Week 11

Relative Motion of Rigid Bodies

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Contents

By the end of this lecture, you are able to:

Relative Motion analysis of Rigid Bodies

- Relative motion analysis provides a systematic way to determine the velocity and acceleration of any point on a rigid body when the motion of another point on the same body is known.
- Relative motion analysis for a particle only considers translation, while for a rigid body, it must also account for rotation through angular velocity and angular acceleration.
- In dynamics, the motion of a rigid body can often be described in terms of the motion of one of its points and the rotation of the body about that point.
- When analyzing rigid-body motion, we commonly separate the motion into translation and rotation components.
- To accurately describe such combined motion, we use relative motion analysis. Depending on how the reference frame moves with the body, we apply one of two approaches [1].

1) Relative Motion Analysis Using a Translating Axis

2) Relative Motion Analysis Using a Rotating Axis

- ***Relative Motion Analysis Using a Translating Axis*** : This method is used when the reference axes move in pure translation with the body (they do not rotate).
- It is suitable when the body mainly translates and has small or negligible rotation, such as in sliding link ages.
- **Graphical approach's** such us The instantaneous center of zero velocity (ICZV) and velocity polygon methods can also be applied under this approach.
- This graphical method simplifies velocity analysis by locating a point on the body that has zero velocity at that instant, allowing all other velocities to be determined through pure rotation about that point.

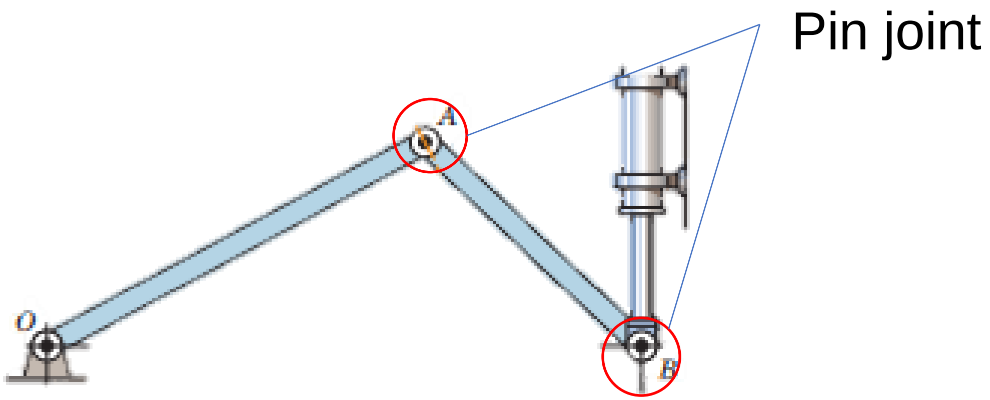


Figure 1. Application area of relative motion using translating axis

- **Relative Motion Analysis Using a Rotating Axis** –This method is applied when the reference axes are attached to the body and hence rotate with it.
- It is essential when analyzing motion where rotation and translation occur together, such as in rotating ar ms or mechanisms with angular motion [1].

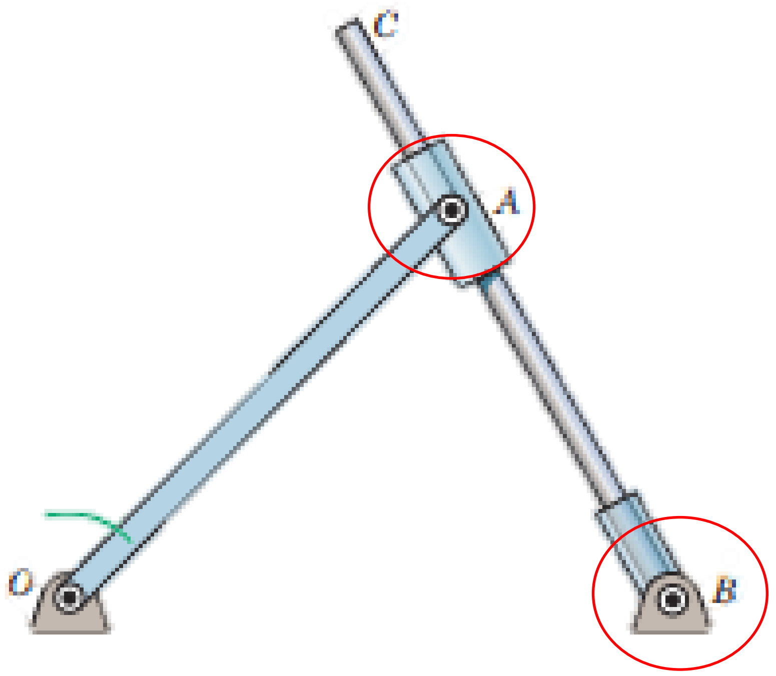


Figure 2 . Application area of relative motion using rotating axis

Relative-Motion Analysis using translating axis

- The general plane motion of a rigid body is a combination of translation and rotation.
- To study these two motion components separately, we use relative motion analysis.
- This analysis involves two sets of coordinate axes:
  - A fixed (x, y) coordinate system — used to measure the absolute position of points on the body [2].
  - A moving (x′, y′) coordinate system — whose origin is attached to a selected base point A on the body [2].

Relative-Motion Analysis using translating axis

- Consider the body on figure 3 Having two points A and B.
- The base point A usually has a known motion.
- The moving axes translate with respect to the fixed frame but do not rotate with the body.
- This setup allows us to express the motion of any point B on the body (e.g., a bar) relative to point A.

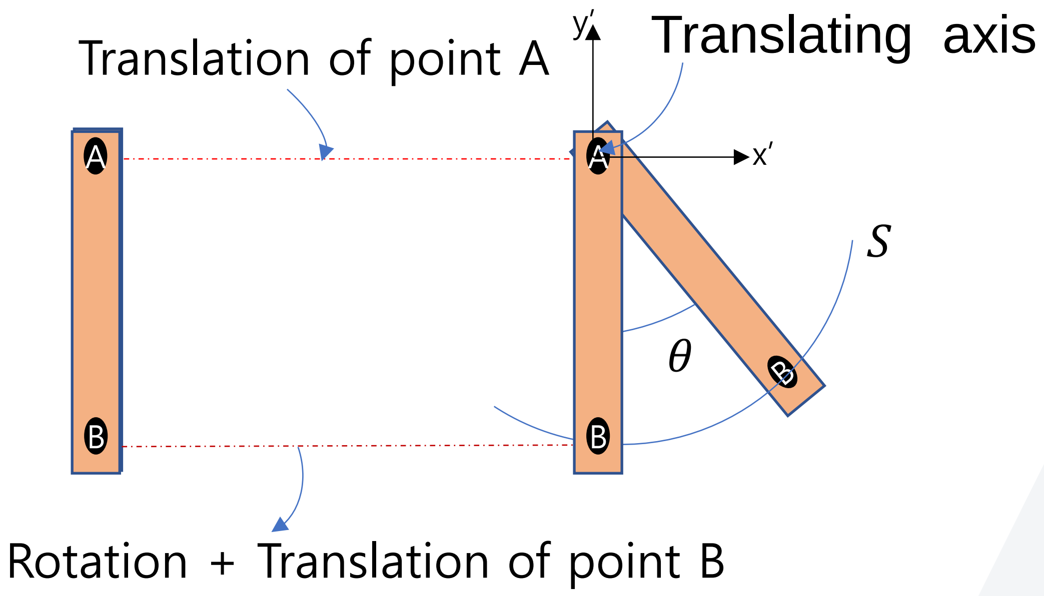


Figure 3. bar

Position

- The position vector r_A specifies the location of the base point A.
- The relative-position vector $r_{B/A}$ locates point B with respect to A.
- By vector addition, the absolute position of point B (r_B) is:

$$\boldsymbol{r_B} = \boldsymbol{r_A} + \boldsymbol{r_{B/A}}$$

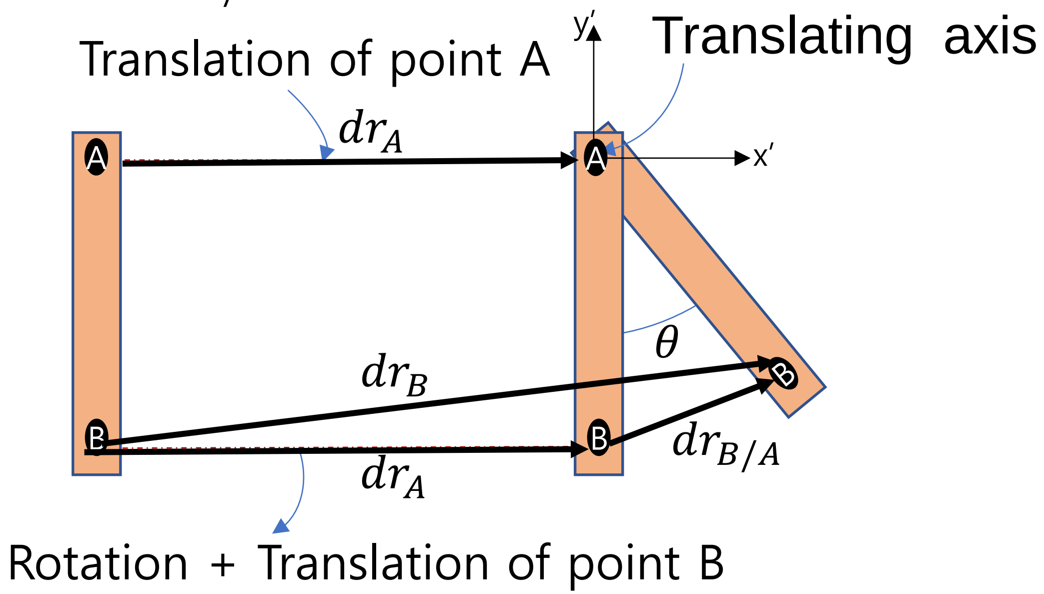


Figure 4. The position vector

- The origin of a second frame of reference x', y', z ' is attached to and moves with particle and only permitted to translate relative to the fixed frame

Displacement

- During a small time interval dt, points A and B move by displacements $\boldsymbol{dr_A}$ and $\boldsymbol{dr_B}$ .The general plane motion can be separated into two parts:

- **Translation of the base point A**

✓The entire body first translates by $\boldsymbol{dr_A}$.

