Statistical Digital Signal Processing

Week 2 Deterministic Signal Modeling (Part-1)

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Topics of Previous Lecture (Week-1)

Overview of Random Process

- Basics of Random Signals
- Random Variables
- Random Process
- Filtering of Stationary Random Process
- Spectral Factorization

Lecture Learning Outcomes

1. Explain fundamental concepts of signal modeling, including the differences between parametric and non-parametric approaches.
2. Analyze the motivations for parametric signal modeling, particularly in terms of efficiency, accuracy, and representation of real-world signals.
3. Apply the systematic steps involved in parametric signal modeling to construct mathematical models from observed signal data.
4. Model deterministic signals using Linear Shift Invariant (LSI) systems, and interpret system behavior through input-output relationships.
5. Implement and evaluate the Least Squares (Direct) Method and Padé Approximation, for accurate signal representation and model parameter estimation.

Week 2: Deterministic Signal Modeling (Part-1)

Outline

- Signal Modeling: Introduction
- Parametric and Non-Parametric Signal Modeling
- Motivations for Parametric Signal Modeling
- Parametric Signal Modeling Steps
- Deterministic Signal Modeling using Linear Shift Invariant Filter
- Least Square or Direct Method
- Padé Approximation

Signal Modeling: Introduction

- **Signal Modeling:** is an important problem encountered across various engineering and scientific applications
- It deals with creating concise and effective representations of signals
- There are different types of signal modeling, the following are among the widely used classifications:

1. Non-Parametric Signal Modeling:

- ✓ a data-driven approach through learning of the signal characteristics directly from the available data.
- ✓ an infinite number of measurements would be needed to represent the signal perfectly
- ✓ Practically, a sufficiently large number of recorded data is required to acceptably represent the system

Introduction

2. Parametric Signal Modeling:

- ✓ a process of representing a signal with a mathematical model that is defined by a fixed number of parameters.
- ✓ It works by assuming the signal can be described by a simpler, parameterized form and then estimating those parameters from data to create an efficient model
- ✓ The goal is to find parameters that produce the "best" approximation of the signal
- ✓ parametric signal modeling is economical and powerful since it uses limited parameters to represent the signal

- There are different motivations behind parametric signal modeling such as:
  - ❖ **Efficient Signal Transmission**
  - ❖ **Efficient Signal Storage**
  - ❖ **Signal Prediction and Estimation**

Signal Modeling for Efficient Transmission and Storage

- **Data compression** is widely employed for efficient signal transmission and storage
- Considering efficient transmission or storage of the signal x(n) consisting N data values is

$$x(n) \rightarrow x(0), x(1), x(2), \ldots, x(N-1)$$

- It is possible to transmit or store x(n) in two ways:

1. Direct or Point-to-Point Basis

✓ Transmit or store all samples of x(n)

2. Coded or Parametrized Method

✓ Model the signal with small number of parameters $k \rightarrow k \ll N$

✓ More efficient to transmit or store those parameters instead of the signal values $\rightarrow$ *Data Compression*

Signal Modeling for Efficient Transmission and Storage

- Consider a signal $x(n)$ which can be given by:

$$x(n) = \alpha \cos(n\omega_0 + \phi) \qquad (1)$$

- The signal values of $x(n)$ can be transmitted or stored in storage directly
- However, the more efficient way is transmitting or storing:

$$\alpha \rightarrow \textit{the amplitude}$$

$$\omega_0 \rightarrow \textit{the frequency}$$

$$\phi \rightarrow \textit{the phase}$$

- The signal $x(n)$ can be recovered from those parameters (α, ω_0, ϕ) using the model given in eq(1).

