



Statistical Digital Signal Processing

Week 7 Optimum Filters: IIR Wiener Filter

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Previous Topic (Week-6)

Optimum Filters: FIR Wiener Filter

- Basics of Optimum Filters
- Wiener Filter
- The FIR Wiener Filter
- Filtering Application
- Linear Prediction
- Noise Cancellation

Contents Here

Lecture Learning Outcomes

1. Explain the fundamental principles and applications of IIR Wiener filters in optimal signal estimation
2. Describe the formulation and characteristics of non-causal IIR Wiener filters, including their theoretical significance
3. Explain the design and operation of the IIR Wiener smoothing filter for noise reduction in signals
4. Analyze the performance of causal IIR Wiener filters
5. Evaluate the concept of causal linear prediction and its relationship to Wiener filtering in signal processing

Week 7: Optimum Filters: IIR Wiener Filter

Outline

- IIR Wiener Filters: Introduction
- Non-Causal IIR Wiener Filters
- The IIR Wiener Smoothing Filter
- Causal IIR Wiener Filters
- Causal Linear Prediction

IIR Wiener Filters: Introduction

- The problem formulation for FIR and IIR Wiener filter is basically similar
- However, there is a significant difference between FIR and IIR Wiener filters which results in different solutions
- In FIR Wiener filters the filters coefficient to be determined are finite while we have infinite filter coefficients to be determined in the IIR Wiener filter
- Hence, in IIR Wiener filter we need to find $h(n)$ for all of n
- In many applications, an IIR (recursive) filter is preferred over an FIR filter for performing linear operations since the design of IIR filter has fewer parameters [1]
- In this session, we will consider two classes of IIR Wiener filter:

- ❖ **Non-Causal IIR Wiener Filters**

- ❖ **Causal IIR Wiener Filters**

Non-Causal IIR Wiener Filters

- For non-causal or unconstrained IIR filter, the goal is to find the unit sample response, $h(n)$, of the IIR filter:

$$H(z) = \sum_{n=-\infty}^{\infty} h(n)z^{-n} \quad (1)$$

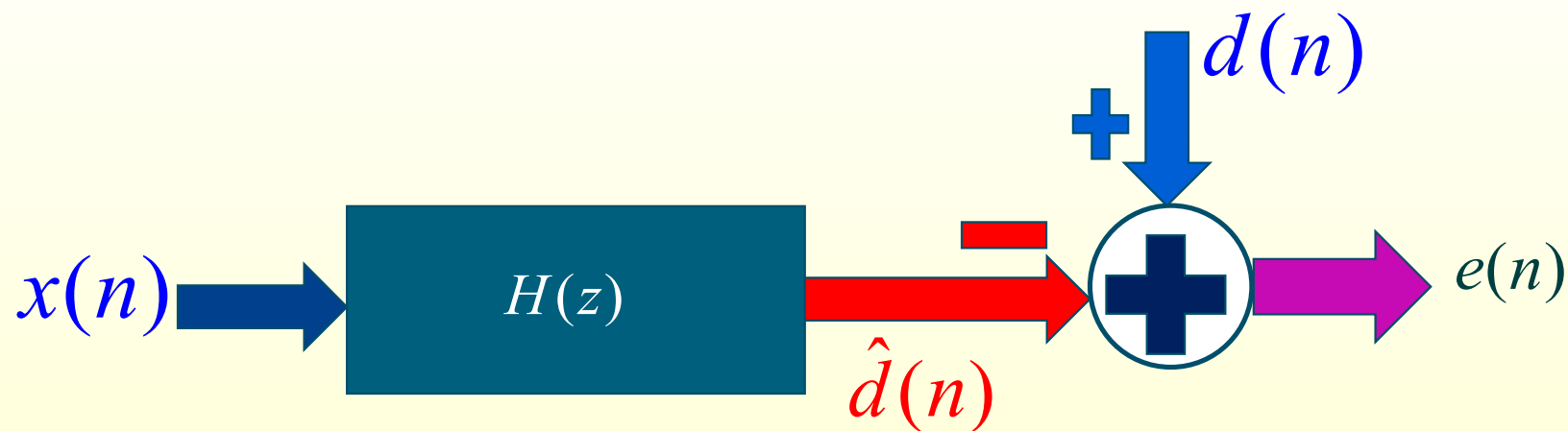


Figure 1: Illustration of linear estimation using non causal IIR filter

Non-Causal IIR Wiener Filters

- The filter should minimize the following mean square error:

$$\xi = E \left\{ |e(n)|^2 \right\} \quad (2)$$

- The estimation error $e(n)$ is given by:

$$e(n) = d(n) - \hat{d}(n) = d(n) - \sum_{l=-\infty}^{\infty} h(l)x(n-l) \quad (3)$$

- Differentiating ξ with respect to $h^*(k)$ similar to what we did for FIR Wiener filter to find the optimum filter parameter:

$$\begin{aligned} \frac{\partial \xi}{\partial h^*(k)} &= E \left\{ e(n) \frac{\partial e^*(n)}{\partial h^*(k)} \right\} \\ &= -E \left\{ e(n)x^*(n-k) \right\} = 0 ; \quad -\infty < k < \infty \quad (4) \end{aligned}$$

Orthogonality Principle

Non-Causal IIR Wiener Filters

- Substituting eq(3) for $e(n)$ in eq(4) and rearranging the terms:

$$\sum_{l=-\infty}^{\infty} h(l)E\{x(n-l)x^*(n-k)\} = E\{d(n)x^*(n-k)\} ; -\infty < k < \infty \quad (5)$$

- Writing eq(5) in terms of autocorrelation and cross correlation functions:

$$\sum_{l=-\infty}^{\infty} h(l)r_x(k-l) = r_{dx}(k) ; -\infty < k < \infty \quad (6)$$

- Eq(6) is **Wiener-Hopf Equations for IIR Wiener filter**, the difference is the **limit of the summation** and the **range of values** for the equation compared to the FIR Wiener-Hopf Equations
- Eq(6) represents an infinite set of linear equations with infinite number of unknowns

Non-Causal IIR Wiener Filters

- However, eq(6) can be solved easily by writing the left side of the equation as convolution between $h(k)$ and $r_x(k)$ as:

$$h(k) * r_x(k) = r_{dx}(k) \quad (7)$$

- Writing eq(7) in frequency domain, we will have:

$$H(e^{j\omega})P_x(e^{j\omega}) = P_{dx}(e^{j\omega}) \quad (8)$$

- Finding the frequency response, $H(e^{j\omega})$, of the non-causal IIR wiener filter from eq(8):

$$H(e^{j\omega}) = \frac{P_{dx}(e^{j\omega})}{P_x(e^{j\omega})} \quad (9)$$

Non-Causal IIR Wiener Filters

- Hence, the system function, $H(z)$, of the non-causal IIR Wiener can be written as

The diagram illustrates the derivation of the system function $H(z)$. It features a red box at the top labeled "Cross-Power Spectral Density" with a red arrow pointing down to the numerator $P_{dx}(z)$ of the fraction. Below the fraction is a blue box labeled "Power Spectral Density" with a blue arrow pointing up to the denominator $P_x(z)$. The entire equation is labeled (10) on the right.

$$H(z) = \frac{P_{dx}(z)}{P_x(z)} \quad (10)$$

- Once we found the system function of the filter, the mean square error, ξ_{\min} , can be found following similar steps we have done for the case of FIR Wiener filter

Non-Causal IIR Wiener Filters: Mean Square Error

- Following similar steps, ξ_{\min} , can be given by:

$$\xi_{\min} = r_d(0) - \sum_{l=-\infty}^{\infty} h(l)r_{dx}^*(l) \quad (11)$$

- Note:** eq(11) differ from the mean square error of FIR Wiener filter only by the limit of the summation
- Applying Parseval's theorem, ξ_{\min} , can be written using frequency domain as:

$$\xi_{\min} = r_d(0) - \frac{1}{2\pi} \int_{-\pi}^{\pi} H(e^{j\omega})P_{dx}^*(e^{j\omega})d\omega \quad (12)$$