✓Point A moves to its new position, and B moves to an intermediate position B'.

- **Rotation about point A**

✓The body then rotates about A by a small angle $d\theta$.

✓Point B' undergoes a relative displacement drB/A, moving to its final position B.

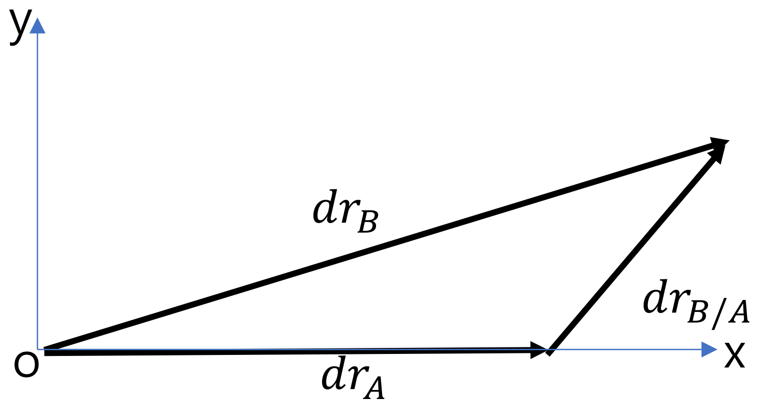


Figure 5. Displacement

- **Resultant Displacement Relation**

✓The total displacement of point B is the vector sum of the two components:

✓Point B' undergoes a relative displacement drB/A, moving to its final position B.

$$\boldsymbol{dr_B = dr_A + dr_{B/A}}$$

Where:

- $\boldsymbol{dr_A}$: due to translation of A
- $\boldsymbol{dr_{B/A}}$:due to rotation about A

Velocity

- To relate the velocities of points A and B, take the time derivative of the position equation:

$$V = \frac{dr}{dt} \qquad \frac{dr_B}{dt} = \frac{dr_A}{dt} + \frac{dr_{B/A}}{dt}$$

$$V_B = V_A + V_{B/A} \quad \text{(m/s)}$$

- Here V_B and V_A refer to absolute velocities, since they are observed from the fixed frame; whereas the relative velocity $V_{B/A}$ is observed from the translating frame [2].
- Since the relative motion is due to rotation of the body about A :

$$V_{B/A} = \boldsymbol{\omega} \times r_{B/A}$$

✓ Its Direction: Perpendicular to $r_{B/A}$

Final Velocity Equation

$$V_B = V_A + \boldsymbol{\omega} \times r_{B/A}$$

The directions of V_A and V_B are always tangent to their paths of motion.

Procedure: For Solving using Relative Motion

- The relative velocity equation can be applied either by using Cartesian vector analysis, or by writing the x and y scalar component equations directly. For application, it is suggested that the following procedure be used.

Step 1. Identify the Type of Motion

- Recognize that the body undergoes general plane motion — a combination of translation and rotation.
- Choose a base point (A) whose motion is known or easily determined.

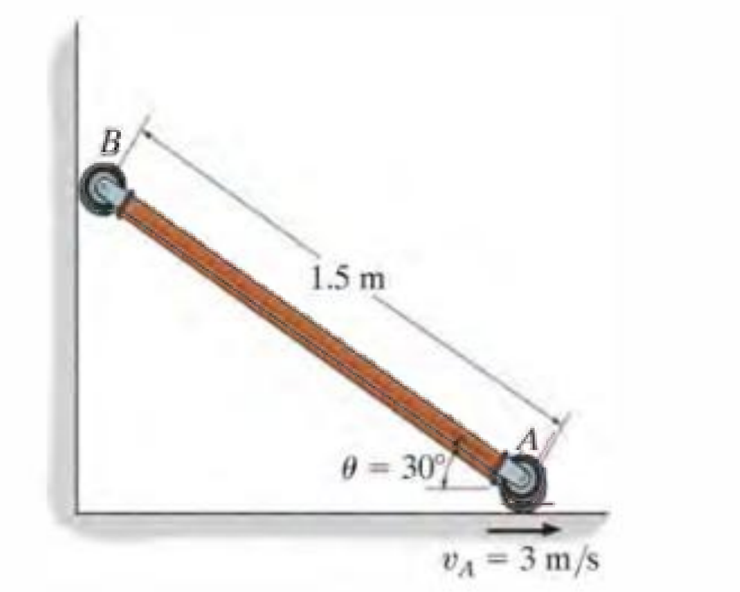


Figure 6. step 1

Example

- Point A moves horizontally, and point B moves vertically relative to the fixed wall.
- Points A and B also rotate relative to each other.
- For this case, point A is taken as the reference point since its velocity is known.

Step 2. Establish Reference Axes

Use two coordinate systems:

- A fixed (x, y) axis : to describe absolute motion.
- A moving or translating (x′, y′) axis :to express the relative motion.

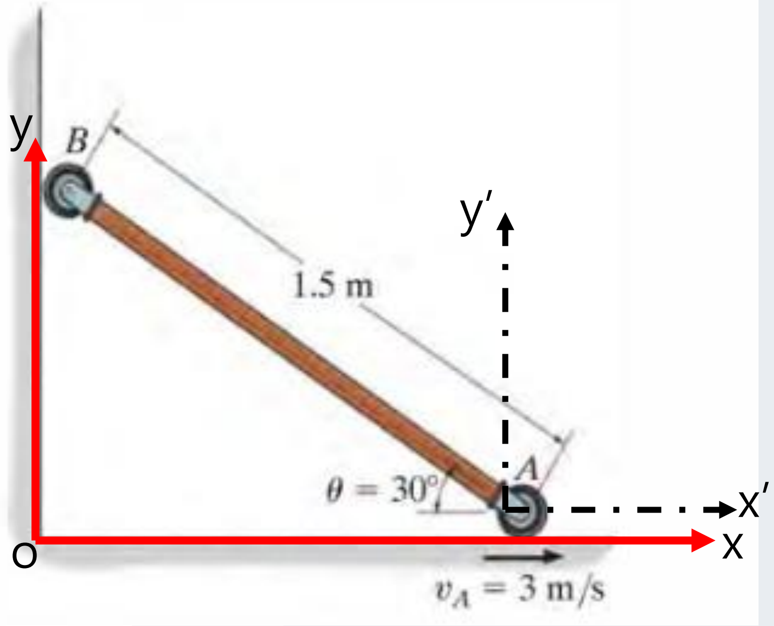


Figure 7. step 2

Example

- The wall is taken as the fixed axis reference point.
- Point A, with a known velocity, is used as the base point for the translating axis.

Step 3. Determine the Direction of Each Velocity and write the velocity equation

- Indicate on it the velocities $\boldsymbol{V_A}$, $\boldsymbol{V_B}$ of points A and B, the angular velocity w, and the relative $\boldsymbol{rB/A}$ position vector
- Velocity is always tangent to the path of motion.
- If the magnitudes of $\boldsymbol{V_A}$ and $\boldsymbol{V_B}$ or w are unknown, the sense of direction of these vectors can be assumed .
- To apply , $\boldsymbol{V_{B/A}} = \boldsymbol{\omega} \times \boldsymbol{rB/A}$, express the vectors in Cartesian vector form and substitute them into the equation.