Signal Modeling for Efficient Transmission and Storage

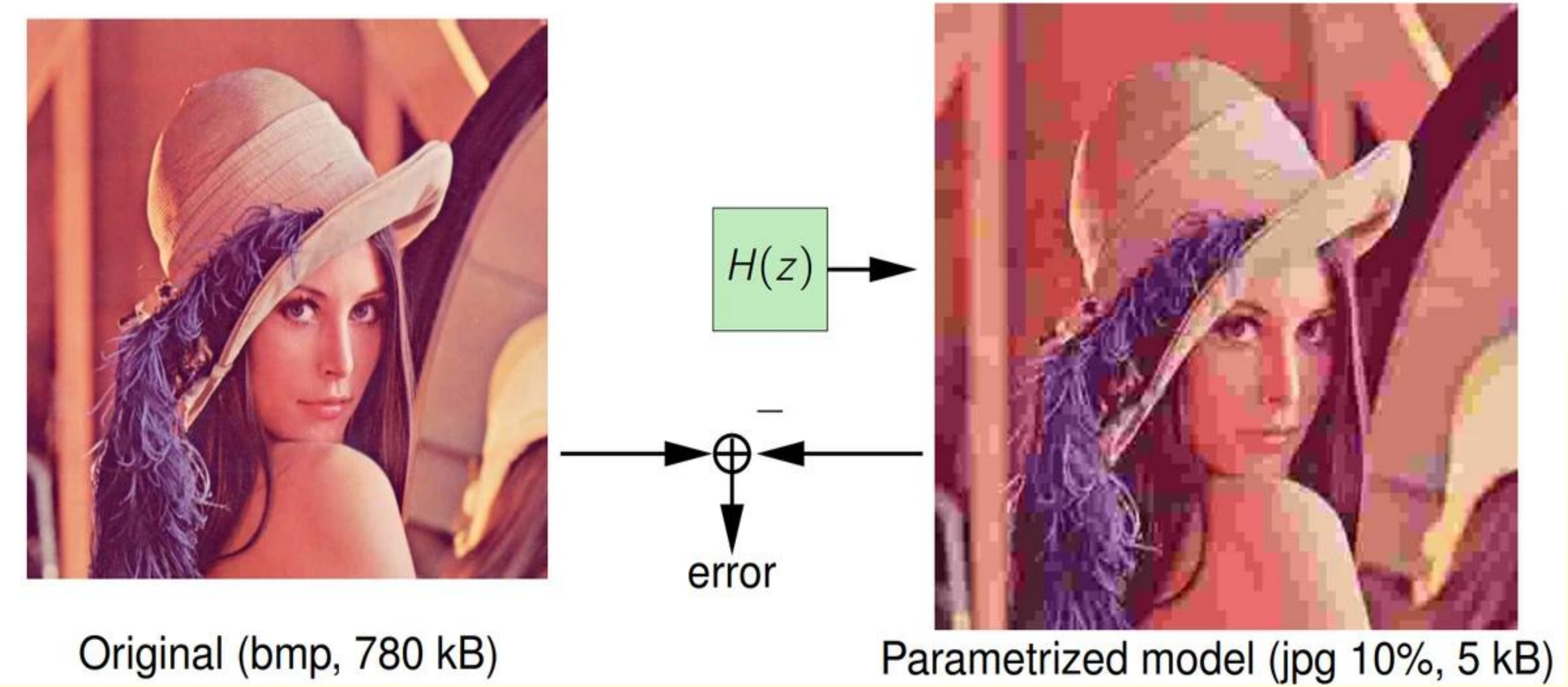

Figure 1: Image compression using parametric signal modeling

Source: *"Digital Signal processing for image compression,"* mystrikingly.com. https://custom-images.strikinglycdn.com/res/hrscywv4p/image/upload/c_limit,fl_lossy,h_9000,w_1200,f_auto,q_auto/24525391/521638_255366.png

Signal Modeling for Prediction

- Signal modeling is also used in prediction and estimation applications
- If the signal $x(n)$ is known over certain interval of time, it is possible to determine the value of the signal over another interval using signal modeling
- **Prediction:** if $x(n)$ is known for $n = 0,1,2,\dots, N-1$, the goal is to predict the next value of the signal $X(N), X(N+1),\dots$
- **Estimation:** if one or more consecutive values are missing or distorted over interval $[N_1, N_2]$ the goal is to use the data outside of the interval to recover the value of $x(n)$ in the interval
- Suppose $x(n)$ is known for $n = 0,1,2,\dots, N-1$, and it can be modeled as:

