us to describe completely the translation of a rigid body

Rotation about a Fixed Axis

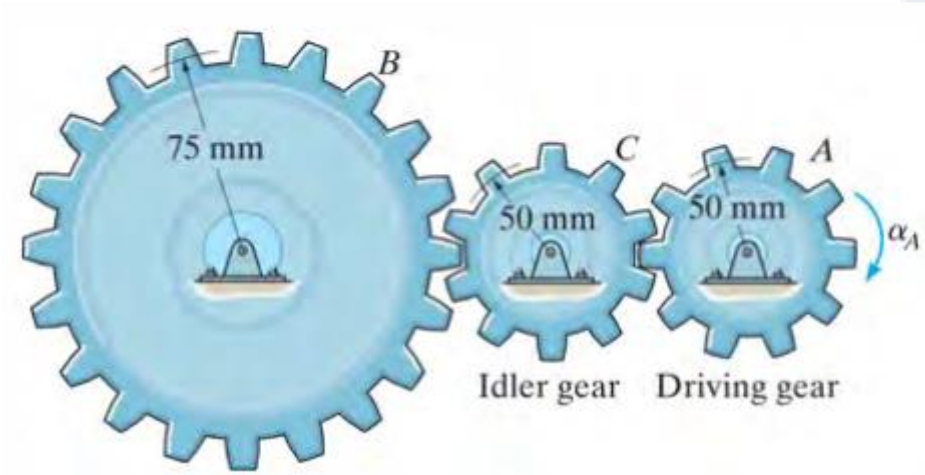
- It is the angular motion about the axis [3].

- It follows that all particles in a rigid body move in circular paths about the axis of rotation.

- Lines in the body which are perpendicular to the axis of rotation (including those which do not pass through the axis) rotate through the same angle in the same time.



a. Rotating fun



b. Rotating gear arrangement

Figure 7. Examples of Rotational motion

Rotation about a Fixed Axis

- For example, Consider a rod rotating about a fixed axis O, and take two points within the rigid body, say points A and B.

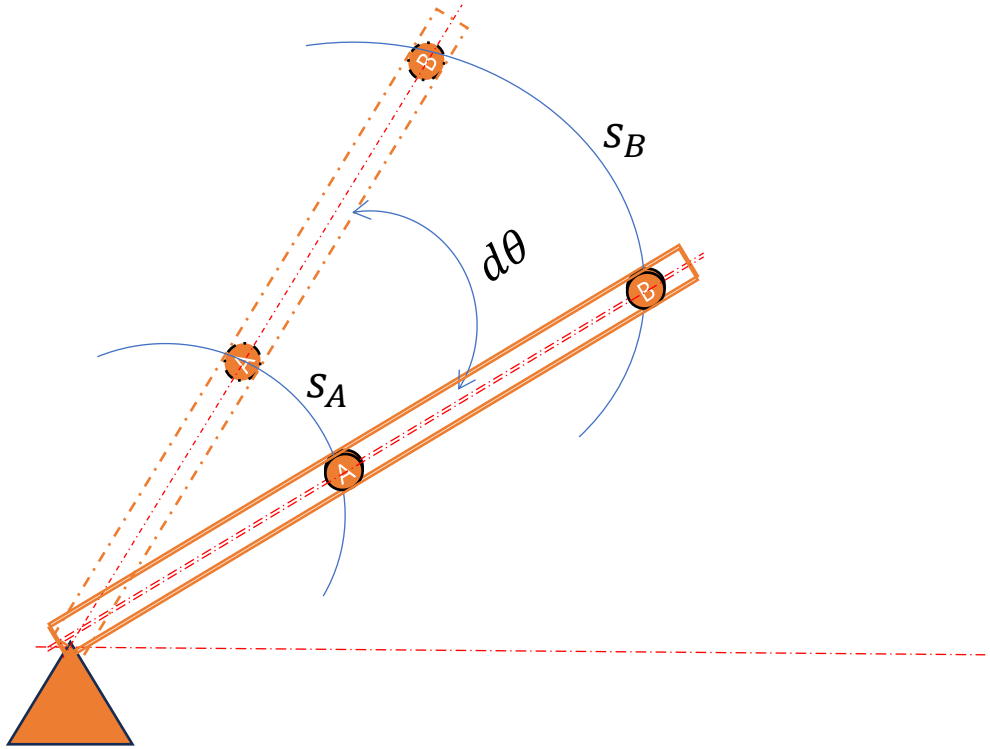
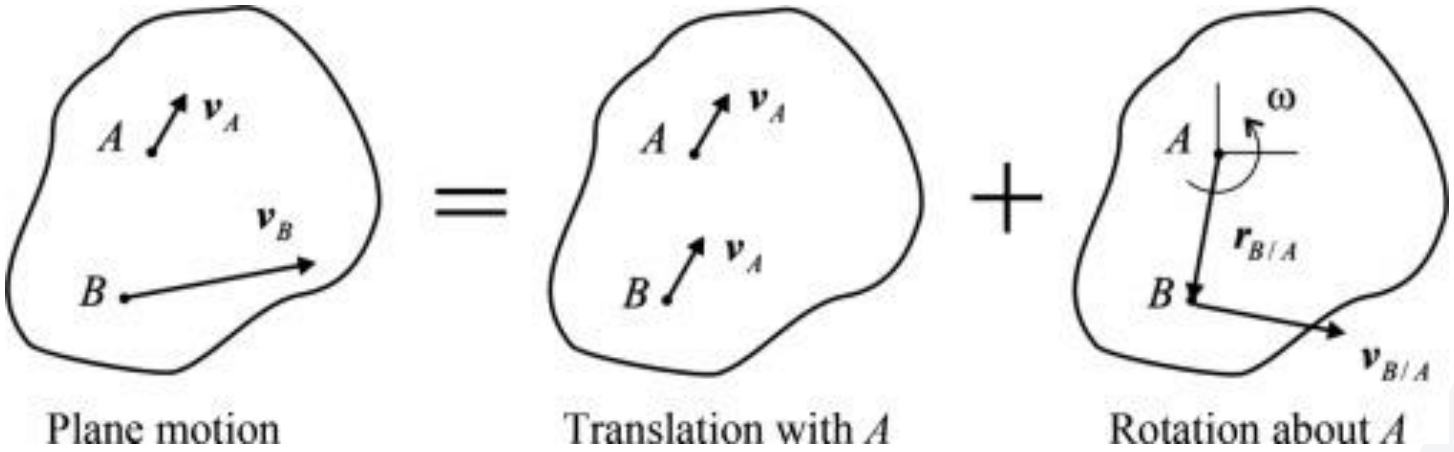