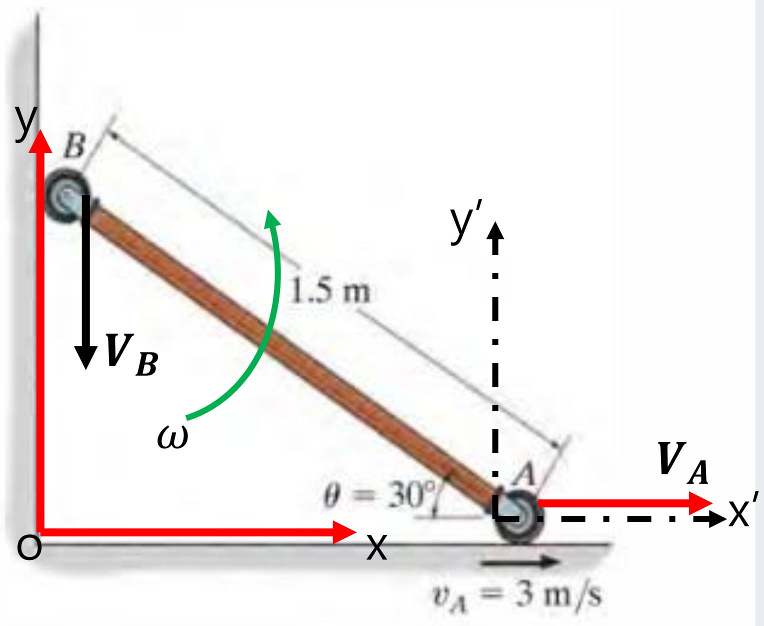


Figure 8. step 3

In the example shown:

- The velocity of point A is horizontal.
- The velocity of point B is vertically downward, following the motion of point A.
- The angular velocity (ω) is assumed to be counterclockwise.
- The position vector from A to B is expressed as:

$$\boldsymbol{rB/A} = -1.5 \cos(30°)\ \mathrm{I} + 1.5\sin(30°)\ \mathrm{j}$$

Step 4. Solve for Unknowns

- Use vector components to find the unknown expressing it with cartesian vector form
- Evaluate the cross product and then equate the respective i and j components to obtain two scalar equations.
- If the solution yields a negative answer for an unknown magnitude, it indicates the sense of direction of the vector is opposite to that shown on the kinematic diagram

In the example shown:

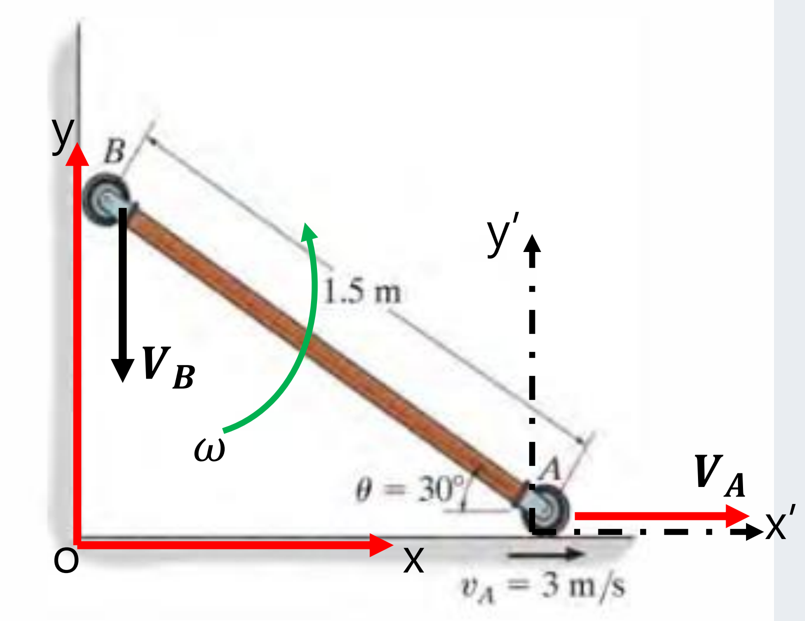


Figure 9. step 4

$$\boldsymbol{V_B} = \boldsymbol{V_A} + \omega_{ab}\boldsymbol{k} \times \boldsymbol{r_{B/A}}$$

$$-V_B\boldsymbol{j} = 3\ \boldsymbol{i} + \omega_{ab}\boldsymbol{k} \times (-1.5\cos 30\ \boldsymbol{i} + 1.5\sin 30\boldsymbol{j})$$

$$\boldsymbol{i}\ term:\ \ 0 = 3 - \omega_{ab}(1.5\sin 30)$$

$$\text{j}\ term: -V_B = 3 - \omega_{ab}(1.5\cos 30)$$

Velocity using Graphical approach

Optionally, the velocity of any point on a rigid body undergoing general plane motion can be found using:

- The instantaneous center of velocity method
- Geometric velocity polygon and trigonometric relationships,

The instantaneous center of velocity method (ICZV)

- The velocity of any point B on a rigid body can be found by selecting a base point A that has zero velocity at the instant considered.

$$V_B = V_A + \omega_{ab} k \times r_{B/A} \qquad if \;\; V_A = 0 \qquad V_B = \omega \times r_{B/A}$$

- This special point A (or IC) is called the Instantaneous Center of Zero Velocity (ICZV).

- The ICZV is perpendicular to the plane of motion, and its intersection with the plane defines the location of the IC.
- At any instant, the rigid body appears to rotate about this instantaneous center.
- The magnitude and direction of velocity at any point B are:

$$\boldsymbol{V_B} = \boldsymbol{\omega r_{B/IC}}$$

$\boldsymbol{V_B}$ and $\boldsymbol{r_{B/IC}}$ are perpendicular to each other

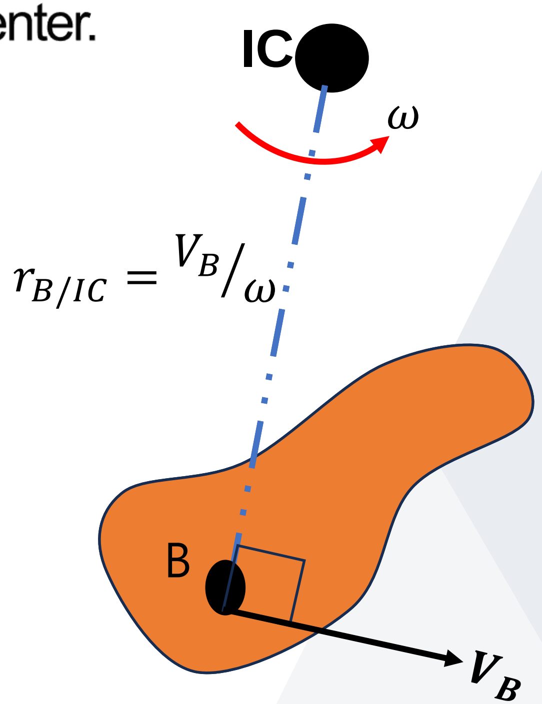


Figure 10 . IC of bodies

- This method provides an intuitive and often simpler way to analyze velocities in general plane motion compared to using full vector equations.

Locating the Instantaneous Center

- To locate the IC we can use the fact that the velocity of a point on the body is always perpendicular to the relative position vector directed from the Ic to the point. Several possibilities exist:

Case 1: Absolute velocities of any two points A and B on the body are known and **are not parallel.**

- Construct at points A and B line segments that are perpendicular to V_A and V_B Extending these perpendiculars to their point of intersection as shown locates the IC at the instant considered.

- If we also know the magnitude of the velocity of one of the points, say, V_A , we may easily obtain the angular velocity the body and the linear velocity of every point in the body.
- Thus, the angular velocity of the body, is:

$$\omega = \frac{V_A}{rA/Ic}$$

which, of course, is also the angular velocity of every line in the body. Therefore, the velocity of B is $V_B = \omega\, rB/Ic$.

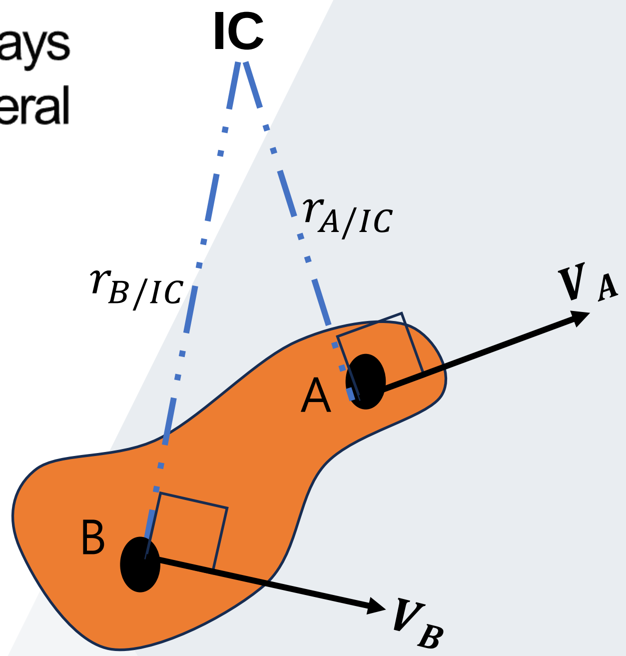


Figure 11 . Non parallel velocity IC

Locating the Instantaneous Center

Case 2: The velocities of two points in a body having plane motion are parallel.

- Here the location of the IC is determined by proportional triangles.