$$x(n) \approx \sum_{k=1}^{p} a_p(k)x(n-k) \qquad (2)$$

- The next value $x(N)$ can be predicted using:

$$\hat{x}(N) \approx \sum_{k=1}^{p} a_p(k)x(N-k) \qquad (3)$$

Parametric Signal Modeling Steps

Choosing the parametric form of the model

↓

Determining the model parameters

↓

Evaluating the model performance

Signal Modeling Using Filter

- Signal can be modeled as an output of linear shift invariant filter that has a rational system function $(H(z))$ given by:

$$H(z) = \frac{B_q(z)}{A_p(z)} = \frac{\sum_{k=0}^{q} b_q(k) z^{-k}}{1 + \sum_{k=1}^{p} a_p(k) z^{-k}} \qquad (4)$$

Input signal

Estimation of the model for $x(n)$

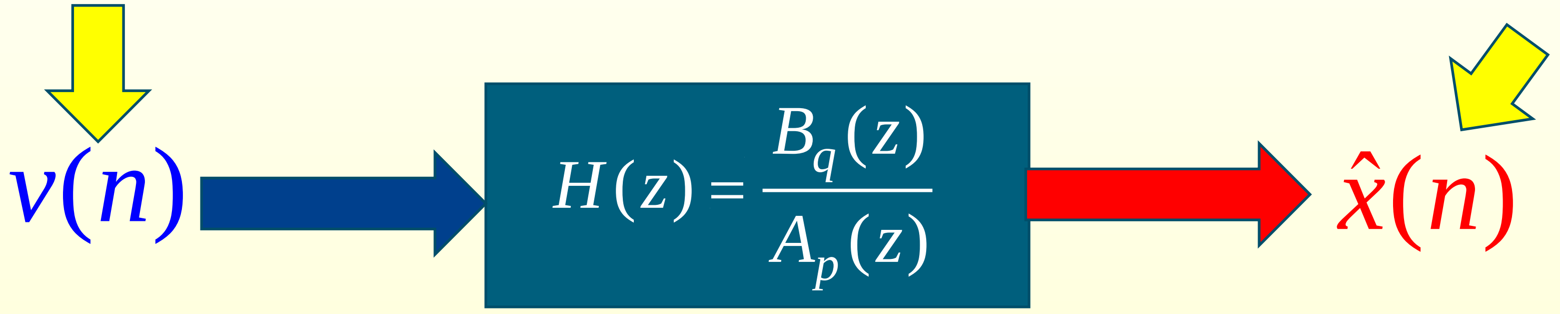

Figure 2: Modeling of a signal as an output of linear shift invariant filter

Signal Modeling Using Filter

- In most cases the signal to be modeled could be deterministic signal
- For the modeling of deterministic signal, the input of the filter is:

$$v(n) = \delta(n) \qquad (5)$$

Where:

$\delta(n)$ is unit sample signal

- After selecting the model, the focus shifts to finding the parameters that best approximate the signal
- Although multiple methods exist for model parameter derivation, choosing a computationally efficient approach is essential
- In this week and next week's lectures, we will explore various approaches to deterministic signal modeling

The Least Squares or Direct Method

- We will consider modeling of deterministic signal $x(n)$ as a unit sample response of linear shift invariant filter $h(n)$ having a rational system function given in Eq (4):

