Week 15

Review and Problem Solving

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Contents

In Today section you will Review :

1. Comprehensive revision of the chapters

Chapter one

Basic Concepts

- **Dynamics: Study of bodies in motion under the action of forces [1].**

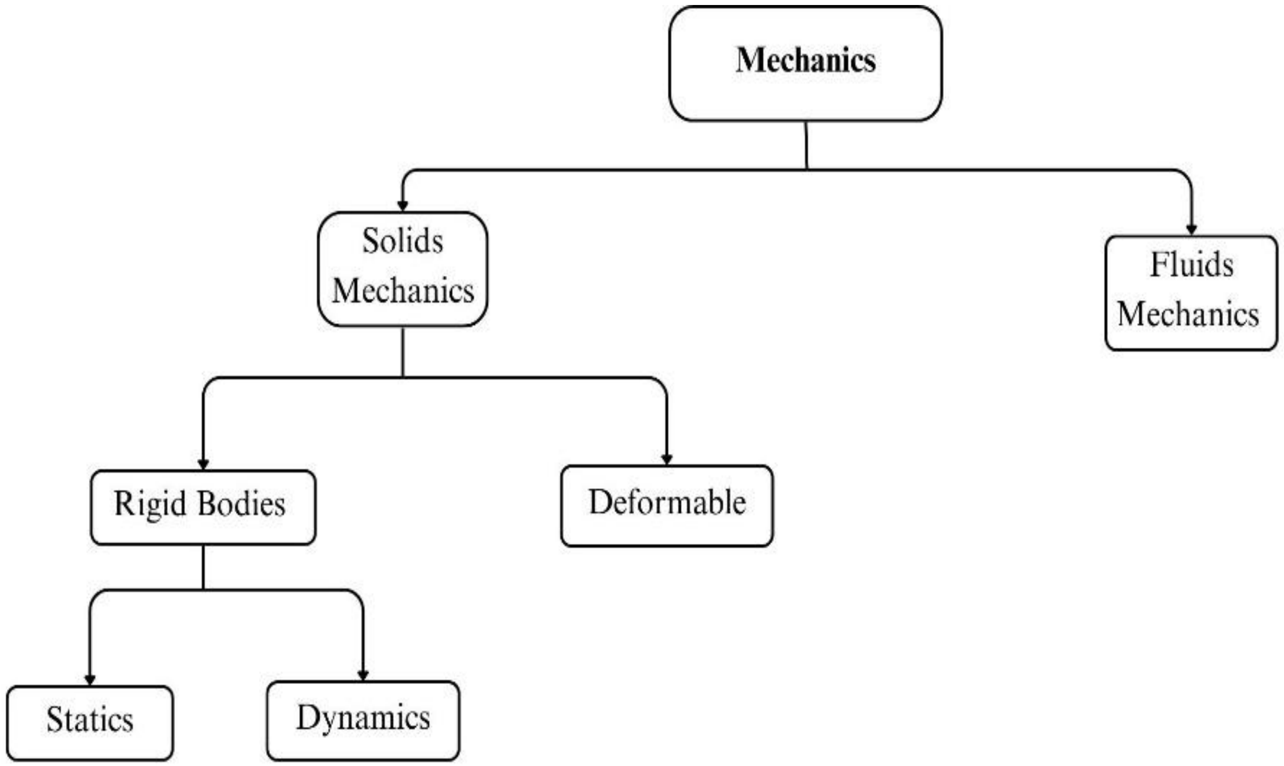


Figure 1. Classifications of engineering mechanics

Branches

•Kinematics: Geometry of motion (without considering forces).

•Kinetics: Relation between motion and forces causing it.

Particle vs. Rigid Body

•Particle: Mass concentrated at a point, no rotation.
•Rigid Body: Size and shape do not change during motion.

Reference Frames

Motion described w.r.t. fixed (inertial) or moving frames.

The course chapters

- **Based on the above concepts and definitions, This course is divided into five chapters (including this chapter or introduction).**

We begin chapter 2 by

- Treating rigid bodies as particles to study translational motion only (**kinematics of particles**).

Then chapter 3,

- Where we connect this translational motion to the forces that cause it (**kinetics of particles).**

After that chapter 4,

- we remove the particle assumption and study rigid bodies in rotational motion as well as combined translational and rotational motion (general plane motion), which is covered under **kinematics of rigid bodies.**

Finally chapter 5,

- we relate these rigid body motions to the forces that produce them (**kinetics of rigid bodies**).