Examples are shown in Fig. 12 and 13. In both cases $r_{A/IC} = \frac{V_A}{\omega}$ and $r_{B/IC} = \frac{V_B}{\omega}$, If d is a known distance between points A and B, then in

Fig.12, $r_{A/IC} + r_{B/IC}$ = d

and in

Fig.13 , $r_{B/IC}$ - $r_{A/IC}$ = d

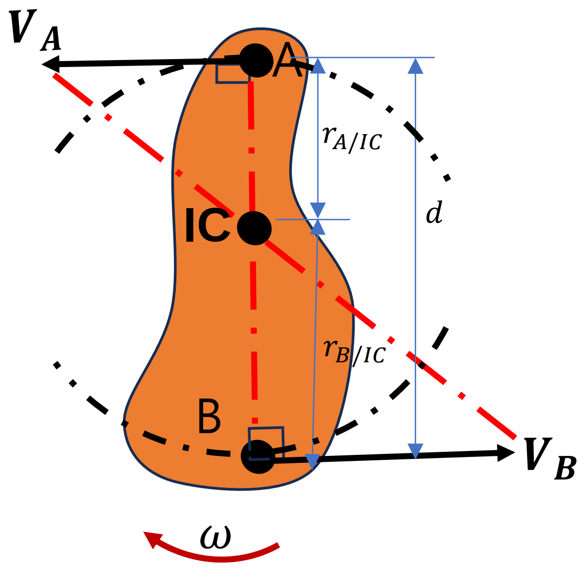


Figure 12 . Parallel opposite velocities IC

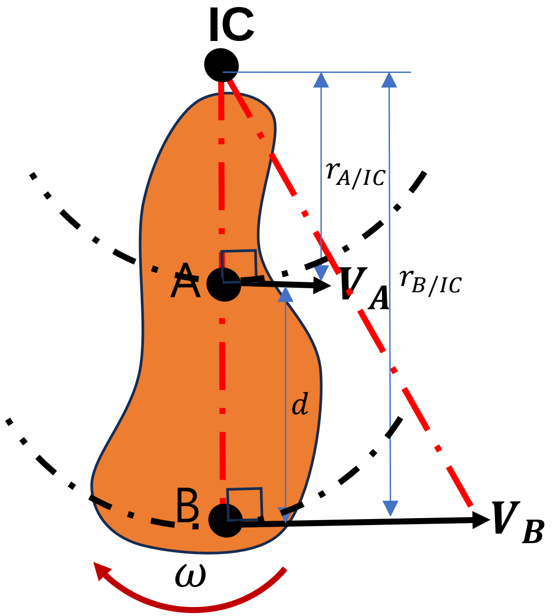


Figure 13. Parallel same velocities direction IC

Procedure

Step 1: Identify the Rigid Body

- From the given mechanism or system, select the link (rigid body) that exhibits general plane motion (i.e., both translation and rotation).

Step 2: Identify the Direction of Motion of Velocities

- Determine the direction of velocity for each given point on the rigid body. The velocity of any point is always:

✓Tangent to the path of motion, and Perpendicular to the line (radius) from the center of rotation or instantaneous center to that point.

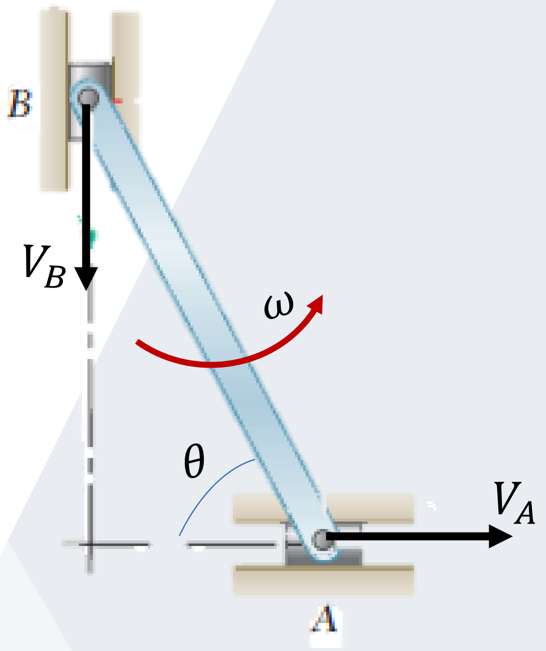


Figure 14: Rod

Example : For the configuration shown in Figure 14, link **AB** is considered as the body undergoing **general plane motion**, where **point A** moves **horizontally**, and **point B** moves **vertically downward**.

Procedure

Step 3: Step 3: Locate the Instantaneous Center (IC)

- The IC is the point on the rigid body that is momentarily at rest.
- For any two points A and B, draw lines perpendicular to their velocity directions.
- The intersection of these perpendiculars gives the location of the IC.
- Once the IC is known, the body can be considered to rotate about this point at that instant.

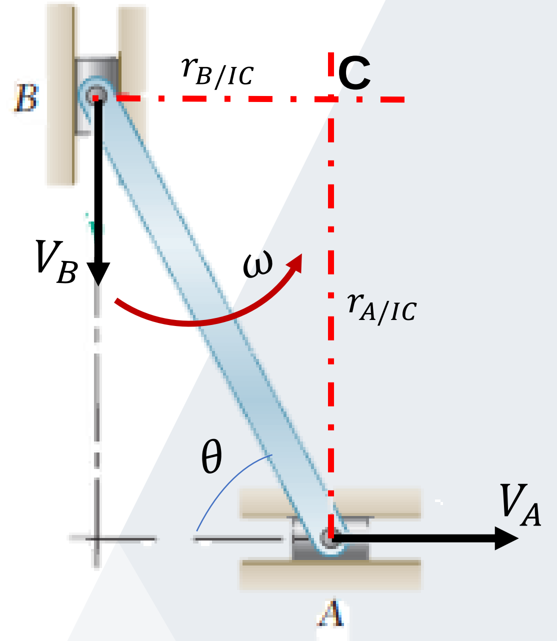


Figure 15. locating IC

Example : Drawing the perpendicular to V_A through *A* and the perpendicular to V_A through *B* (Fig. 15), we obtain the instantaneous center *C*.

At the instant considered, the velocities of all the particles of the rod are thus the same as if the rod rotated about *C*.

Procedure

Step 4: Find the unknown

- Once the (IC) is identified for the given instant, triangle formed. Using trigonometric relationships, the unknowns can be determined, including the distances of the points from the IC..

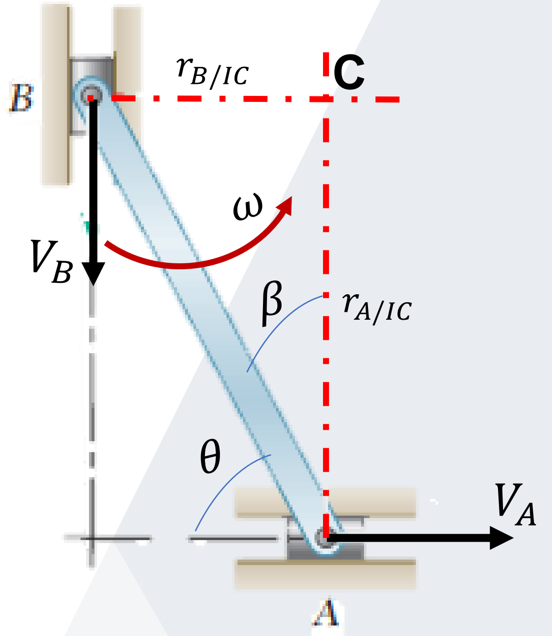


Figure 16. locating IC

Example : Now, if we know the magnitude V_A of the velocity of *A*, we can obtain the magnitude v of the angular velocity of the rod from:

$$\omega = \frac{V_A}{r_{A/IC}} = \frac{V_A}{AB\cos\beta}$$

Then we obtain the magnitude of the velocity of *B* as:

$$V_B = r_{B/IC}\,\omega$$

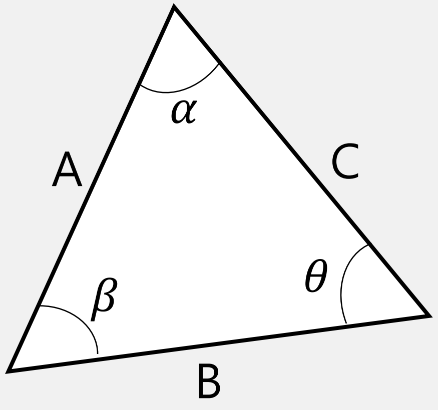
Note that we used only *absolute* velocities in the computation.

Using velocity polygon Method

- If the directions of velocities of two points, A and B, are known, and the magnitude of one velocity is given, then the directions of the velocity vectors, V_A, V_B and $V_{B/A}$,the forms a triangle [1].

- Using the sine and cosine laws, the unknown velocities can be determined. .

- As review, their formulations are provided below.



Law of Sines:

$$\frac{A}{Sin\theta} = \frac{B}{sin\alpha} = \frac{C}{sin\beta}$$

Law of Cosines:

C= $\sqrt{A^2 + B^2 - 2ABcos\beta}$

B= $\sqrt{A^2 + C^2 - 2ACcos\alpha}$

A= $\sqrt{C^2 + B^2 - 2BCcos\theta}$

Figure 17. Triangle

- Draw vectors V_B and V_A from a common point. Apply the laws of sines and cosines to determine $V_{B/A}$.