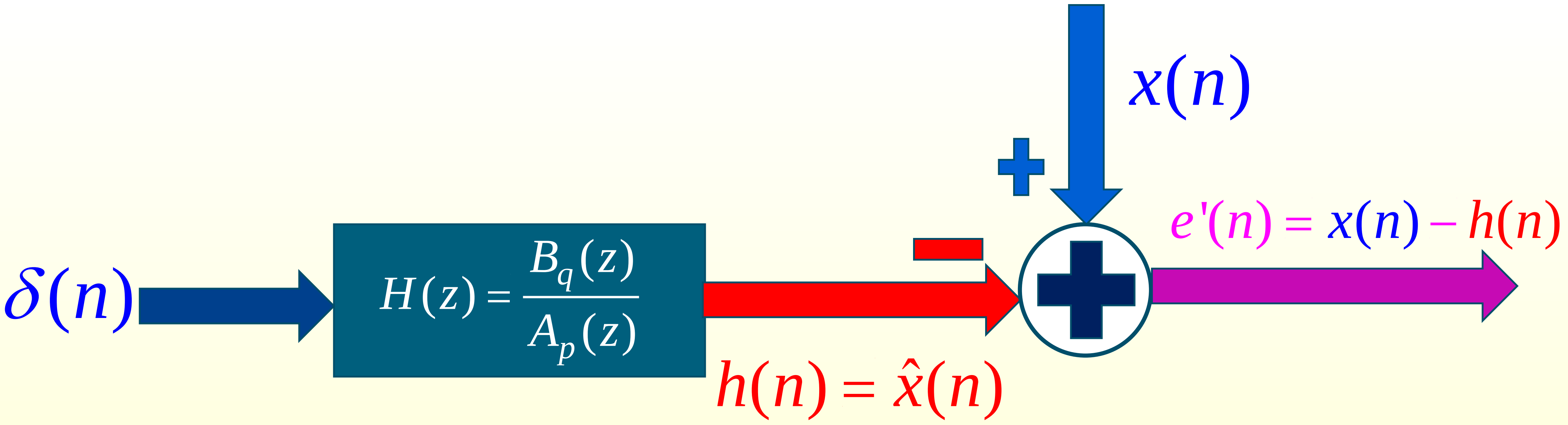

Figure3: Modeling of a signal as a unit sample response of linear shift invariant filter

The Least Squares or Direct Method

- With out loss of generality, it is assumed that $x(n)=0,\ for\ n<0$ and $h(n)$ is causal
- The modeling error $e'(n)$ is given by;

$$e'(n)=x(n)-h(n)=x(n)-\hat{x}(n) \qquad (6)$$

- The objective is to find the filter parameters $a_p(k)$ and $b_q(k)$ that can minimize $e'(n)$
- In least square method, the error that is to be minimized is the squared error (ε_{LS}):

$$\varepsilon_{LS}=\sum_{n=0}^{\infty}|e'(n)|^2=\sum_{n=0}^{\infty}e'(n)e'^*(n) \qquad (7)$$

- Optimization theory will be applied to reduce the above error as follows:

$$\frac{\partial\varepsilon_{LS}}{\partial a_p^*(k)}=0;\quad k=1,2,\ldots,p \qquad (8)$$

$$\frac{\partial\varepsilon_{LS}}{\partial b_q^*(k)}=0;\quad k=0,1,2,\ldots,q \qquad (9)$$

The Least Squares or Direct Method

- The least square error (ε_{LS}) can be written using Parseval's theorem:

$$\varepsilon_{LS} = \frac{1}{2\pi}\int_{-\pi}^{\pi} |E'(e^{j\omega})|^2 \, d\omega \qquad (10)$$

Where:

$$\underbrace{E'(e^{j\omega})}_{\text{Fourier transform of } e'(n)=x(n)-h(n)} = X(e^{j\omega}) - H(e^{j\omega}) = X(e^{j\omega}) - \frac{B_q(e^{j\omega})}{A_p(e^{j\omega})} \qquad (11)$$

Fourier transform of $e'(n) = x(n) - h(n)$

- Using eq(8) and eq(10), we will have:

$$\frac{\partial \varepsilon_{LS}}{\partial a_p^*(k)} = \frac{1}{2\pi}\int_{-\pi}^{\pi} \frac{\partial}{\partial a_p^*(k)} \left[\underbrace{E'(e^{j\omega}) E'^*(e^{j\omega})}_{|E'(e^{j\omega})|^2} \right] d\omega = 0 \qquad (12)$$