Figure 8. Rotational motion

- When the rod rotates through some angle θ, both points A and B undergo the same angular displacement, but the linear distance traveled by each point is different. This illustrates the typical behavior of a rigid body undergoing rotation about a fixed axis
- NB. We will discuss about this in detail in our next class

General Plane Motion.

- When a body is subjected to general plane motion, it undergoes a combination of translation and rotation.
- Every point experiences both linear and angular motion.



Source: https://www.sciencedirect.com/topics/engineering/plane-motion

Figure 9. General plane motion

- The translation occurs within a reference plane, and the rotation occurs about an axis perpendicular to the reference plane.

General Plane Motion.

- For example consider, The figure 10,which illustrates the general plane motion of a rigid rod having two points, A and B.

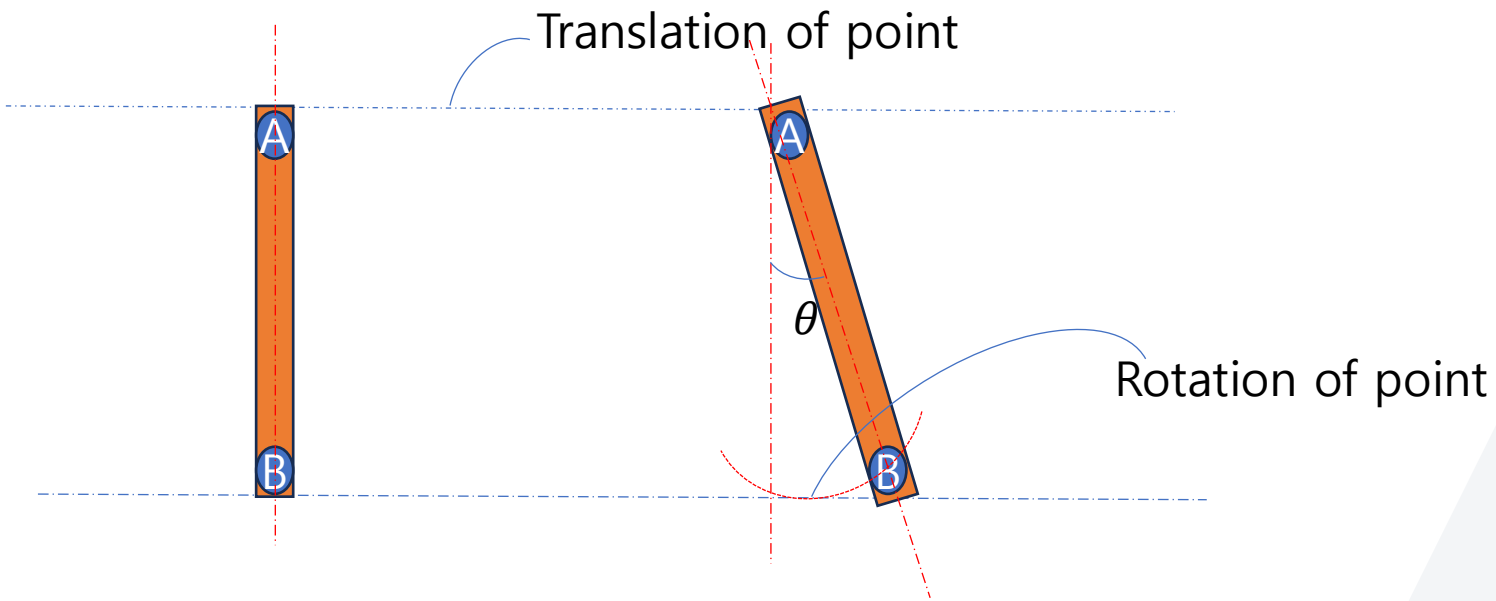


Figure 10. General plane motion

- As the rod moves, point A translates horizontally, while at the same time point B traces a curved path rotating through an angle θ about a translating axis A.
- This combined movement represents the typical nature of general plane motion in rigid bodies.

Analyzing General Plane Motion