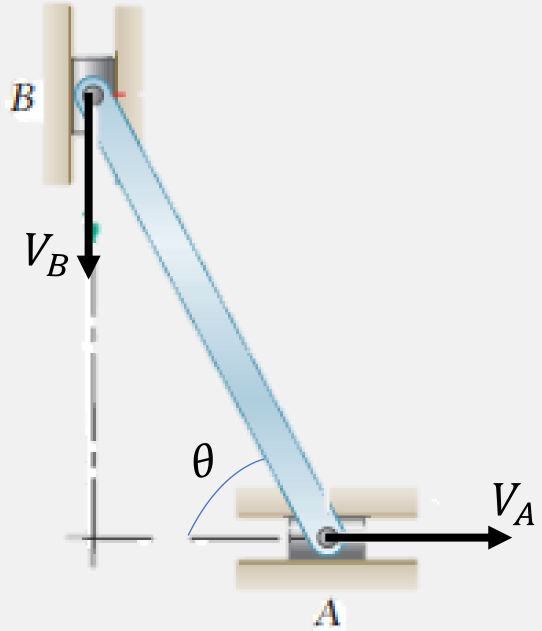


Figure 18. Rod

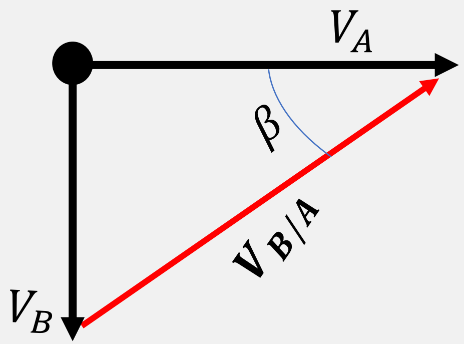


Figure 19. Velocity polygon

- Alternatively, if V_B and V_A are drawn to the correct scale, the relative velocity $V_{B/A}$ can be obtained directly from the velocity polygon, without the need to apply the sine or cosine laws.

Relative Acceleration

- Consider the equation $V_B = V_A + V_{B/A}$,which describes the relative velocities of two points A and B in plane motion in terms of nonrotating reference axes.
- By differentiating the equation with respect to time, we may obtain the relative-acceleration equation [2].

$$a_B = a_A + a_{B/A} \quad (m/s^2)$$

- The terms a_B and a_A are measured with respect to a set of fixed x, y axes and represent the absolute accelerations of points B and A.
- Here $a_{B/A}$ is the acceleration of B as seen by the observer located at A and translating with the x ', y', Z' reference frame.
- It was shown that to this observer point B appears to move along a circular arc that has a radius of curvature $r_{B/A}$.
- Consequently, $a_{B/A}$ can be expressed in terms of its tangential and normal components;

$$a_{B/A} = a_{(B/A)t} + a_{(B/A)n} \ (m/s^2)$$

$$a_{(B/A)t} = \alpha \times r_{B/A}$$

$$a_{(B/A)t} = -\omega^2 \ r_{B/A}$$

$$a_B = a_A + a_{(B/A)t} + a_{(B/A)n} \ (m/s^2)$$

Relative Acceleration

$$\boldsymbol{a}_B = \boldsymbol{a}_A + \boldsymbol{a}_{(B/A)t} + \boldsymbol{a}_{(B/A)n} \quad \left({}^{m}/_{s^2}\right)$$

- N.B: The magnitude of $\boldsymbol{a}_{(B/A)t} = \boldsymbol{\alpha} \times \boldsymbol{r}_{B/A}$ and the direction is perpendicular to $\boldsymbol{r}_{B/A}$

  The magnitude of $\boldsymbol{a}_{(B/A)n} = \boldsymbol{\omega}^2\, \boldsymbol{r}_{B/A}$ and the direction is always from B towards A
- The terms in above equations are represented graphically in Fig. 20.
- At a given instant, the acceleration of point B is obtained by combining the bar's translational acceleration at A with its rotational motion about A, resulting in both tangential and normal components as shown.

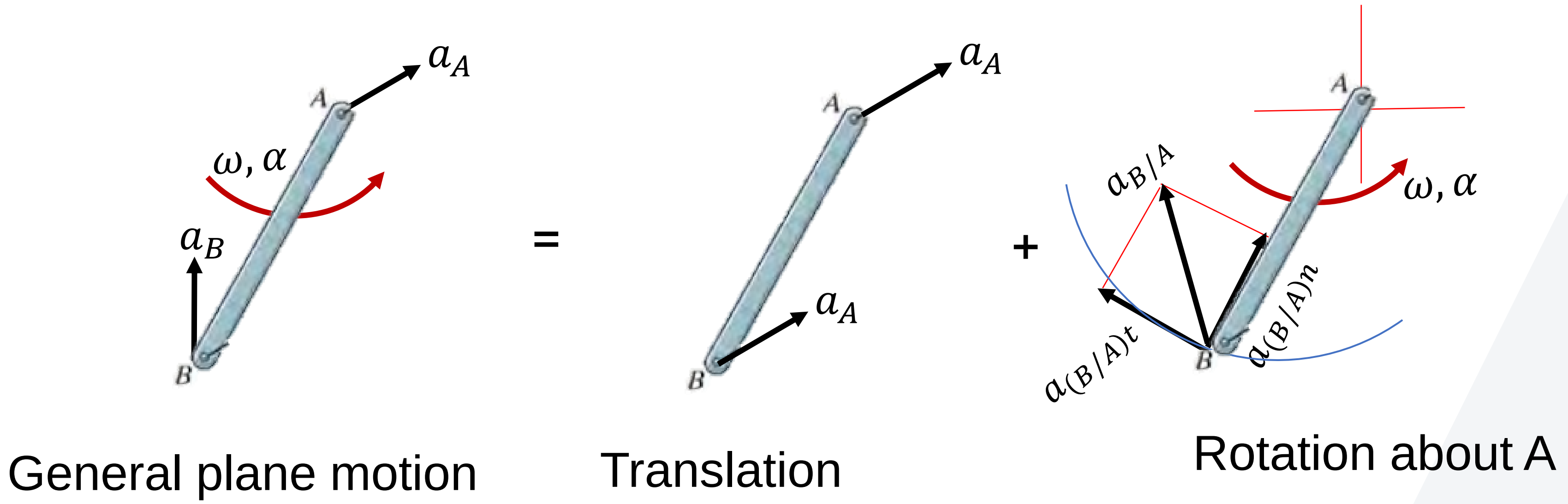


Figure 20. Acceleration of B

Procedure: For Solving using Relative Motion

- The relative Acceleration equation can be applied either by using Cartesian vector analysis. For application, it is suggested that the following procedure be used.

Step 1. Identify the Type of Motion

- Recognize that the body undergoes general plane motion — a combination of translation and rotation.
- Choose a base point (A) whose motion is known or easily determined.

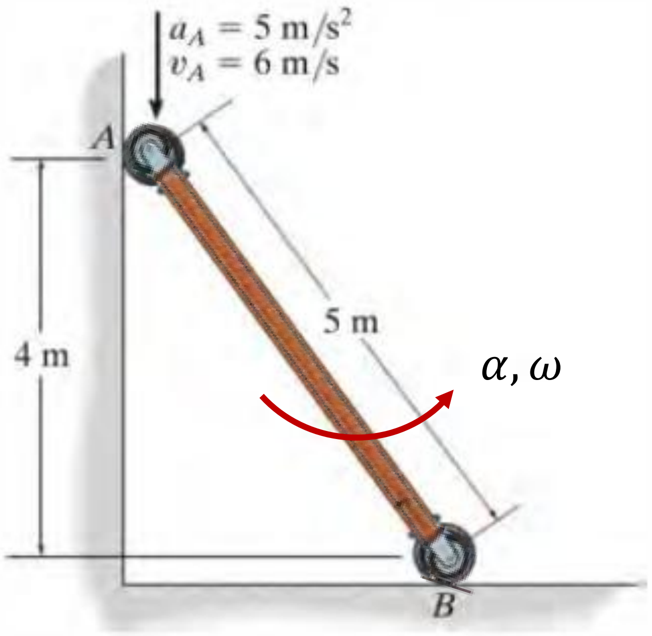


Figure 21. step 1

Example

- Point A moves horizontally, and point B moves vertically relative to the fixed wall.
- Points A and B also rotate relative to each other.
- For this case, point A is taken as the reference point since its velocity and acceleration is known.

Step 2. Establish Reference Axes

Use two coordinate systems:

- A fixed (x, y) axis : to describe absolute motion.
- A moving or translating (x′, y′) axis :to express the relative motion.

Example

- The wall is taken as the fixed axis reference point.
- Point A, with a known velocity, is used as the base point for the translating axis.

