SOLUTION FOR FINAL EXAMINATION

I) Solution for Multiple choice question

1. A 2. B 3. B 4. B 5. C 6. B 7.C

II) Solution for numerical problem

Solution for number 8

STRATEGY:

A lot of things are going on in this problem, so you need to break the motion into steps.

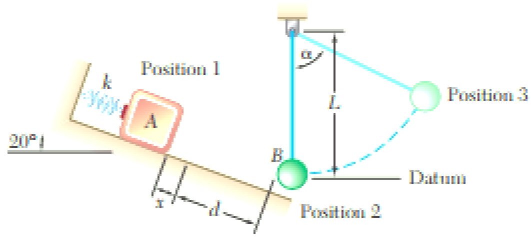

Figure 1. Three positions of interest for this problem

Step 1: Block *A* slides down the incline, so there are two positions. Therefore, use the work–energy principle between position 1 and position 2 to find the velocity of *A* just before it strikes ball *B.*

Step 2: Block *A* hits *B*, so an impact occurs. Therefore, use impulse-momentum and the equation for the coefficient of restitution.

Step 3: Ball *B* is swinging up, so you have two positions (position 2 and position 3). You are asked to find the speed at position 3, therefore, use the conservation of energy.

Step 4: To find the tension when $\theta = 40°$, use Newton's second law with normal and tangential coordinates

Let us find the velocity of particle A before the impact.

- Applying work-energy equation.

$$T_{A_1} + \sum U_{1-2} = T_2$$

$$\sum U_{1-2} = U_{g_{1-2}} - U_{F_{1-2}} - U_{S_{1-2}}$$

$$U_{g_{1-2}} = mgh,\ h = (x + d)\sin(20)$$

$$= 2 \times 9.81 \times (0.1 + 1.5) sin(20)$$

$$U_{F_{1-2}} = F_N(x + d), F_N = mgcos\theta$$

$$= 2 \times 9.81 \times (0.1 + 1.5) cos(20)$$

$$U_{Spring_{1-2}} = -\frac{1}{2}K(s_2^2 - s_1^2)$$

$s_2 = 0$ (Because at state 2 or when the block hit the ball it is already detached from spring, as result there is no spring at state 1.)

$$U_{Spring_{1-2}} = -\frac{1}{2}K(0 - s_1^2) = -\frac{1}{2} \times 800(0.1^2)$$

So V_A Solving for gives: V_A =5 2.973 m/s

- Once we get the velocity values of the Block A and the ball be for impact, we can apply principle of conservation of momentum and coefficient of restitution to get the velocity after impact.
- Before state 1 and state 2

$$\mathrm{M_A V_{(A)1} + M_B V_{(B)1} = M_A V_{A2} + M_B V_{B2}} \quad (1)$$

$$e = \frac{\mathrm{V_{B2} - V_{A2}}}{\mathrm{V_{(A)1} - V_{(B)1}}} \ldots \quad (2)$$

Solving both Equation 1 and Equation 2, we get both $\mathrm{V_{B2}} = 3.63 m/s\ and\ V_{A2} = 1.03 m/s$

- Now based on the velocity value of the ball after impact (V_{B2}), we can find the velocity, to get the tension value. The result can be obtained using
  a) *Conservation of energy between state 2 and state 3*
  b) *The combination of equation of motion and kinematic*

Applying conservation of energy principle between state 2 and state 3:

$T_2 + v_2 = T_3 + T_3$

$\frac{1}{2}mV_b^2 = M_b gh + 0 + \frac{1}{2}M_b V_{b3}^2$

$V_{b3} = \sqrt{((V_{b2})^2 - 2g(L - L\cos 40))}$=*2.94m/s*

Then to find the tension we use equation of motion for curvilinear motion using normal tangent coordinate.

$\sum F_n = ma_n$

$T - mg\,cos\,40 = m_b\left(\frac{(V_{b3})^2}{2}\right)$

T=16.14N

Solution For number 9

9. Kinematic Diagram: from the geometry, $\theta = \tan^{-1}\left(\frac{1.5}{9}\right) = 9.462^o$ and $r_{BE} = \sqrt{9^2 + 1.5^2}$ = 9.124ft. Since crank CD and beam BE are rotating about fixed points D and E, then V_C and V_B are always perpendicular to crack CD and beam BE, respectively. The magnitude of V_C and V_B are $V_C = \omega_{CD} r_{CD}$ = 6(8) = 18$\frac{ft}{s}$ and $V_B = \omega_{BE} r_{BE}$ = 9.124(ω_{BE}). At the instant shown, V_C is directed vertically while V_B is directed with an angle 9.462^o with the vertical.

Instantaneous Center: The instantaneous center of zero velocity of link BC at the instant shown is located at the intersection of extended lines drawn perpendicular from V_C and V_B. From the geometry:

$r_{B/IC} = \frac{10}{\sin 9.462^0}$ = 60.83ft and $r_{C/IC} = \frac{10}{\tan 9.462^0}$ = 60.0ft

The angular velocity of link BC is given by

$\omega_{BC} = \frac{v_c}{r_{C/IC}}$ = $\frac{18}{60}$ = 0.300 rad/s

Thus, the angular velocity of beam BE is given by

$$V_B = \omega_{BC} r_{B/IC}$$

9.124ω_{BC}= 0.300(60.83)

ω_{BE} = 2.00 rad/s

The speed of rod hanger H is given by

$$V_H = \omega_{BE} r_{EA} = 2.00(9) = 18.0\ ft/s$$