The Least Squares or Direct Method

- The previous equation, eq(12), boils down to:

$$\frac{\partial \varepsilon_{LS}}{\partial a_p^*(k)} = \frac{1}{2\pi}\int_{-\pi}^{\pi}\left[X(e^{j\omega}) - \frac{B_q(e^{j\omega})}{A_p(e^{j\omega})}\right]\frac{B_q^*(e^{j\omega})}{[A_p^*(e^{j\omega})]^2}e^{jk\omega}d\omega = 0 \qquad (13)$$

$$for\ \ k = 1, 2, \ldots, p$$

- Similarly for $b_q(k)$, we have:

$$\frac{\partial \varepsilon_{LS}}{\partial b_q^*(k)} = \frac{1}{2\pi}\int_{-\pi}^{\pi}\left[X(e^{j\omega}) - \frac{B_q(e^{j\omega})}{A_p(e^{j\omega})}\right]\frac{e^{jk\omega}}{A_p^*(e^{j\omega})}d\omega = 0 \qquad (14)$$

$$for\ \ k = 0, 1, 2, \ldots, q$$

- As revealed by eq(13) and eq(14), it is required to solve p+q+1 non linear equations to find the optimum set of filter/model parameters

The Least Squares or Direct Method

- Due to the requirement of solving p+q+1 non-linear equations, the Least square method is not mathematically tractable
- Even though least square method is efficient in minimizing the modeling error, it is computationally intensive
- Due to the significant delay introduced by computation time, direct/least square method is hardly applicable for real time applications
- Therefore, the next focus will be on the indirect methods of signal modeling
- In indirect methods, the modeling problem will be changed somewhat to easily find the model parameters

Padé Approximation

- Unlike the least square method, Padé approximation only requires solving a set of linear equations [1]
- In Padé approximation, the signal $x(n)$ is model as an output of linear time invariant filter using an input $\delta(n)$
- Padé approximation force the filter output $h(n)$ to be equal to $x(n)$ for $p+q+1$ values or for $n = 0,1,2,\ldots, p+q$ [1]
- From:

$$H(z) = \frac{B_q(z)}{A_p(z)} = \frac{\sum_{k=0}^{q} b_q(k) z^{-k}}{1+\sum_{k=1}^{p} a_p(k) z^{-k}} \qquad (15)$$

- From eq (15), we have,

$$A_p(z)H(z) = B_q(z) \qquad (16)$$

Padé Approximation

- In time domain, the previous equation, eq(16), becomes:

$$a_p(n) * h(n) = b_q(n) \qquad (17)$$

- eq(17) can be simplified as:

$$\sum_{k=0}^{p} a_p(k)h(n-k) = b_q(n) \qquad (18)$$

$$\Big\downarrow \; a_p(0) = 1$$

$$h(n) + \sum_{k=1}^{p} a_p(k)h(n-k) = b_q(n) \qquad (19)$$

- Enforcing $h(n) = x(n)$ for $q + p + 1$ values of n, we have:

$$x(n) + \sum_{k=1}^{p} a_p(k)x(n-k) = b_q(n), \; for \; n = 0,1,\ldots, p+q \qquad (20)$$

Padé Approximation

- the previous equation, eq(20), can be written in the following form:

$$x(n) + \sum_{k=1}^{p} a_p(k)x(n-k) = \begin{cases} b_q(n), & for\ n = 0,1,\ldots,q \\ 0, & for\ n = q+1, q+2, \ldots, p+q \end{cases} \tag{21}$$

- the above equation, eq(21), can be written in matrix form as follows:

$$\begin{bmatrix} x(0) & 0 & \cdots & 0 \\ x(1) & x(0) & \cdots & 0 \\ \vdots & \vdots & \cdots & \vdots \\ x(q) & x(q-1) & \cdots & x(q-p) \\ \hdashline x(q+1) & x(q) & \cdots & x(q-p+1) \\ \vdots & \vdots & & \vdots \\ x(q+p) & x(q+p-1) & \cdots & x(q) \end{bmatrix} \begin{pmatrix} 1 \\ a_p(1) \\ a_p(2) \\ \vdots \\ a_p(p) \end{pmatrix} = \begin{pmatrix} b_q(0) \\ b_q(1) \\ b_q(2) \\ \vdots \\ b_q(q) \\ \hdashline 0 \\ \vdots \\ 0 \end{pmatrix} \tag{22}$$