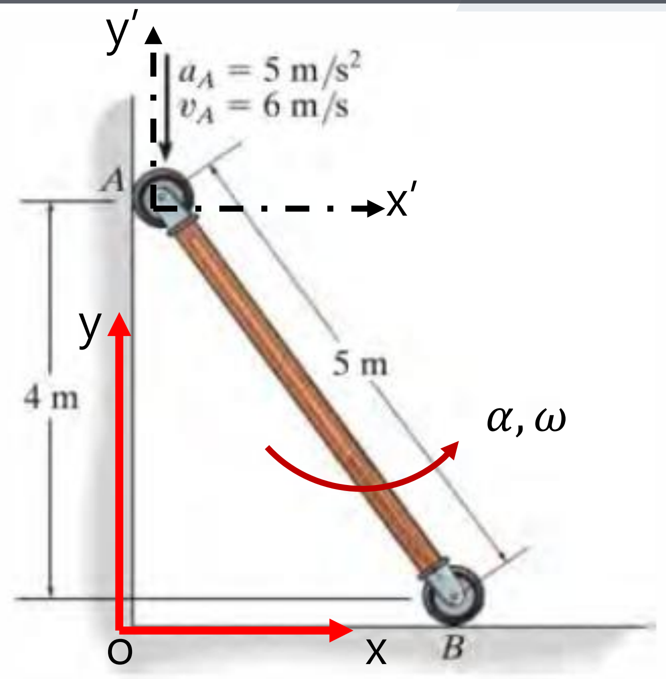


Figure 22. step 2

Step 3. Determine the linear Velocity of the points and angular velocity of the body using the procedures and any method discussed for velocity.

Step 4: Write the relative-acceleration formula relating two points of interest on the body being analyzed.

- To apply $\boldsymbol{a_B} = \boldsymbol{a_A} + \boldsymbol{a_{(B/A)t}} + \boldsymbol{a_{(B/A)n}}$, express the vectors in Cartesian vector form and substitute them into the equation.
- Evaluate the cross product and then equate the respective i and j components to obtain two scalar equations.

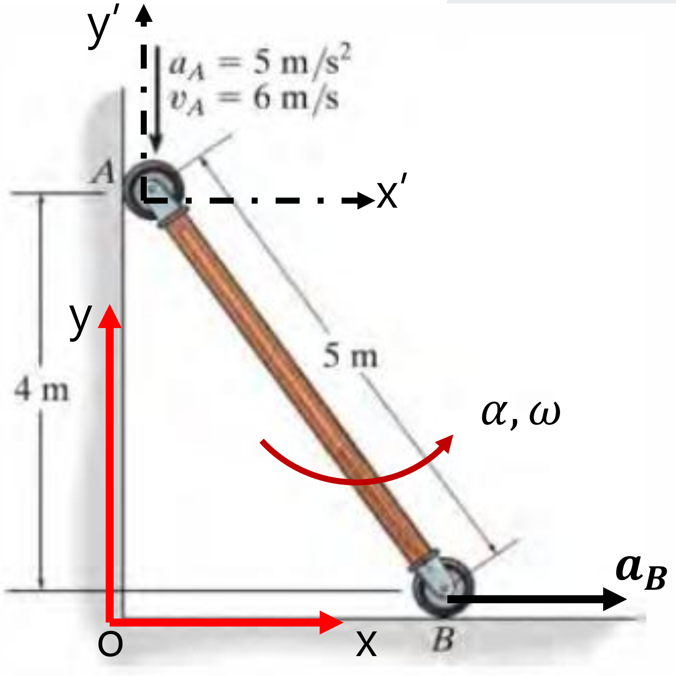


Figure 23. step 4

Example, for the case shown on figure 23 , the unknow point be acceleration can be expressed as:

$$\boldsymbol{a_B} = \boldsymbol{a_A} + \boldsymbol{a_{(B/A)t}} + \boldsymbol{a_{(B/A)n}}$$

In Cartesian vector form:

$$\boldsymbol{a_B i} = -\boldsymbol{a_A j} + (\boldsymbol{\alpha k} \times \boldsymbol{r_{B/A}}) + (-\ \boldsymbol{\omega^2}\ \boldsymbol{r_{B/A}})$$

Relative Motion Using Rotating Axes

- helps determine velocities and accelerations of points on the same or connected rigid bodies. For mechanisms with sliding at connections, kinematic analysis is best done using a coordinate system that both translates and rotates.

- Use of rotating reference axes greatly facilitates the solution of many problems in kinematics where motion is generated within a system or observed from a system which itself is rotating.

- Furthermore, this frame of reference is useful for analyzing the motions of two points on a mechanism which are not located in the same body and for specifying the kinematics of particle motion when the particle moves along a rotating path [1].

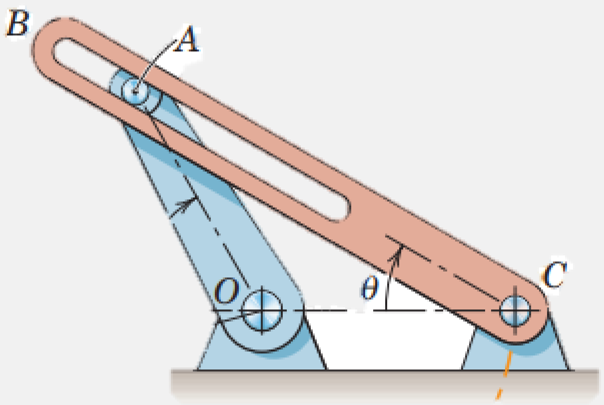


Figure 24. sliding and rotating link

- Here point A is translating and rotating relative to c, and it is called rotating axis

Position

- Consider the two points A and B shown in Fig. 25. Their location is specified by the position vectors r_A and r_B which are measured with respect to the fixed X, Y, Z coordinate system.
- As shown in the figure, the "base point" A represents the origin of the x, y, z coordinate system, which is assumed to be both translating and rotating with respect to the X, Y, Z system.
- At the instant considered, point A has a velocity V_A and an acceleration a_A , while the angular velocity and angular acceleration of the x, y axes are Ω (omega) and $\dot{\Omega}$, respectively.
- Using vector addition, the three position vectors in 22 are related by the equation

$$r_B = r_A + r_{B/A}$$

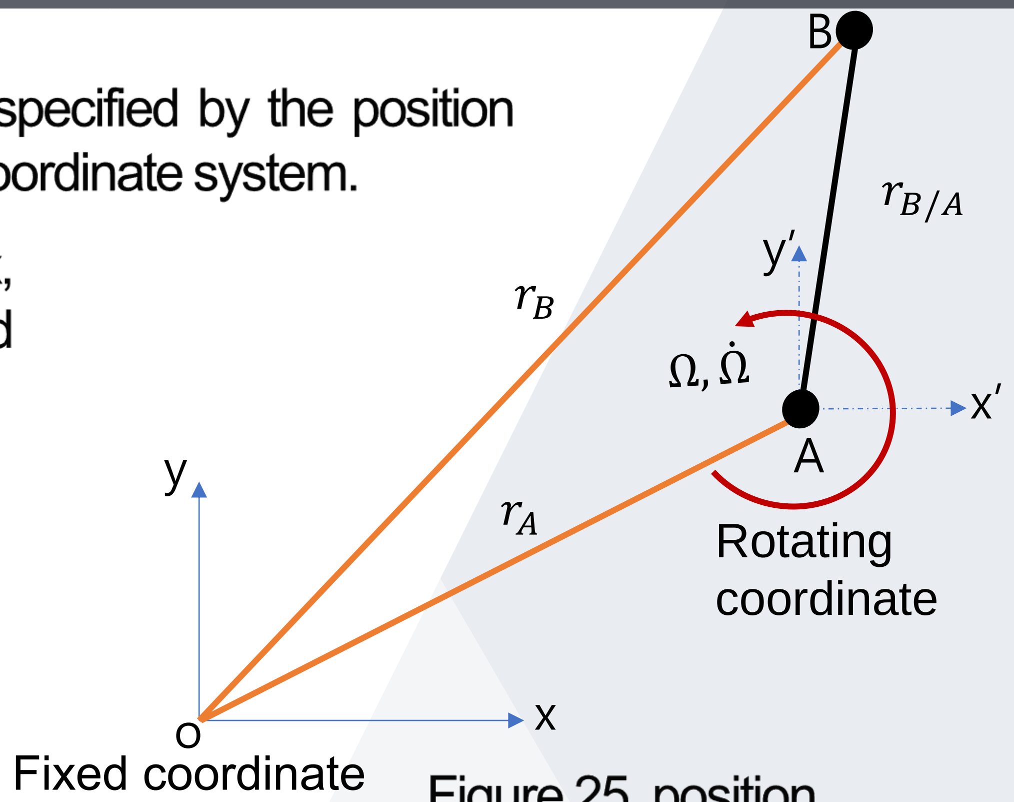


Figure 25. position

$r_{B/A}$ will be measured with respect to the rotating x, y frame of referen ce and expressed as:

$$r_{B/A} = x_{B/A} i + y_{B/A} j$$

Velocity

- The velocity of point B is determined by taking the time derivative of position , which yields [3].

$$V_B = V_A + \frac{dr_{B/A}}{dt}$$

- The last term in this equation is evaluated as follows:

$$\frac{dr_{B/A}}{dt} = \frac{d}{dt}(x_{B/A}i + y_{B/A}j)$$

$$\frac{dr_{B/A}}{dt} = \underbrace{\left(\frac{d}{dt}(x_{B/A})i + \frac{d}{dt}(y_{B/A})j\right)}_{V_{(B/A)_{x,y,z}}} + \underbrace{\left(\frac{di}{dt}(x_{B/A}) + \frac{dj}{dt}(y_{B/A})\right)}_{\Omega \times r_{B/A}}$$

The two terms in the first set of parentheses represent the components of velocity of point B as measured by an observer attached to the moving x, y, z coordinate system and will be denoted by vector

In the second set of parentheses the instantaneous time rate of change of the unit vectors i and j is measured by an observer located in the fixed X, Y, Z coordinate system.