Chapter 2. kinematics particles

- **Translational motion can be classified as rectilinear and curvilinear**

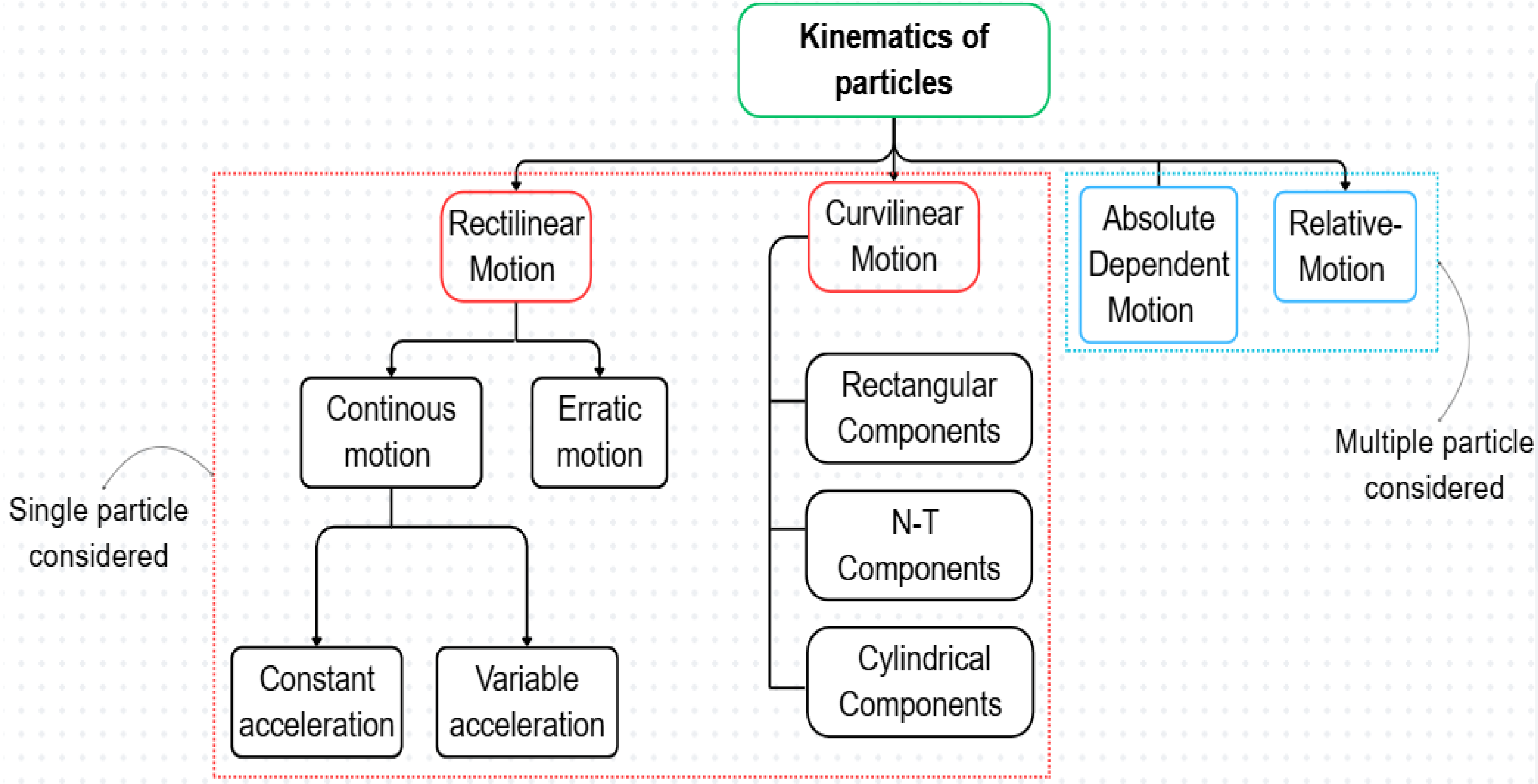


Figure 2. Chapter 2 structure

Summary of Important in rectilinear motion

Continuous

Variable acceleration

$\boldsymbol{v}= \frac{\boldsymbol{ds}}{\boldsymbol{dt}}$

$a = \frac{\boldsymbol{dv}}{\boldsymbol{dt}}$

$a = v\frac{dv}{ds}$ Or $a\ ds = v\ dv$

Constant acceleration

$V = V_O + \boldsymbol{a_c t}$

$S = s_O + v_O t + \frac{1}{2}(a_c t^2)$

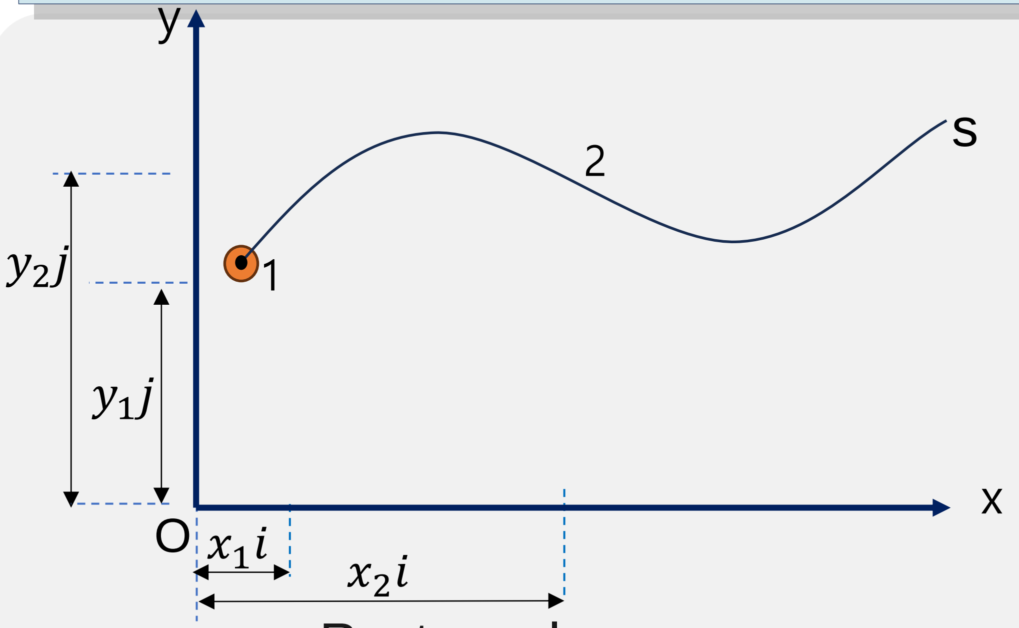
$V^2 = (V_O)^2 + 2\boldsymbol{a_c}(S - s_O)$

Erratic motion

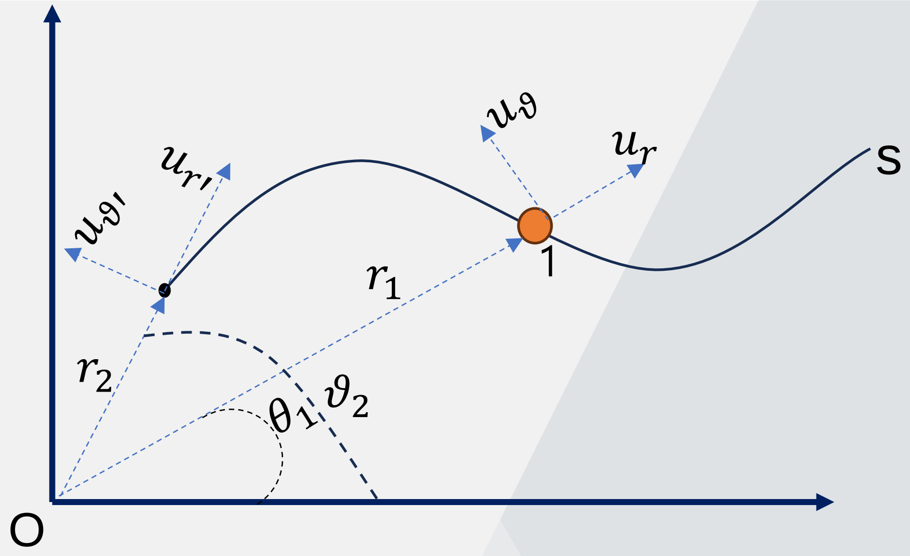
Given Graph	Required Graph
S-t	• V-t • a-t
V-t	• s-t • a-t
a-t	• s-t • v-t
a-s	• v-s
V-s	• a-s