- We can obtain the velocity and acceleration of any particle of the rigid body using the following Four methods:

1) Absolute Motion Analysis

2) Relative Velocity and acceleration analysis using translating axis

3) Instantaneous center of rotation

4) Relative Velocity and acceleration analysis using Rotating axis

Analyzing General Plane Motion

- Each method has its own advantages and is applied differently, yet all produce the same result.
- The first three methods which are:

  Absolute Motion Analysis

  Relative Velocity and acceleration analysis using translating axis

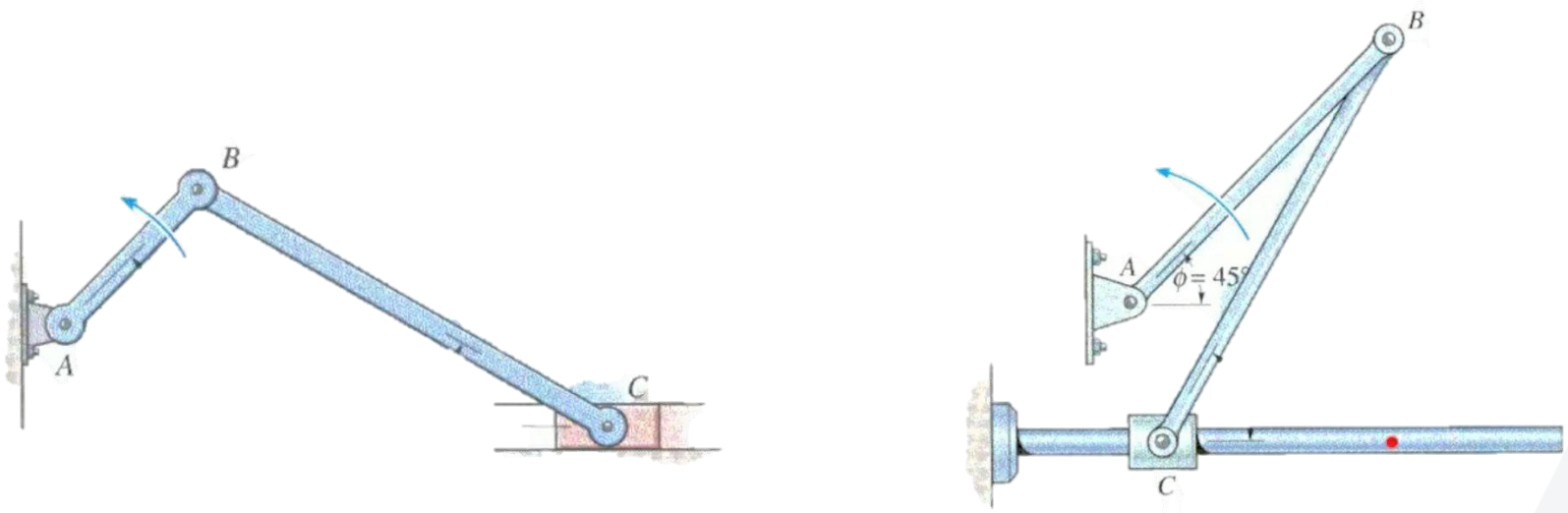
  Instantaneous center of rotation

  are useful for analyzing motion involving pin connections,

- While the last case which is Relative Velocity and acceleration analysis using Rotating axis used for a special situation.

- Each method has its own advantages and is applied differently, yet all produce the same result.

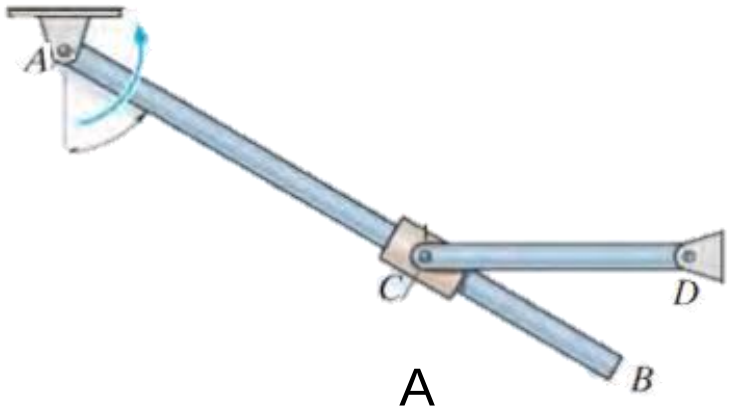
- The first three methods are very useful for determining the motion of points on the same rigid body, or the motion of points located on several pin-connected bodies.
- For example, they can be used to solve the type of problem shown below, where the distance between two points on the rigid body remains constant during motion.