Velocity

- The velocity substituting the values , yields[3].

$$V_B = V_A + \Omega \times r_{B/A} + V_{(B/A)_{x,y,z}}$$

- where:

V_B= velocity of B, measured from the X, Y, Z reference:

V_A = velocity of the origin A of the x, y, z reference, measured from the X, Y, Z reference

$V_{(B/A)_{x,y,z}}$ = velocity of "B with respect to A," as measured by an observer attached to the rotating x, y, z reference

Ω = angular velocity of the x, y, z reference, measured from the X, Y, Z reference

$r_{B/A}$ = position of B with respect to A

Comparing equation for translating axis , it can be seen that the only difference between these two equation is represented by the term, $V_{(B/A)_{x,y,z}}$

Acceleration

- The acceleration of B, observed from the X, Y, Z coordinate system, may be expressed in terms of its motion measured with respect to the rotating system of coordinates by taking the time derivative of velocity equation [3].

$$\underbrace{\frac{dV_B}{dt}}_{a_B} = \underbrace{\frac{dV_A}{dt}}_{a_A} + \underbrace{\frac{d}{dt}(\Omega \times r_{B/A})}_{(\dot{\Omega} \times r_{B/A})(\Omega \times r_{B/A} + V_{(B/A)_{x,y,z}})} + \underbrace{\frac{d}{dt}(V_{(B/A)_{x,y,z}})}_{a_{(B/A)_{x,y,z}} + \Omega \times V_{(B/A)_{x,y,z}}}$$

Substituting this and rearranging terms, we get:

$$a_B = a_A + (\dot{\Omega} \times r_{B/A}) + (\Omega \times r_{B/A}) + 2\Omega \times V_{(B/A)_{x,y,z}} + a_{(B/A)_{x,y,z}}$$

Where:

a_B = acceleration of B, measured from the X, Y, Z reference

a_A = acceleration of A, measured from the X, Y, Z reference

$(a_{(B/A)_{x,y,z}}), (V_{(B/A)_{x,y,z}})$ =acceleration and velocity of B with respect to A, as measured by an observer attached to the rotating x,y, z reference

$\Omega, \dot{\Omega}$ = angular acceleration and angular velocity of the x, y, z reference, measured from the X, Y, Z reference

Acceleration

$$a_B = a_A + (\dot{\Omega} \times r_{B/A}) + (\Omega \times r_{B/A}) + 2\Omega \times V_{(B/A)_{x,y,z}} + a_{(B/A)_{x,y,z}}$$

Where:

a_B = acceleration of B, measured from the X, Y, Z reference

a_A = acceleration of A, measured from the X, Y, Z reference

$(a_{(B/A)_{x,y,z}})$, $(V_{(B/A)_{x,y,z}})$=acceleration and velocity of B with respect to A, as measured by an observer attached to the rotating x,y, z reference

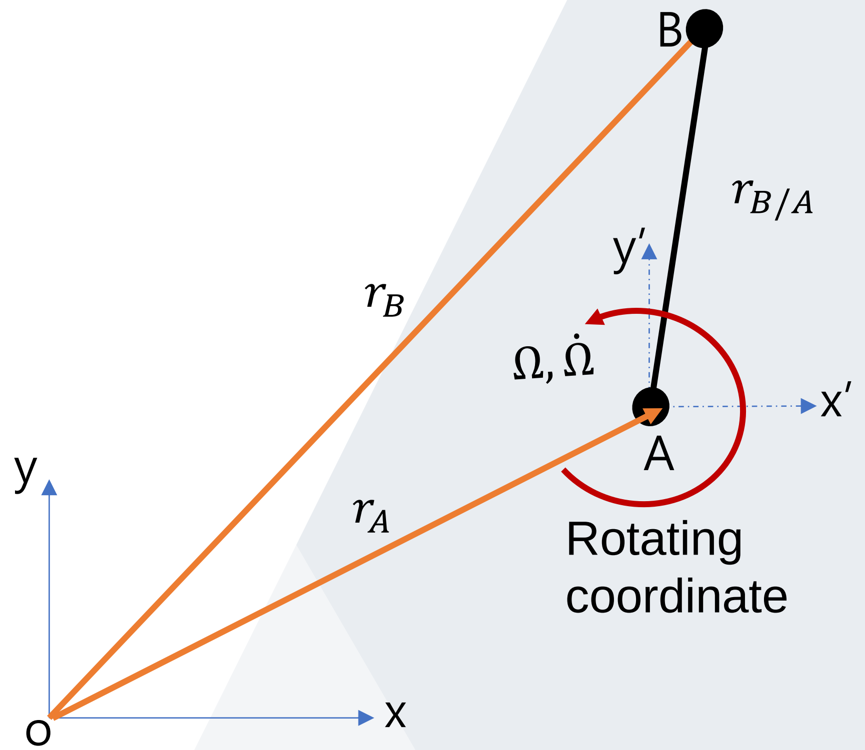


Figure 26. rotating axis

$\dot{\Omega}$, $\Omega \doteq$ angular acceleration and angular velocity of the x, y, z reference, measured from the X, Y, Z reference

$2\Omega \times V_{(B/A)_{x,y,z}}$:is known as the **Coriolis acceleration**.

Summary on Relative motion

Translating axis

- when translating axes are placed at the "base point" A, the relative motion of point B with respect to A is simply circular motion of B about A
- The following equations apply to two points A and B located on the same rigid body

$$\boldsymbol{V_B} = \boldsymbol{V_A} + \boldsymbol{\omega_{ab} k} \times \boldsymbol{r_{B/A}}$$
$$\boldsymbol{a_B} = \boldsymbol{a_A} + \boldsymbol{a_{(B/A)t}} + \boldsymbol{a_{(B/A)n}} \quad \left({}^{m}/{}_{s^2}\right)$$

- The velocity can also be solved using instantaneous zero velocity method .
- The velocity and acceleration can be also solved using graphical method .

Rotating axis

- When the velocity and acceleration can also be obtained using rotating axis for sliding connections

$$V_B = V_A + \Omega \times r_{B/A} + V_{(B/A)_{x,y,z}}$$

$$a_B = a_A + (\dot{\Omega} \times r_{B/A}) + (\Omega \times r_{B/A}) + 2\Omega \times V_{(B/A)_{x,y,z}} + a_{(B/A)_{x,y,z}}$$

Problem 2

Rod AB has the angular motion shown. Determine the the velocity and acceleration of the collar C at this instant[1].

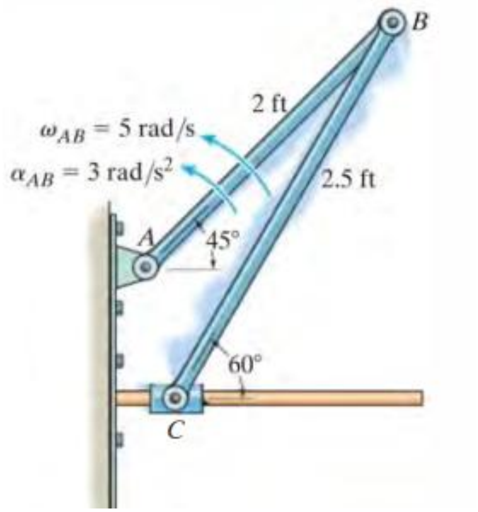


Figure 27.problem 1

Given

- $\omega_{AB} = 5\, {}^{rad}/_{s}$ (ccw)
- $\alpha_{AB} = 3\, {}^{rad}/_{s^2}$
- $\theta_{AB} = 45^0$
- $\theta_{Bc} = 60^0$

Required

V_C=?

a_C=?