Understand curvilinear motion

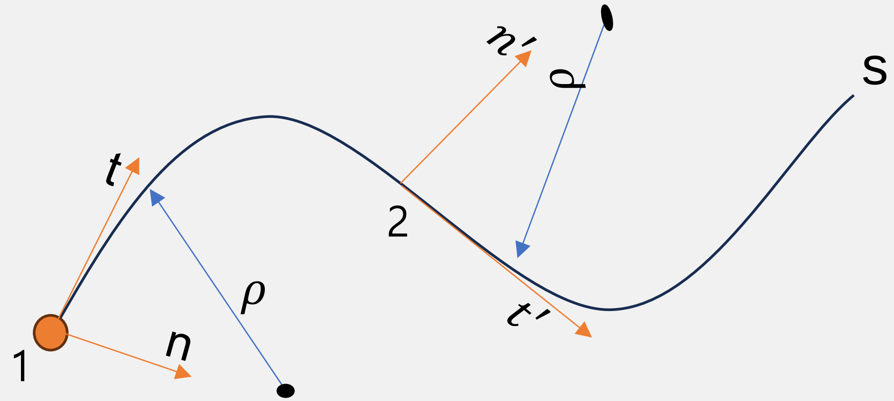
Coordinate systems



Rectangular



Polar Coordinates



Normal and Tangential Coordinates

Figure 3. Coordinate systems

Summary on the Important Equations

Rectangular

Position $\boldsymbol{r} = \boldsymbol{x i} + \boldsymbol{y j} + \boldsymbol{z k}$

Velocity $V = V_x \boldsymbol{i} + V_y j + V_z k$ $\quad v = \sqrt{(\boldsymbol{v_x})^2 + (\boldsymbol{v_y})^2 + (\boldsymbol{v_z})^2}$

Acceleration a= $a_x i + a_y j + a_z k$ $\quad \boldsymbol{a} = \sqrt{(\boldsymbol{a_x})^2 + (\boldsymbol{a_y})^2 + (\boldsymbol{a_z})^2}$

Motion of a Projectile

Horizontal Motion ($a_x = 0$)

$X = X_O + (v_o)_x t$

$V_x = constant$

Vertical Motion (a_y) $= -g$

$V_y = (v_o)_y - \boldsymbol{gt}$

$Y = y_O + (v_o)_y t - \frac{1}{2} g t^2)$

$V_y{}^2 = (V_O)^2 - 2\boldsymbol{g}\,(Y - y_O)$

Cylindrical (Polar-coordinate)

Velocity

V=$\dot{r}\, u_r + r\dot{\theta} u_\theta$ where $v_r = \dot{r}$

v= $\sqrt{(\boldsymbol{v_r})^2 + (\boldsymbol{v_\theta})^2}$ $\quad v_\theta = r\dot{\theta}$

Acceleration

$a = a_r u_r + a_\theta\, u_\theta$ where $a_\theta = 2\dot{r}\dot{\theta} - r\,\ddot{\theta}$

a= $\sqrt{(\boldsymbol{a_r})^2 + (\boldsymbol{a_\theta})^2}$ $\quad a_r = \ddot{r} - r\dot{\theta}^2$

r is radial coordinate $\quad$ θ is *transverse* coordinate

Normal-tangent

Velocity **V=** $\boldsymbol{v\, u_t}$

Acceleration $\boldsymbol{a} = \boldsymbol{a_t\, u_t} + \boldsymbol{a_n u_n}$

$\boldsymbol{a_n} = \frac{\boldsymbol{v^2}}{\boldsymbol{\rho}}$ $\quad \rho = \frac{[1 + (\frac{dy}{dx})^2]^{\frac{3}{2}}}{\frac{d^2y}{dx^2}}$

$\boldsymbol{a} = \sqrt{(\boldsymbol{a_n})^2 + (\boldsymbol{a_t})^2}$

If $\boldsymbol{a_t}$ is varies, $\quad \boldsymbol{a_t} = \frac{dv}{dt}$

$\boldsymbol{a_t}\, ds = v\, dv$

If $\boldsymbol{a_t}$ is constant, $\quad V = V_O + \boldsymbol{a_t t}$

$S = s_O + v_O t + \frac{1}{2}(a_t t^2)$

$V^2 = (V_O)^2 + 2\boldsymbol{a_t}\,(S - s_O)$

Understand Motion of multiple Particles

- As shown in Figure 10, the motion of the two planes is independent as they move freely. In contrast, in Figure 11, the movement of block A affects the motion of block B because they are connected by a rope [1].

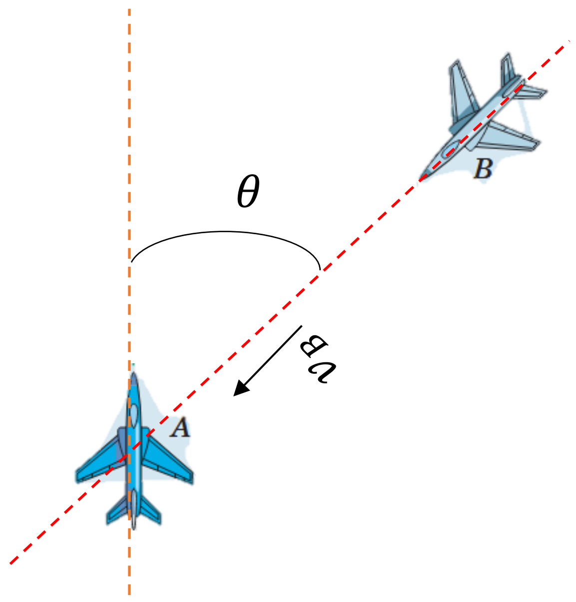


Figure 4. Independent motion

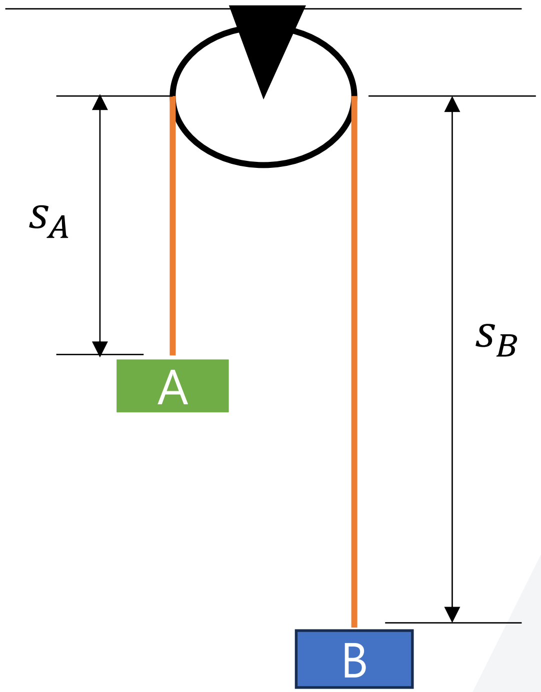


Figure 5. Dependent (constrained) motion

- Constraint equations and relative motion principles are key tools for solving problems involving interconnected particles.