Source : Engineering Mechanics - Dynamics, Meriam J.L.,John Wiley & Sons, 9th ed., 2020.,page 370

Figure 11. applications of Relative Velocity and acceleration analysis using translating axis

- In some problems, however, rigid bodies (mechanisms) are constructed such that sliding will occur at their connections.
- The kinematic analysis for such cases is best performed if the motion is analyzed using a coordinate system which both translates and rotates.
- Furthermore, this frame of reference is useful for analyzing the motions of two points on a mechanism which are not located in the same body and for specifying the kinematics of particle motion when the particle moves along a rotating path



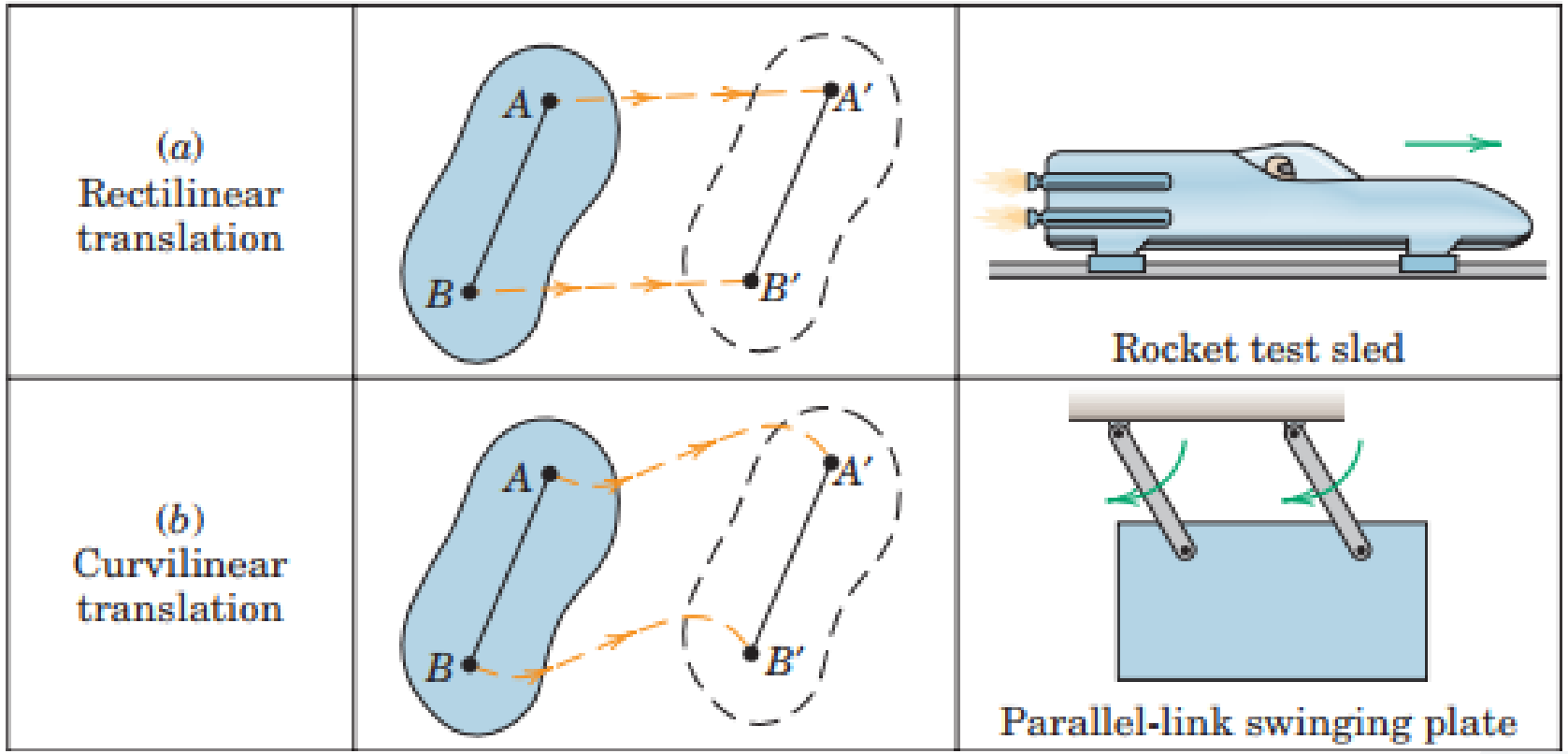
A

B

Source : Engineering Mechanics - Dynamics, Meriam J.L.,John Wiley & Sons, 9th ed., 2020.,page 377