Padé Approximation

- The last p equations will be used to find $a_p(k)$:

$$\begin{bmatrix} x(q+1) & x(q) & \cdots & x(q-p+1) \\ \vdots & \vdots & & \vdots \\ x(q+p) & x(q+p-1) & \cdots & x(q) \end{bmatrix} \begin{pmatrix} 1 \\ a_p(1) \\ a_p(2) \\ \vdots \\ a_p(p) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{pmatrix} \qquad (23)$$

- From the above equation, we have:

$$\begin{pmatrix} x(q+1) \\ x(q+2) \\ \vdots \\ x(q+p) \end{pmatrix} + \begin{bmatrix} x(q) & \cdots & x(q-p+1) \\ \vdots & & \vdots \\ x(q+p-1) & \cdots & x(q) \end{bmatrix} \begin{pmatrix} a_p(1) \\ a_p(2) \\ \vdots \\ a_p(p) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{pmatrix} \qquad (24)$$

Padé Approximation

- The previous eq (24) can be re-written as:

$$\underbrace{\begin{pmatrix} x(q) & x(q-1) & \cdots & x(q-p+1) \\ x(q+1) & x(q) & \cdots & x(q-p+2) \\ \vdots & \vdots & \cdots & \vdots \\ x(q+p-1) & x(q+p) & \cdots & x(q) \end{pmatrix}}_{\mathbf{X}_q} \underbrace{\begin{pmatrix} a_p(1) \\ a_p(2) \\ \vdots \\ a_p(p) \end{pmatrix}}_{\bar{\mathbf{a}}_p} = -\underbrace{\begin{pmatrix} x(q+1) \\ x(q+2) \\ \vdots \\ x(q+p) \end{pmatrix}}_{-\mathbf{x}_{q+1}} \qquad (25)$$

- Then, in compact form we will have:

$$\mathbf{X}_q \bar{\mathbf{a}}_p = -\mathbf{x}_{q+1} \qquad (26)$$

Where:

$\mathbf{X}_q$ Is a $p \times p$ non-symmetric Toeplitz matrix

Padé Approximation

- In eq (25) , we have p unknowns and p linear equations
- Eq(25) can be solved to find the unknown filter parameter $\bar{\mathbf{a}}_p$
- If $\mathbf{X}_q$ is invertible $\bar{\mathbf{a}}_p$ can be found by:

$$\bar{\mathbf{a}}_p = -\mathbf{X}_q^{-1}\mathbf{x}_{q+1} \qquad (27)$$

- If $\mathbf{X}_q$ is singular matrix and no solution exist that can satisfy eq(26), we will make a slight change on the denominator of $H(z)$ which was used in Padé formulation
- Originally $A_p(z)$ was written by assuming $a_p(0)=1$, now we will consider $a_p(0)=0$ and $A_p(z)$ changes as follows

$$A_p(z) = \underbrace{1}_{a_p(0)} + \sum_{k=1}^{p} a_p(k)z^{-k} \implies A_p(z) = \sum_{k=1}^{p} a_p(k)z^{-k} \qquad (28)$$

Padé Approximation

- Once $\bar{a}_p$ is found, b_q can be found from the upper matrix given at eq(22) as:

$$\underbrace{\begin{pmatrix} x(0) & 0 & & \cdots & 0 \\ x(1) & x(0) & 0 & \cdots & 0 \\ x(2) & x(1) & x(0) & \cdots & 0 \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ x(q) & x(q-1) & x(q-2) & \cdots & x(q-p) \end{pmatrix}}_{\mathbf{X}_0} \underbrace{\begin{pmatrix} 1 \\ a_p(1) \\ a_p(2) \\ \vdots \\ a_p(p) \end{pmatrix}}_{\mathbf{a}_p} = \underbrace{\begin{pmatrix} b_q(0) \\ b_q(1) \\ b_q(2) \\ \vdots \\ b_q(q) \end{pmatrix}}_{\mathbf{b}_q} \qquad (29)$$