System of particles motion analysis

Relative motion

- If two particles A and B undergo independent motions, then these motions can be related to their relative motion using a translating set of axes attached to one of the particles (A).

$$V_B = V_A + V_{B/A}$$

$$a_B = a_A + a_{B/A}$$

- For planar motion, each vector equation produces two scalar equations, one in the x, and the other in the y direction.
- For solution, the vectors can be expressed in Cartesian form, or Graphical methods are used .

Constrained / dependent motion

- The dependent motion of blocks that are suspended from pulleys and cables can be related by the geometry of the system.
- First we establishing datum for position coordinates,
- Using geometry and/or trigonometry, the coordinates are then related to the cable length to get position equation.
- The first time derivative of this equation gives a relationship between the velocities of the blocks, and a second gives their accelerations.

Chapter 3. Understand the kinetics of particles

- kinetics of particle is defined as the study of translational motion of rigid bodies with the causing force

The three general approaches to the solution of kinetics problems are:

(A) direct application of Newton's second law (called the force- mass-acceleration method),

(B) use of work and energy principles, and

(C) solution by impulse and momentum methods

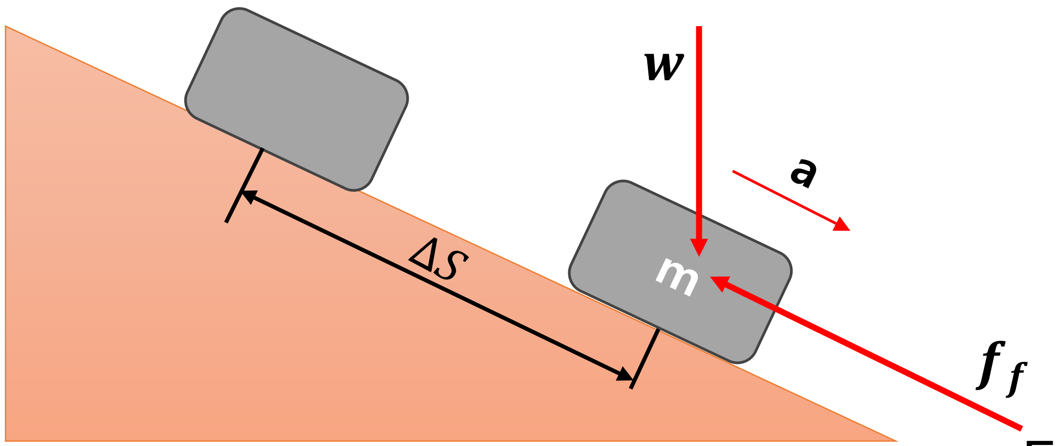


Figure 6. Typical kinetics system

- **Chapter 3 is subdivided into Sections A, B, and C, according to these three methods of solution**