Figure 12. applications of Relative Velocity and acceleration analysis using rotating axis

Summery of Motion of rigid bodies



Source : Engineering Mechanics - Dynamics, Meriam J.L.,John Wiley & Sons, 9th ed., 2020.,page 350

Figure 13. motion of rigid body

Summery of Motion of rigid bodies

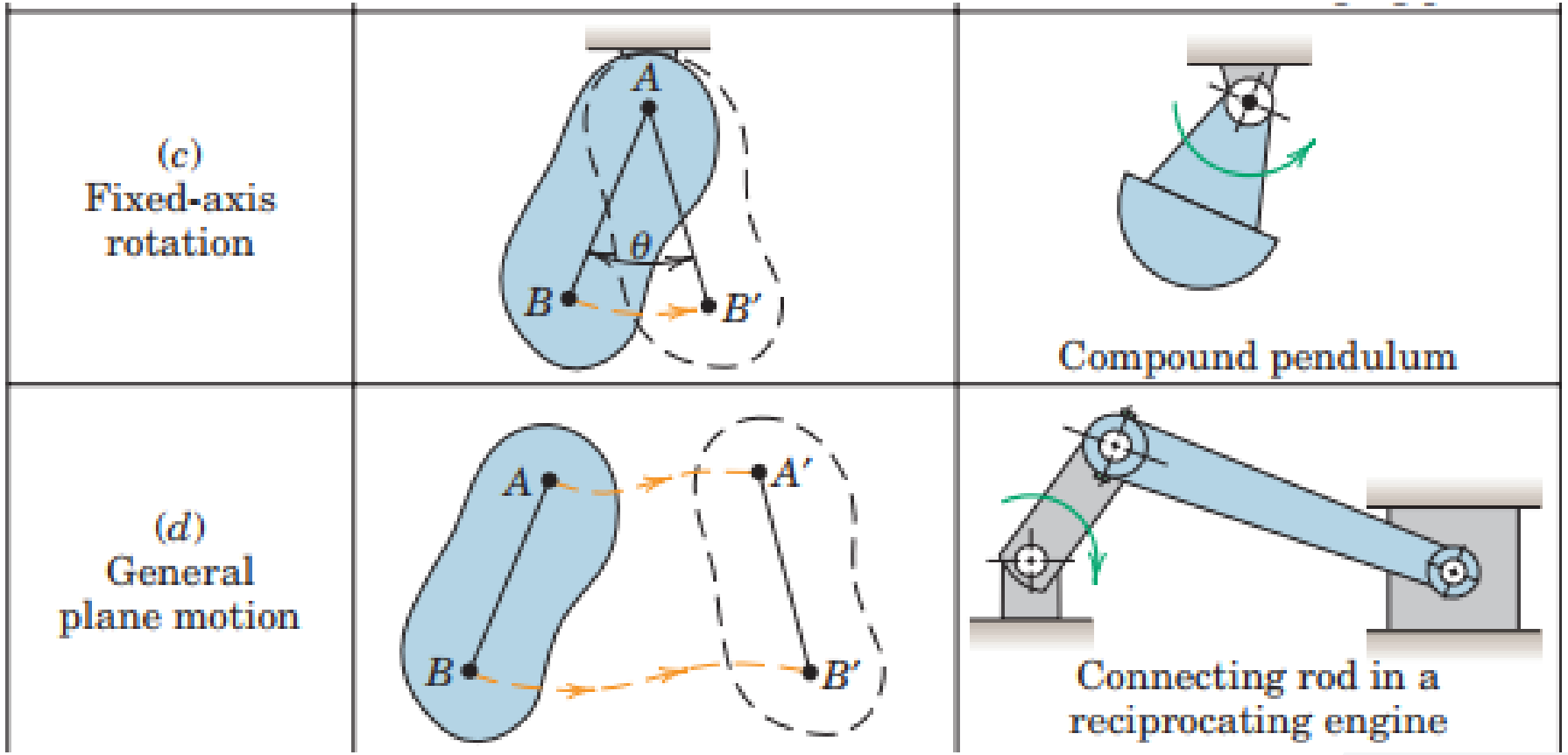
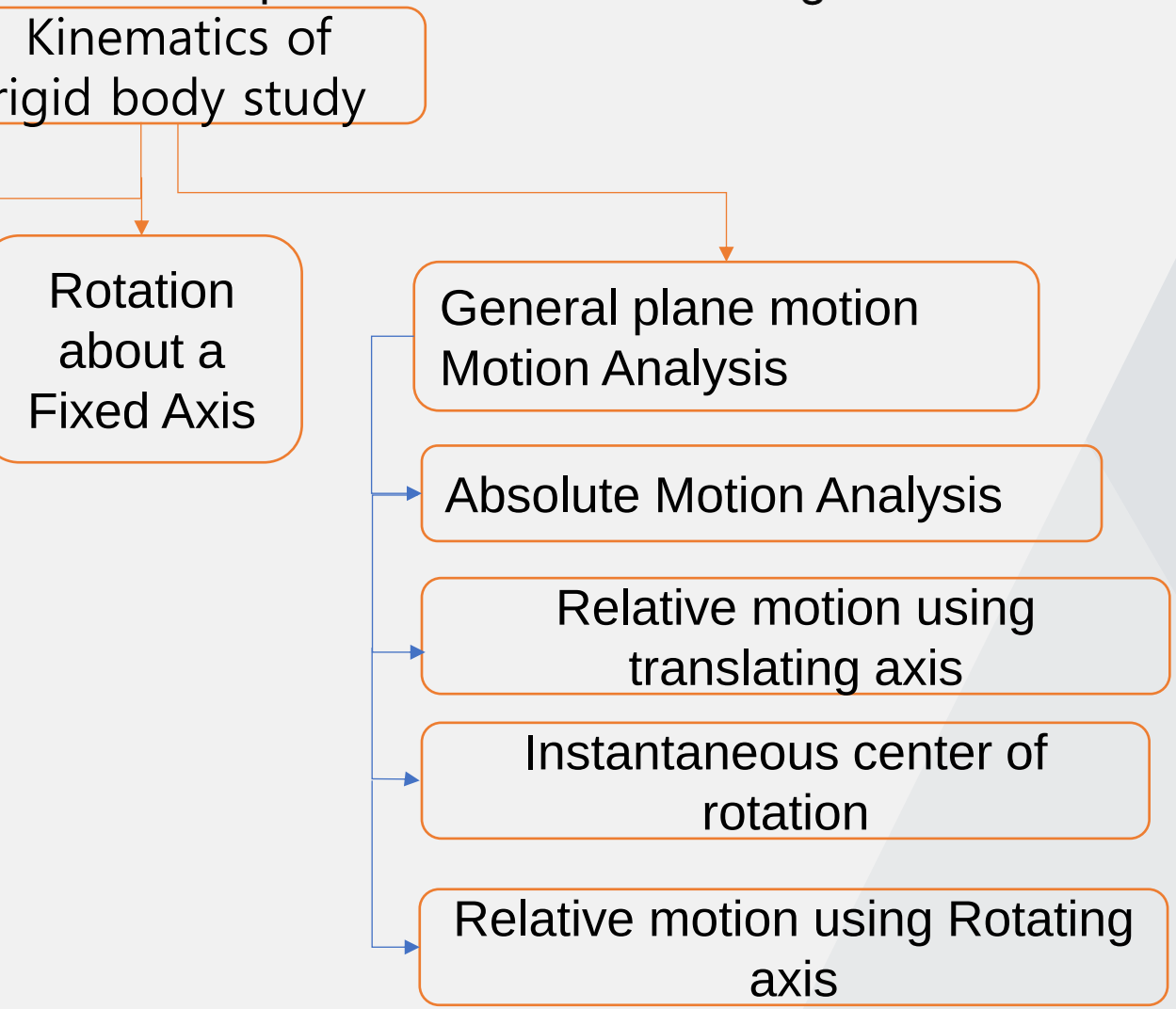