- Then we will have:

$$\mathbf{X}_0 \mathbf{a}_p = \mathbf{b}_q \qquad (30)$$

Padé Approximation: Example

- Consider a signal X whose first six values are as follows [2]:

$$\mathrm{X} = [1,\ 1.500,\ 0.750,\ 0.375,\ 0.1875,\ 0.0938]^T$$

- Use the Padé Approximation to find:

1. Second order all pole model $(p = 2, q = 0)$
2. Second order moving average model $(p = 0, q = 2)$
3. Model with one pole and one zero $(p = 1, q = 1)$

Solution:

1. We will solve the following equations to find the second order all pole model with $(p = 2, q = 0)$

$$\begin{pmatrix} x(0) & 0 & 0 \\ x(1) & x(0) & 0 \\ x(2) & x(1) & x(0) \end{pmatrix} \begin{pmatrix} 1 \\ a_p(1) \\ a_p(2) \end{pmatrix} = \begin{pmatrix} b_q(0) \\ 0 \\ 0 \end{pmatrix}$$

Padé Approximation: Example

- Solving the last two equations, we will have

$$\begin{pmatrix} x(1) & x(0) & 0 \\ x(2) & x(1) & x(0) \end{pmatrix} \begin{pmatrix} 1 \\ a_p(1) \\ a_p(2) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\Downarrow$$

$$\begin{pmatrix} x(0) & 0 \\ x(1) & x(0) \end{pmatrix} \begin{pmatrix} a_p(1) \\ a_p(2) \end{pmatrix} = -\begin{pmatrix} x(1) \\ x(2) \end{pmatrix}$$

$$\Downarrow$$

$$\begin{pmatrix} 1 & 0 \\ 1.5 & 1 \end{pmatrix} \begin{pmatrix} a_p(1) \\ a_p(2) \end{pmatrix} = -\begin{pmatrix} 1.5 \\ 0.75 \end{pmatrix}$$

Padé Approximation: Example

- Solving the previous equation, we will have

$$a_p(1) = -1.5 \quad \text{and} \quad a_p(2) = 1.5$$

- From the upper equation, we will have

$$b_q(0) = x(0) = 1$$

- Then, the system function $(H(z))$ of the filter becomes:

$$H(z) = \frac{B_q(z)}{A_P(z)} = \frac{\sum_{k=0}^{q} b_q(k) z^{-k}}{1 + \sum_{k=0}^{p} a_p(k) z^{-k}} = \frac{b_q(0)}{1 + a_p(1) z^{-1} + a_p(2) z^{-2}}$$

$$= \frac{1}{1 - 1.5 z^{-1} + 1.5 z^{-2}}$$

Padé Approximation: Example

- From previous equation we have:

$$H(z) = 1 + 1.5H(z)z^{-1} - 1.5H(z)z^{-2}$$

- In time domain, the above equation becomes:

$$h(n) = x(n) = \delta(n) + 1.5h(n-1) - 1.5h(n-2)$$

- Using the above equation, the model produce the following approximation for $x(n)$:

$$\hat{\mathrm{X}} = [\underbrace{1,\ 1.5,\ 0.75}_{\text{Correctly model } x(n) \text{ for } n=0,1,2,..,q+p},\ \underbrace{-1.125,\ -2.8125,\ -2.5312}_{\text{Does not give guaranty for values of } x(n) \text{ for } n>q+p}]$$

Padé Approximation: Example

2. For second order all zero model with $(p=0, q=2)$, the following equations will be solved to find the filter/model parameters:

$$\begin{pmatrix} x(0) \\ x(1) \\ x(2) \end{pmatrix} = \begin{pmatrix} b_q(0) \\ b_q(1) \\ b_q(2) \end{pmatrix}$$

- From the above equation, we have:

$$b_q(0) = 1, \quad b_q(1) = 1.5, \quad b_q(2) = 0.75$$

- The system function of the filter becomes:

$$H(z) = 1 + 1.5z^{-1} + 0.75z^{-2}$$

Padé Approximation: Example

- In time domain, we have:

$$H(z)=\sum_{n=0}^{\infty} h(n)z^{-n} = h(0)+h(1)z^{-1}+h(2)z^{-2}+h(3)z^{-3}+\ldots$$
$$=1+1.5z^{-1}+0.75z^{-2}$$