- Writing the autocorrelation function as the inverse DTFT of power spectral density:

$$r_d(k) = \frac{1}{2\pi} \int_{-\pi}^{\pi} P_d(e^{j\omega})e^{jk\omega} d\omega \xrightarrow{\text{for } k=0} r_d(0) = \frac{1}{2\pi} \int_{-\pi}^{\pi} P_d(e^{j\omega})d\omega \quad (13)$$

Non-Causal IIR Wiener Filters: Mean Square Error

- Substituting eq(13) for $r_d(0)$ in eq(12), we have:

$$\begin{aligned}\xi_{\min} &= \frac{1}{2\pi} \int_{-\pi}^{\pi} P_d(e^{j\omega}) d\omega - \frac{1}{2\pi} \int_{-\pi}^{\pi} H(e^{j\omega}) P_{dx}^*(e^{j\omega}) d\omega \\ &= \frac{1}{2\pi} \int_{-\pi}^{\pi} \left[P_d(e^{j\omega}) - H(e^{j\omega}) P_{dx}^*(e^{j\omega}) \right] d\omega\end{aligned}\quad (14)$$

- Similarly the ξ_{\min} can also be expressed using z domain as:

$$\xi_{\min} = r_d(0) - \frac{1}{2\pi j} \oint_C H(z) P_{dx}^*(1/z^*) z^{-1} dz \quad (15)$$

- Writing the autocorrelation function as the inverse z transform of power spectral density:

$$r_d(k) = \frac{1}{2\pi j} \oint_C P_d(z) z^k \frac{dz}{z} \xrightarrow{\text{for } k=0} r_d(0) = \frac{1}{2\pi j} \oint_C P_d(z) z^{-1} dz \quad (16)$$

Non-Causal IIR Wiener Filters: Mean Square Error

- Using eq(15 & 16) ξ_{\min} can be written as follows:

$$\begin{aligned}\xi_{\min} &= \frac{1}{2\pi j} \oint_C P_d(z) z^{-1} dz - \frac{1}{2\pi j} \oint_C H(z) P_{dx}^*(1/z^*) z^{-1} dz \\ &= \frac{1}{2\pi j} \oint_C \left[P_d(z) - H(z) P_{dx}^*(1/z^*) \right] z^{-1} dz\end{aligned}\quad (17)$$

- The contour C in eq(17) can be taken as a unit circle since we are considering a stable system and the region of convergence includes the unit circle

The IIR Wiener Smoothing Filter

- One of the problem which can be addressed by Wiener filtering framework is **Smoothing**

Smoothing

- ❖ This is similar to the filtering problem except that the filter is allowed to be non-causal
 - ❖ The Wiener smoothing filter may be designed to estimate the desired signal from all available data of the noisy observation
- Therefore, the goal of the IIR Wiener smoothing filter is for producing minimum mean-square estimate of the desired signal $d(n)$ from the noisy observation $x(n)$ for all n , where $x(n)$ is given by:

$$x(n) = d(n) + v(n) \quad (18)$$

Where:

$v(n)$ Is a zero mean noise uncorrelated with $d(n)$

The Wiener Smoothing Filter

- To find the optimal Wiener smoothing filter transfer function, $H(e^{j\omega})$, what we required is finding $P_x(e^{j\omega})$ and $P_{dx}(e^{j\omega})$ since $H(e^{j\omega}) = P_{dx}(e^{j\omega}) / P_x(e^{j\omega})$ according to eq(9)
- The autocorrelation of $x(n)$ is given as:

$$\begin{aligned} r_x(k) &= E \left\{ x(n)x^*(n-k) \right\} \\ &= E \left\{ [d(n) + v(n)][d(n-k) + v(n-k)]^* \right\} \\ &= r_d(k) + r_v(k) \end{aligned} \quad (19)$$

- Then $P_x(e^{j\omega})$ becomes:

$$P_x(e^{j\omega}) = P_d(e^{j\omega}) + P_v(e^{j\omega}) \quad (20)$$

The IIR Wiener Smoothing Filter

- Similarly, the cross correlation $r_{dx}(k)$ is given by:

$$\begin{aligned} r_{dx}(k) &= E \left\{ d(n) x^*(n-k) \right\} \\ &= E \left\{ d(n) [d(n-k) + v(n-k)]^* \right\} \\ &= r_d(k) \end{aligned} \quad (21)$$