Figure 14. motion of rigid body

Chapter structure

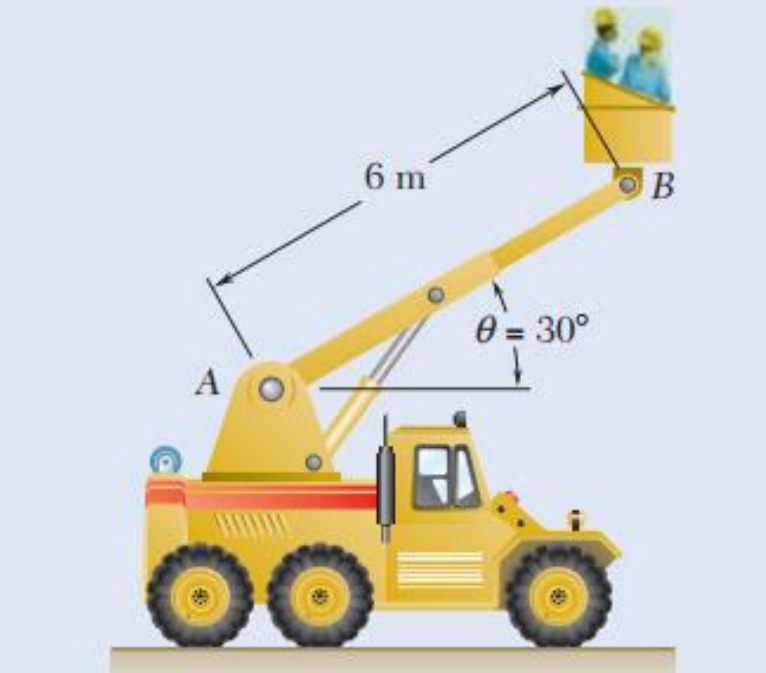
- The structure and flow of topics covered in this chapter are illustrated in the diagram below.