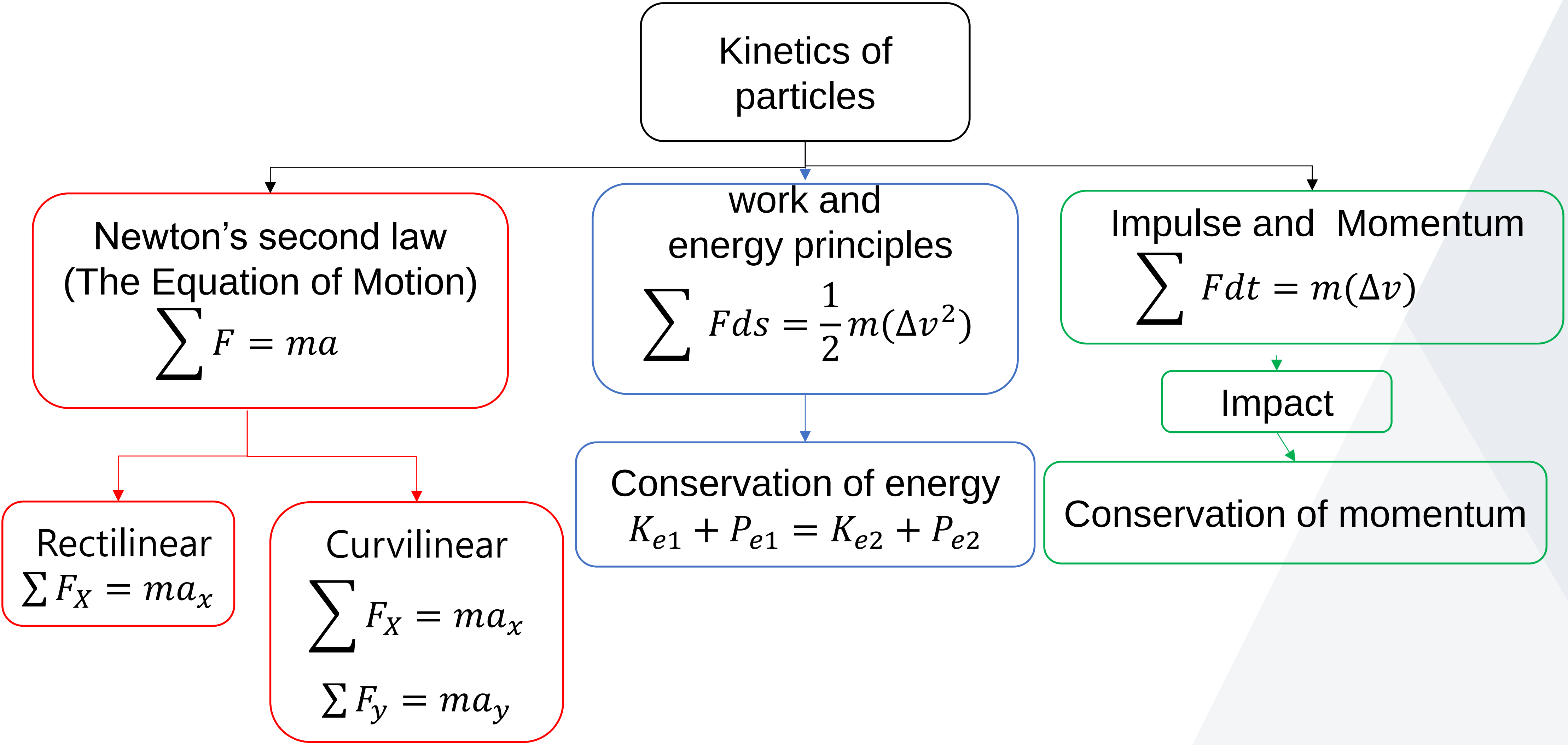


Figure 7. Chapter 3 structure

Summary on equation of motion

- Kinetics is the study of the relation between forces and the acceleration they cause.
- This relation is based on Newton's second law of motion, expressed mathematically as

$$\sum F = ma$$

- Before applying the equation of motion, it is important to first draw the particle's FBD and kinetic diagram.

Rectilinear motion

Equation of motion only applied in 1D and in other direction are in equilibrium

$$\sum F_x = ma_x \qquad \sum F_y = 0 \qquad \sum F_z = 0$$

Curvilinear motion

Polar Co-ordinate

$$\sum F_\theta = ma_\theta \qquad a_r = (\ddot{r} - r\dot{\theta}^2)$$

$$\sum F_r = ma_r \qquad a_\theta = (r\ddot{\theta} - 2\dot{r}\dot{\theta})$$

$$\sum F_{bn} = 0$$

Normal-tangent Co-ordinate

$$\sum F_t = ma_t \qquad a_t = \frac{dv}{dt}$$

$$\sum F_n = ma_n \qquad (a_n = \frac{v^2}{\rho})$$

$$\sum F_{bn} = 0$$

Summary on work energy

Work –energy equation

- useful for solving problems that involve force, velocity, and displacement
- Equation $$T_1 + \sum U_{1-2} = T_2$$
- Applied in all cases.

Initial kinetic energy: $T_1 = \frac{1}{2} m\, v_1{}^2$

work of a constant force: $U_{1\rightarrow2} = (P\cos\theta)(x_2 - x_1)$

work of a weight: $U_{1\rightarrow2} = -w(y_2 - y_1)$

work of an elastic spring: $U_{1\rightarrow2} = \frac{1}{2}(-k)(x_2{}^2 - x_1{}^2)$

Final kenetic energy: $\mathrm{T}_2 = \frac{1}{2} m v_2{}^2$

Conservation of energy

- A conservative force does work that is independent of its path. Two examples are the weight of a particle and the spring force.
- The work done by a conservative force depends upon its position relative to a datum.
- When this work is referenced from a datum, it is called potential energy..
- Equation $$T_1 + V_1 = T_2 + V_2$$
- Applied in certain cases.

Summary on Impulse momentum

- This is a vector equation that can be resolved into rectangular components and used to solve problems that involve force, velocity, and time.
- Expressed mathematically as:

$$mv_1 + \sum \int_{t_1}^{t_2} F dt = mv_2$$

- where: mv is referred to us momentum and $\sum \int_{t_1}^{t_2} F dt$ is the sum of impulse forces over a given time interval

System of particle

$$\sum mv_1 + \sum \int_{t_1}^{t_2} F dt = \sum mv_2$$

- The conservation of momentum

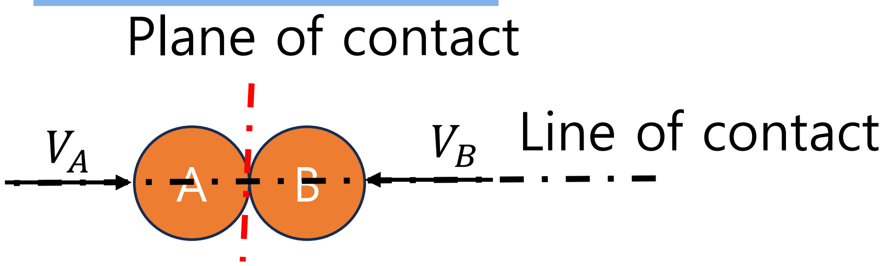
$$m_A v_{(A)_1} + m_B v_{(B)_1} = m_A v_{(A)_2} + m_B v_{(B)_2}$$

- The Coefficient of Restitution

$$e = \frac{v_{(B)_2} - v_{(A)_2}}{v_{(A)_1} - v_{(B)_1}}$$

Impact

Central impact

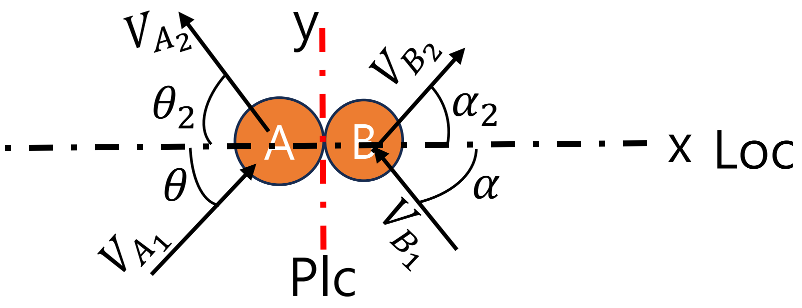


- Along Line of contact.

$$m_A v_{(A)_1} + m_B v_{(B)_1} = m_A v_{(A)_2} + m_B v_{(B)_2}$$

$$e = \frac{v_{(B)_2} - v_{(A)_2}}{v_{(A)_1} - v_{(B)_1}}$$

Oblique impact



- Along plane of contact.

$$v_{(A_1)_y} = v_{(A_2)_y} \qquad v_{(B_1)_y} = v_{(B_2)_y}$$

- Along Line of contact.

$$m_A v_{(A_1)_x} + m_B v_{(B_1)_x} = m_A v_{(A_2)_x} + m_B v_{(B_2)_x}$$

$$e = \frac{v_{(B_2)_x} - v_{(A_2)_x}}{v_{(A_1)_x} - v_{(B_1)_x}}$$

Choosing the Right Method

- Work & Energy Method – faster and simpler in many cases. Has limitations – cannot determine acceleration or normal force.
- $\Sigma F = ma$ is required when acceleration or normal force is unknown.
- For non-impulsive motion, both $\Sigma F = ma$ and Work–Energy give quick solutions.
- For impact or collision problems, only Impulse–Momentum is practical, because:

✓ $\Sigma F = ma$ becomes too complex.