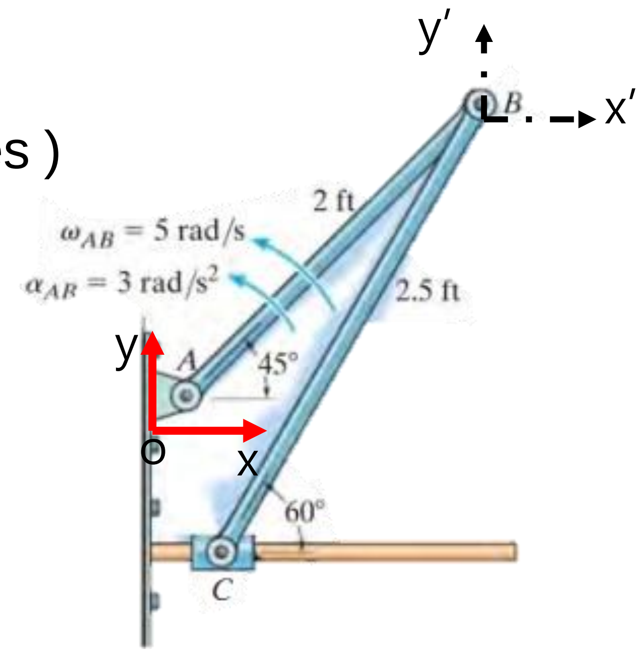
Solution **Velocity** **:** Using Relative motion equation

Step 1. Identify the Type of Motion

Link Ab- Pure rotation about A and link BC is exhibiting general plane motion (point c translate and point b rotates)

Step 2. Establish Reference Axes

Step 3. Determine the linear Velocity of the points and angular



Link AB

$V_B = \omega_{AB} K \times r_{AB} = 5K \times (2COS\,45\, i + 2\sin 45\, j)$

$V_B = -7.07\, i + 7.07\, j$ $[V_B]$ =10m/s

Link BC

$V_C = V_B + \omega_{bc} k \times r_{C/B}$

Expressing in Cartesian coordinate and substituting the values

$V_C i = (-7.07\, i + 7.07\, j) + \omega_{bc} k \times (-2.5\cos 60°\mathbf{i} - 2.5\sin 60°\mathbf{j})$

Equating the i and j components gives:

j $term$: ω_{bc} =5.6 rad/s

i $term$: V_c =5.18 m/s

Method 2

Solution

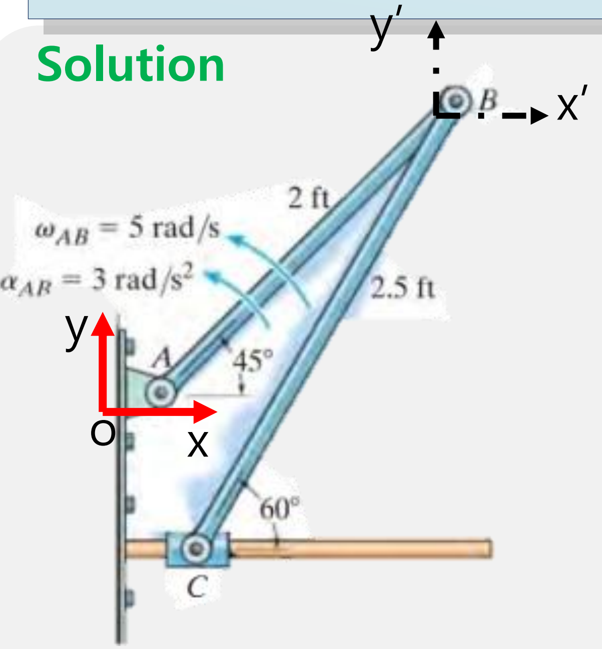


Figure 27.problem 1

Given

- $\omega_{AB} = 5\ ^{rad}/_{s}$ (ccw)
- $\alpha_{AB} = 3\ ^{rad}/_{s^2}$
- $\theta_{AB} = 45^0$
- $\theta_{Bc} = 60^0$

Required

V_C=?

a_C=?

Velocity : Using Instantaneous center

Link Ab- Pure rotation about A and link BC is exhibiting general plane motion (point c translate and point b rotates)

$V_B = \omega_{AB}\ \ r_{AB} = 10$ m/s

Radius to instantaneous center (IC)

$$\frac{r_{B/IC}}{\sin 30^0} = \frac{2.5}{\sin 135^0}$$

$$r_{B/IC} = 1.7678ft$$

$$\frac{r_{C/IC}}{\sin 15^0} = \frac{2.5}{\sin 135^0}$$

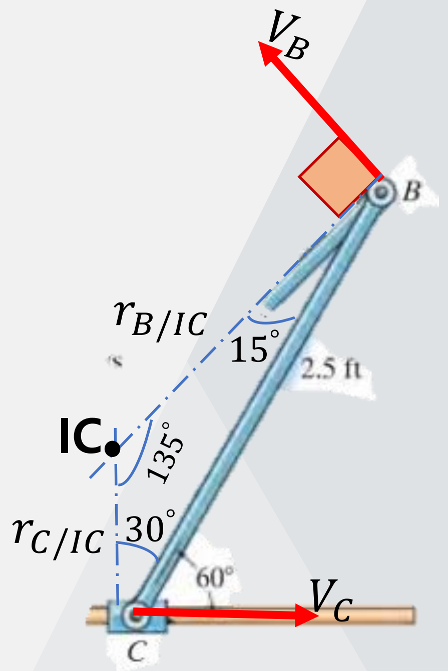
$$r_{B/IC} = 0.915ft$$

Angular velocity of BC and velocity of C

$$\omega = \frac{V_B}{r_{B/IC}} = 5.66rad$$

$$V_C = \omega\ r_{C/IC}$$

$$V_C = 5.18\ m/s$$



Solution

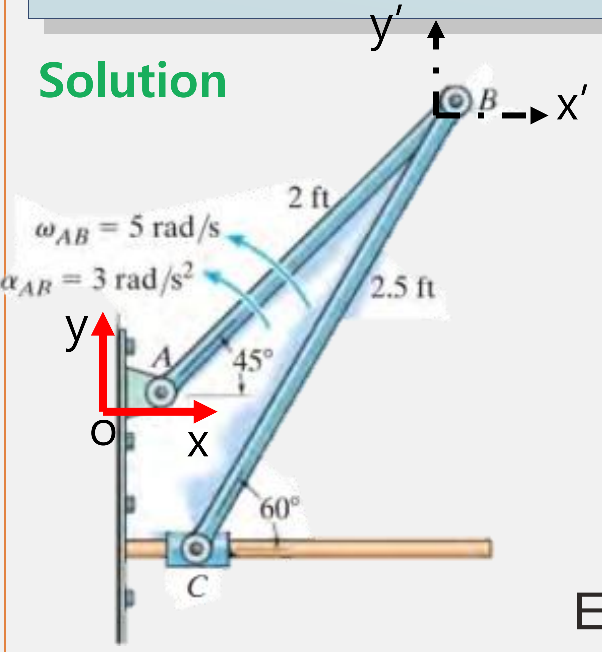


Figure 27.problem 1

Given

- $\omega_{AB} = 5\, {}^{rad}/_{s}$ (ccw)
- $\alpha_{AB} = 3\, {}^{rad}/_{s^2}$
- $\theta_{AB} = 45^0$
- $\theta_{Bc} = 60^0$

Required

V_C=?

a_C=?

Acceleration: using relative motion equation

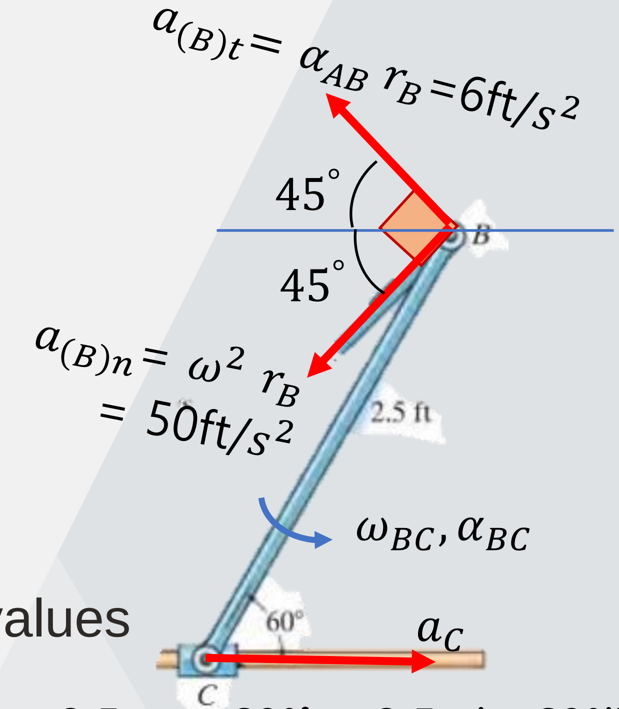
Link AB

$a_B = a_{(B)n} + a_{(B)t}$

$a_B = (-50\cos 45\, i - 50\sin 45\, j) - 6\cos 45\, i + 6\sin 45\, j)$

Link BC

$\boldsymbol{a_C} = \boldsymbol{a_B} + (\boldsymbol{\alpha k} \times \boldsymbol{r_{C/B}})$ **+ (-** $\boldsymbol{\omega^2}\ \boldsymbol{r_{C/B}}$)



Expressing in Cartesian coordinate and substituting the values

$a_C i = (-50\cos 45\, i - 50\sin 45\, j) - 6\cos 45\, i + 6\sin 45\, j) + (\alpha_{BC} k \times (-2.5\cos 60°\mathbf{i} - 2.5\sin 60°j)$
$+ (-5.66)^2 \times (-2.5\cos 60°i - 2.5\sin 60°j)$

Equating the i and j components gives:

$$\alpha_{BC} = 30.5\, rad/s^2$$

$$a_C = 66.5\, m/s^2$$

Summary

In This Lecture We Covered:

1. Introduction to relative motion for rigid bodies
2. Relative motion using translating axis
3. Relative motion using Rotating axis
4. Steps to Solve problems → description

References

[1] Dynamics, Hibbeler, Rusel M.,Prentice Hall,10th ed., 2003

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[3] Engineering Mechanics - Dynamics, Meriam J.L.,John Wiley & Sons, 9th ed., 2020.,

[4] Vector Mechanics for Engineers: Dynamics, Johnston, E. R., & Clausen, W. E., McGraw-Hill, 11th ed., 2015