These concepts have already been covered in our study of kinematics of particles in Chapters 2 and 3.

Activity 1

What type of motion does link AB undergo as the cylinder extends?



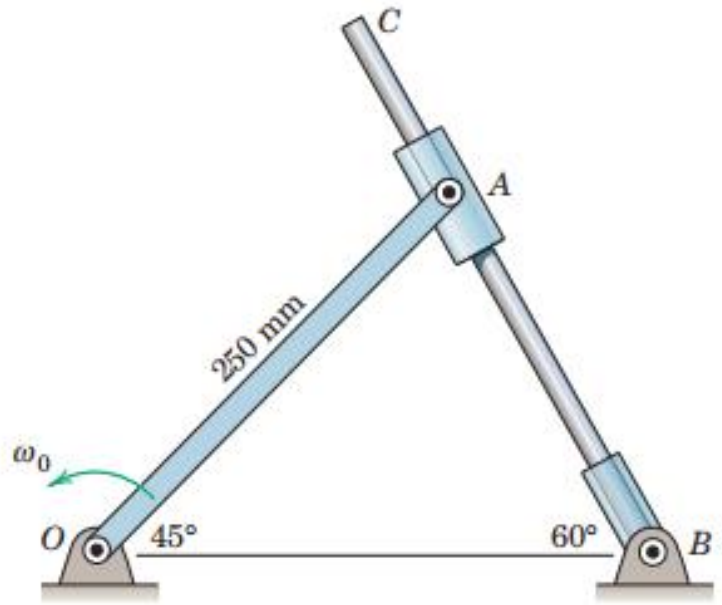
Answer: *Link **AB** undergoes **general plane motion**, since point **A** rotates about a fixed pivot while point **B** moves along a curved path — meaning the link experiences both rotation and translation.*

Source : Engineering Mechanics - Dynamics, Meriam J.L.,John Wiley & Sons, 9th ed., 2020.,page 365

Figure 15. Activity 1

Activity 2

What is the difference between determining the motion of AB and Ao in terms of the approach used to find their velocities?".



Answer: *Link AB can be solved using relativ e motion rotating axis equation and AO can b e solve by using fixed axis motion equation.*

Source : Engineering Mechanics - Dynamics, Meriam J.L.,John Wiley & Sons, 9th ed., 2020.,page 366

Figure 16. Activity 2

Activity 3

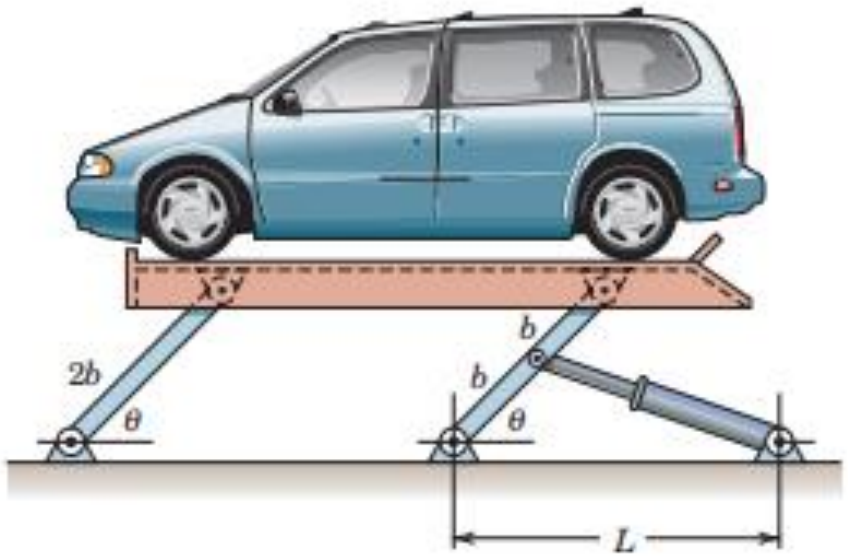
A basketball player shoots a free throw in such a way that his shoulder can be considered a pin joint at the moment of release as show. Determine the motion type of upper arm and forearm.