✓ Work–Energy cannot be used (energy loss during impact).

- Some problems combine methods:

✓ Impact phase → use Impulse–Momentum & relative velocity.

✓ Before/After impact → use Work–Energy.

✓ Normal force calculation → use $\Sigma F = ma$.

Chapter 4. Understand the Kinematics of Rigid Bodies

- The structure and flow of topics covered in this chapter are illustrated in the diagram below.

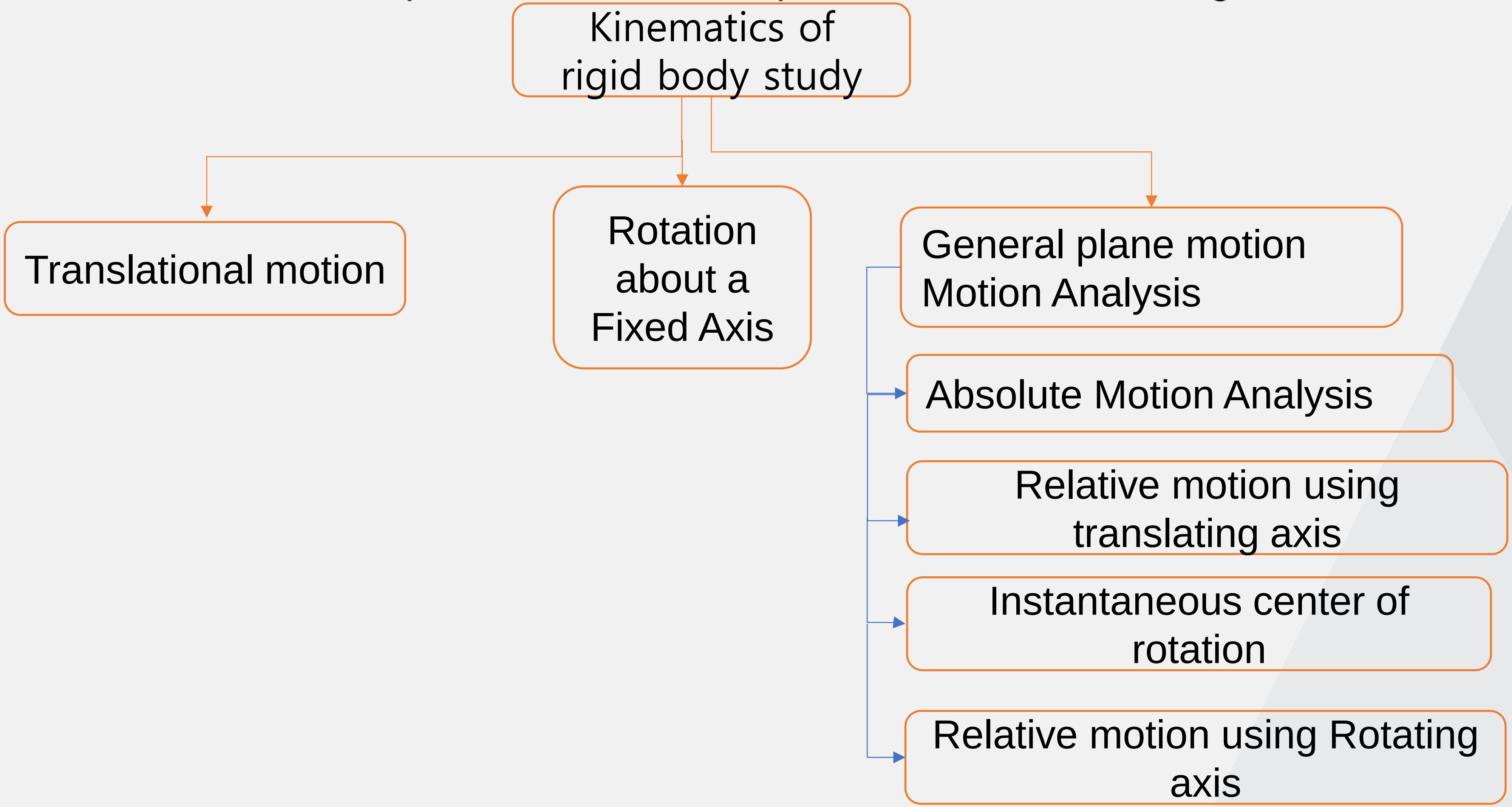


Figure 11. Classifications of Chapter 4

Summary on Translational Motion of rigid bodies

- Consider a rigid body which is subjected to either rectilinear or curvilinear translation in the x-y plane

- Position: $r_B = r_A + r_{B/A}$
- Velocity ($\frac{dr_B}{dt}$): $V_A = V_B$
- Acceleration : $a_A = a_B$