- Hence, we have: $x(0)=h(0)=1,\ x(1)=h(1)=1.5,\ x(2)=h(2)=0.75$

- Using the above equation, the model produce the following approximation for $x(n)$:

$$\hat{X}=[\underbrace{1,\ 1.5,\ 0.75}_{\text{Correctly model } x(n) \text{ for } n=0,1,2,..,q+p},\ \underbrace{0,\ 0,\ 0}_{\text{Does not give guaranty for values of } x(n) \text{ for } n>q+p}]$$

Correctly model $x(n)$ for $n=0,1,2,..,q+p$

Does not give guaranty for values of $x(n)$ for $n>q+p$

Padé Approximation: Example

3. For pole- zero model with $(p=1, q=1)$, the following equations will be solved to find the filter/model parameters:

$$\begin{pmatrix} x(0) & 0 \\ x(1) & x(0) \\ x(2) & x(1) \end{pmatrix} \begin{pmatrix} 1 \\ a_p(1) \end{pmatrix} = \begin{pmatrix} b_q(0) \\ b_q(1) \\ 0 \end{pmatrix}$$

- $a_p(1)$ can be found from the last normal equation as

$$x(2) + a_p(1)x(1) = 0 \Rightarrow a_p(1) = -\frac{x(2)}{x(1)} = -\frac{0.75}{1.5} = -0.5$$

- Using the first two equation to find $b_q(0)$ and $b_q(1)$:

$$\begin{pmatrix} x(1) & x(0) \\ x(2) & x(1) \end{pmatrix} \begin{pmatrix} 1 \\ a_p(1) \end{pmatrix} = \begin{pmatrix} b_q(0) \\ b_q(1) \end{pmatrix}$$

Padé Approximation: Example

- Using the first two equation to find $b_q(0)$ and $b_q(1)$:

$$\begin{pmatrix} 1 & 0 \\ 1.5 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ -0.5 \end{pmatrix} = \begin{pmatrix} b_q(0) \\ b_q(1) \end{pmatrix}$$

- Using the above equation:

$$b_q(0) = 1, \quad b_q(1) = 1$$

- The system function of the filter becomes:

$$H(z) = \frac{1 + z^{-1}}{1 - 0.5z^{-1}}$$

- In time domain:

$$h(n) = \delta(n) + 1.5(0.5)^{n-1}u(n-1)$$

Time Shifted Unit step function

Padé Approximation: Example

- Using the previous equation, the model produce the following approximation for $x(n)$:

$$\hat{X} = [1,\ \underbrace{1.500,\ 0.750,\ 0.375,\ 0.1875,\ 0.0938}]$$

The model gives all of the exact signal values. However, this will not be the case in general. For $n > q + p$, sometimes the model may give good approximation and some times bad approximation. There is no guaranty!

Summary

- **Motivation For Parametric Signal Modeling:**
  - ✓ Signal compression for efficient transmission and storage
  - ✓ Capability of signal estimation and prediction
- **Deterministic Signal Modeling: Using Linear Shift Invariant Filter**
  - ✓ **Least Square/Direct Method**
    - ❖ Requires solving of non-linear equations
    - ❖ It is not mathematically tractable
    - ❖ Computationally intensive and not practical for real time applications
  - ✓ **Padé Approximation**
    - ❖ Produce an exact fit for data in the **[0,q+p]** interval
    - ❖ Does not guarantee for data values for **n>q+p**
    - ❖ The model generated is not stable

References

[1] Ramiro S. Barbosa, J. A. Tenreiro Machado, Isabel M. Ferreira,‘’ pole-zero approximations of digital fractional-order integrators and differentiators using signal modeling techniques’’, Institute of Engineering of Porto, Pp.3-4, 2005.

[2] Monson H. Hayes, *“Statistical Digital Signal Processing and Modeling”,* John Wiley and sons, Pp.139-140, 1996.

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Thank You!