Source : Vector Mechanics for Engineers: Dynamics, Johnston, E. R., & Clausen, W. E., McGraw-Hill, 11th ed., 2015, page 1085

Figure 17. Activity 3

Activity 4

Identify the motion of the car rest on the platform due to the extension of the cylinder

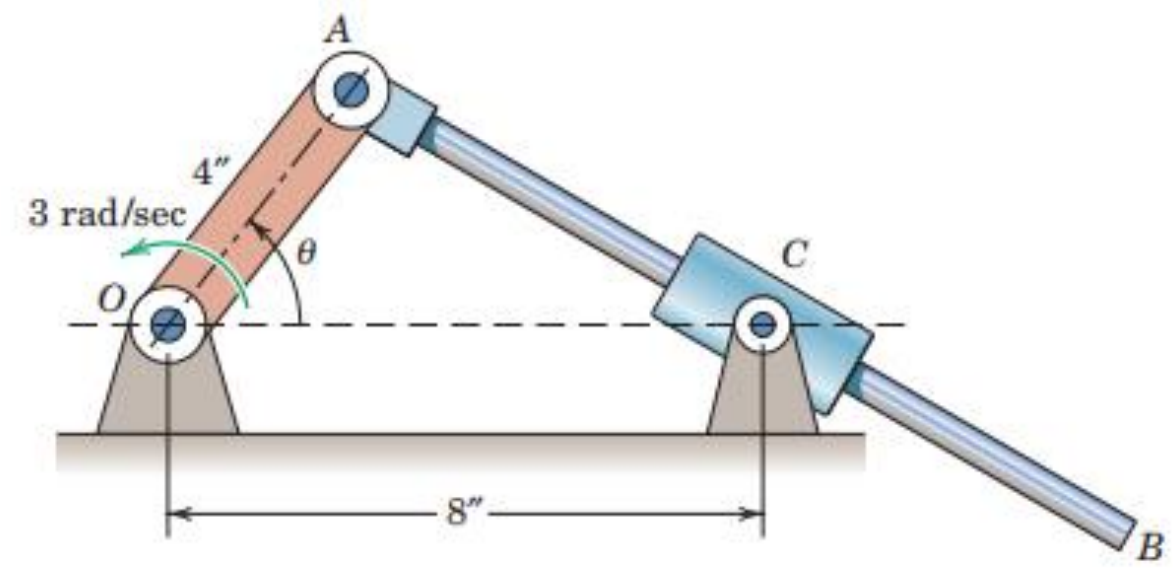


Source : Engineering Mechanics - Dynamics, Meriam J.L.,John Wiley & Sons, 9th ed., 2020.,page 365

Figure 18. Activity 4

Activity 5

Identify which approach or equation is used to find the velocity and acceleration between points A and C.

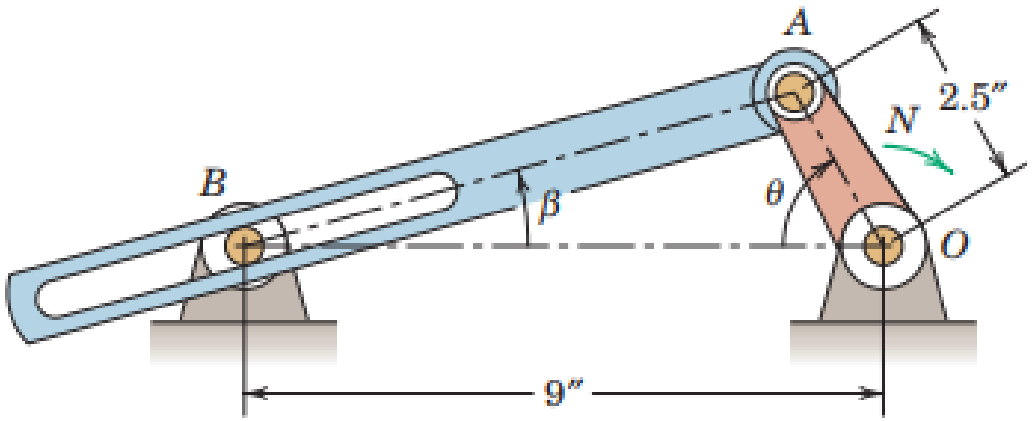


Source : Engineering Mechanics - Dynamics, Meriam J.L.,John Wiley & Sons, 9th ed., 2020.,page 366

Figure 19. Activity 5

Activity 6

Identify which approach or equation is used to find the velocity and acceleration between points A and B.



Source : Engineering Mechanics - Dynamics, Meriam J.L.,John Wiley & Sons, 9th ed., 2020.,page 366

Figure 20. Activity 6

summary

In today class, we covered:

1. Introduction to Kinematics of Rigid Bodies
2. Types of Motion of rigid bodies
3. General plane motion
4. Overview of Methods of solving General plane motion
5. Activities on identifying motion types

References

[1] Dynamics, Hibbeler, Rusel M.,Prentice Hall,10th ed., 2003

[2] Cengel, Yunus, and John Cimbala. *Ebook: Fluid mechanics fundamentals and applications (si u nits)*. McGraw Hill, 2013.

[3] Engineering Mechanics - Dynamics, Meriam J.L.,John Wiley & Sons, 9th ed., 2020.,

[4] Vector Mechanics for Engineers: Dynamics, Johnston, E. R., & Clausen, W. E., McGraw-Hill, 11th ed., 2015