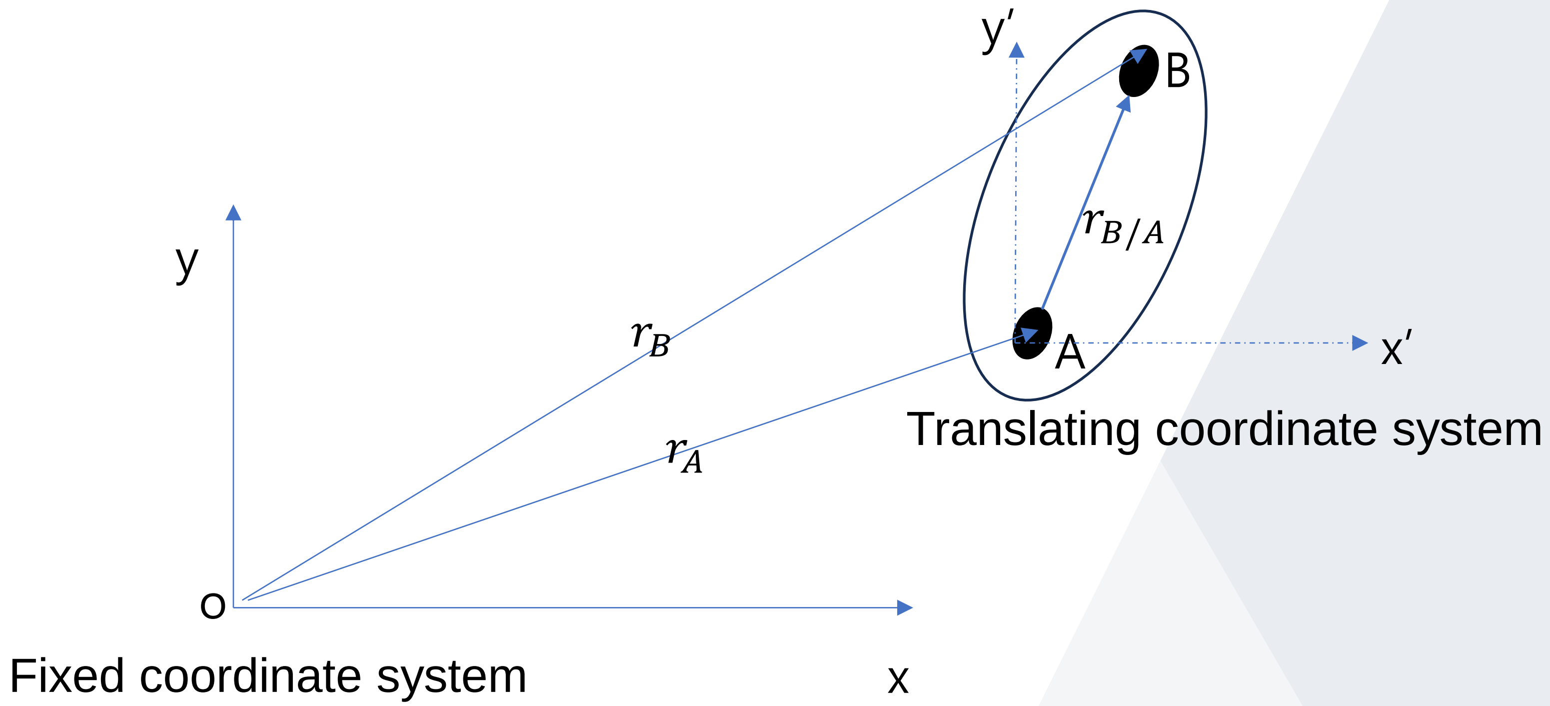


Figure 12. Translational motion Expression

- Thus, when a rigid body is in translation, all the points of the body have the same velocity and the same acceleration at any given instant .

These concepts have already been covered in our study of kinematics of particles in Chapters 2 and 3.

Summary on Rotational motion

- Points located on a body that rotates about a fixed axis follow circular paths
- All the points on a rotating body move with the same angular velocity and acceleration but different linear velocity and acceleration

$$\omega = \frac{d\theta}{dt} \qquad \alpha = \frac{d\omega}{dt} \qquad \alpha d\theta = \omega d\omega$$

- If the body's angular acceleration is constant, then the following equations can be used:

$$\omega = \omega_O + \boldsymbol{\alpha}_c t$$

$$\theta = \theta_O + \omega_O t + \frac{1}{2}(\boldsymbol{\alpha}_c t^2)$$

$$\omega^2 = (\omega_O)^2 + 2\boldsymbol{\alpha}_c(\theta - \theta_O)$$

time dependent

- In most cases the velocity of P and its two components of acceleration can be determined from the scalar equations

$$v = \boldsymbol{\omega r}$$

$$a_n = \boldsymbol{v}^2/\boldsymbol{r} = \boldsymbol{\omega}^2\boldsymbol{r} \qquad a_t = \boldsymbol{\alpha}\,\boldsymbol{r}$$

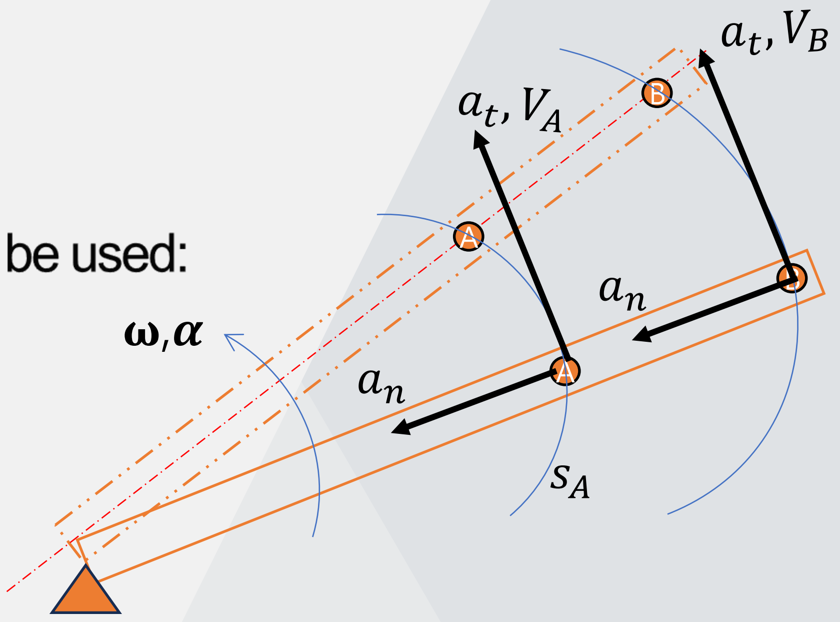


Figure 13. Rotation

If the geometry of the problem is difficult to visualize, the following vector equations should be used:

$$v = \boldsymbol{\omega} \times \boldsymbol{r} \qquad a_t = (\alpha \times r) \qquad a_n = \omega \times \omega \times r = -w^2 r$$

General plane motion of rigid body

Absolute Motion Analysis

- The absolute-motion approach to rigid-body kinematics is quite straightforward, provided the configuration lends itself to a geometric description which is not overly complex [2].
- By direct application of the time-differential equations $v = \frac{ds}{dt}$, $a = \frac{dv}{dt}$, $\omega = \frac{d\theta}{dt}$, and $\alpha = \frac{d\omega}{dt}$, the motion of the point and the angular motion of the line can then be related.
- This procedure is similar to that used to solve dependent motion problems involving pulleys where the geometric relations were quite simple, and no angular quantities had to be considered.
- Here In rigid-body motion, geometric relations involve both linear and angular variables, so their relations of time derivatives include both linear and angular velocities and accelerations.
- To do so, First, we wrote an equation which describes the general geometric configuration of a given problem in terms of knowns and unknowns. Then we differentiated this equation with respect to time to obtain velocities and accelerations, both linear and angular.

Summary on Relative motion

Translating axis

- when translating axes are placed at the "base point" A, the relative motion of point B with respect to A is simply circular motion of B about A
- The following equations apply to two points A and B located on the same rigid body

$$\boldsymbol{V_B} = \boldsymbol{V_A} + \boldsymbol{\omega_{ab} k} \times \boldsymbol{r_{B/A}}$$

$$\boldsymbol{a_B} = \boldsymbol{a_A} + \boldsymbol{a_{(B/A)t}} + \boldsymbol{a_{(B/A)n}} \quad \left({}^{m}/_{s^2}\right)$$

- The velocity can also be solved using instantaneous zero velocity method .
- The velocity and acceleration can be also solved using graphical method .

Rotating axis

- When the velocity and acceleration can also be obtained using rotating axis for sliding connections

$$V_B = V_A + \Omega \times r_{B/A} + V_{(B/A)_{x,y,z}}$$

$$a_B = a_A + (\dot{\Omega} \times r_{B/A}) + (\Omega \times r_{B/A}) + 2\Omega \times V_{(B/A)_{x,y,z}} + a_{(B/A)_{x,y,z}}$$

Understand the kinetics of rigid bodies

- The kinetics of rigid bodies deals with the relationships between external forces and moments acting on a body and the translational and rotational motions produced by these forces[1].
- While the kinematics of rigid bodies (studied earlier) was only concerned with describing motion ,such as velocity, acceleration, and angular velocity — kinetics focuses on why that motion occurs.[1].
- Previously, in the kinetics of particles, we treated the entire mass as concentrated at a single point. Here, however, the shape, size, and mass distribution of the body become important.
- For a rigid body:
  - Forces may act at different points
  - The body can both translate and rotate.
  - The mass moment of inertia plays a key role in rotational motion.

scope Cont'd…

- Therefore, in rigid-body kinetics, we require three equations of motion:
  - Two force equations (for translation in x and y directions), and
  - One moment equation (for rotation about a point or the mass center).
- Kinetics of rigid bodies is analyzed using three major methods:
  - Force and Acceleration Method
  - Work and Energy Method
  - Impulse and Momentum Method
- Each method is applied to three types of motion:
  1. Pure Translation,
  2. Rotation about a Fixed Axis, and
  3. General Plane Motion (a combination of both).

Summary

- The Equations of Motion relate the external forces and moments acting on a rigid body to its linear and angular accelerations.
- They form the foundation of rigid-body dynamics for both simple and complex motions.
- Used to analyze: Translation, Rotation about a fixed axis, General plane motion

✓ Before applying the equations of motion, always draw afree-body diagram in order to identify all the forces acting on the body

✓ A kinetic diagram may also be drawn in order to represent ma_G and $I_G\alpha$ graphically.

Summary

- Two Fundamental Equations of Motion are used and applied differently depending on the motion of the rigid body

Pure Translation:

$$\sum M_G = I\alpha_G = 0$$

Rectilinear $\sum F_x = ma_{(G)_x}$

Curvilinear $\sum F_t = ma_{(G)_t} = m(r_G\ \alpha)$ $\sum F_n = ma_{(G)_n} = m(\frac{{V_G}^2}{r_G}) = \omega^2\ r_G$

Rotation about Fixed Axis:

$$\sum M_o = I_O\alpha$$

$$\sum F_n = ma_{(G)_n} \qquad \sum F_t = ma_{(G)_t}$$

General Plane Motion: Combination of translation (of G) + rotation (about G).

Summary

Work –energy equation

- useful for solving problems that involve force, velocity, and displacement
- Equation $T_1 + \sum U_{1-2} = T_2$
- Kinetic energy $T = \frac{1}{2} m\, V_G{}^2 + \frac{1}{2} I_G \omega^2$
- Work is due to forces and couple

$$U_{1 \rightarrow 2} = \int_1^2 ((F\cos\theta))ds$$

$$U = M(\theta_2 - \theta_1)$$

- Applied in all cases.

Conservation of energy

- A conservative force does work that is independent of its path. Two examples are the weight of a particle and the spring force.
- The work done by a conservative force depends upon its position relative to a datum.
- When this work is referenced from a datum, it is called potential energy..
- The potential energy is the sum of the body's gravitational and elastic potential energies.
- Equation $T_1 + V_1 = T_2 + V_2$

$$V = mgh_g + \frac{1}{2} k\, S^2$$

Summary on Impulse momentum

- The principles of linear and angular impulse and momentum are used to solve problems involving force, velocity, and time.
- Before applying the equations, draw a free-body diagram in order to identify all the forces which cause linear and angular impulses on the body..
- Also, establish the directions of the velocity of the mass center and the angular velocity of the body just before and just after the impulses are applied.

$$mv_{(Gx)_1} + \sum \int_{t_1}^{t_2} F_x dt = mv_{Gx(_2)} \qquad mv_{(Gy)_1} + \sum \int_{t_1}^{t_2} F_y dt = mv_{Gy(_2)} \qquad I_G\omega_1 + \sum \int_{t_1}^{t_2} M_G dt = I_G\omega_2$$

Conservation of Momentum

- If non impulsive forces or no impulsive forces act on the body in a particular direction, or if the motions of several bodies are involved in the problem, then consider applying the conservation of linear or angular momentum for the solution.

$$\sum mv_{(G)_1} = \sum mv_{(G)_2} \qquad H_{(G)1} = H_{(G)2}$$

References

[1] Dynamics, Hibbeler, Rusel M.,Prentice Hall,10th ed., 2003

[2] Engineering Mechanics - Dynamics, Meriam J.L.,John Wiley & Sons, 9th ed., 2020.,

[3] Vector Mechanics for Engineers: Dynamics, Johnston, E. R., & Clausen, W. E., McGraw-Hill, 11th ed., 2015

[4] Cengel, Yunus, and John Cimbala. *Ebook: mechanics fundamentals and applications (si units)*. Mc Graw Hill, 2013.