- Hence, from eq(21), we can have:

$$P_{dx}(e^{j\omega}) = P_d(e^{j\omega}) \quad (22)$$

- Using eq(9, 20, and 22), The IIR Wiener smoothing filter becomes:

$$H(e^{j\omega}) = \frac{P_{dx}(e^{j\omega})}{P_x(e^{j\omega})} = \frac{P_d(e^{j\omega})}{P_d(e^{j\omega}) + P_v(e^{j\omega})} \quad (23)$$

The IIR Wiener Smoothing Filter: Mean Square Error

- Similarly, using eq (14, 20, and 22) the minimum mean square error becomes:

$$\begin{aligned}\xi_{\min} &= \frac{1}{2\pi} \int_{-\pi}^{\pi} \left[P_d(e^{j\omega}) - H(e^{j\omega}) P_{dx}^*(e^{j\omega}) \right] d\omega \\ &= \frac{1}{2\pi} \int_{-\pi}^{\pi} \left[P_d(e^{j\omega}) - H(e^{j\omega}) P_d^*(e^{j\omega}) \right] d\omega\end{aligned}\quad (24)$$

- Since power spectrum is real and non negative, $P_d^*(e^{j\omega}) = P_d(e^{j\omega})$
- Therefore, using eq(23), eq(24) boils down to:

$$\begin{aligned}\xi_{\min} &= \frac{1}{2\pi} \int_{-\pi}^{\pi} P_d(e^{j\omega}) \left[1 - H(e^{j\omega}) \right] d\omega \\ &= \frac{1}{2\pi} \int_{-\pi}^{\pi} P_d(e^{j\omega}) \left[1 - \frac{P_d(e^{j\omega})}{P_d(e^{j\omega}) + P_v(e^{j\omega})} \right] d\omega\end{aligned}$$

The IIR Wiener Smoothing Filter: Mean Square Error

$$\begin{aligned} &= \frac{1}{2\pi} \int_{-\pi}^{\pi} P_d(e^{j\omega}) \left[\frac{P_v(e^{j\omega})}{P_d(e^{j\omega}) + P_v(e^{j\omega})} \right] d\omega \\ &= \frac{1}{2\pi} \int_{-\pi}^{\pi} P_v(e^{j\omega}) \left[\frac{P_d(e^{j\omega})}{P_d(e^{j\omega}) + P_v(e^{j\omega})} \right] d\omega \\ &= \frac{1}{2\pi} \int_{-\pi}^{\pi} P_v(e^{j\omega}) H(e^{j\omega}) d\omega \end{aligned} \quad (25)$$

- Alternatively expressing in z domain, we will have:

$$\xi_{\min} = \frac{1}{2\pi j} \oint_C P_v(z) H(z) z^{-1} dz \quad (26)$$

Causal IIR Wiener Filters

- So far we have seen unconstrained type of IIR Wiener filter
- Now, we will put a constraint on the IIR Wiener filter to be causal
- Hence, the unit sample response, $h(n)$, of the filter is equal to zero for $n < 0$
- In case of causal IIR Wiener filter, the estimate of the desired signal can be written as:

$$\hat{d}(n) = h(n) * x(n) = \sum_{l=0}^{\infty} h(l)x(n-l) \quad (27)$$

- Similar to what we have seen for non-causal IIR Wiener filter case, the causal IIR Wiener filter should also minimize:

$$\xi = E \left\{ |e(n)|^2 \right\} \quad (28)$$

Where:

$$e(n) = d(n) - \hat{d}(n) = d(n) - \sum_{l=0}^{\infty} h(l)x(n-l) \quad (29)$$

Causal IIR Wiener Filters

- Taking the partial derivative of ξ with respect to $h^*(k)$ for $k \geq 0$ to obtain the optimal filter parameter, we will get the following equation:

$$\sum_{l=0}^{\infty} h(l)r_x(k-l) = r_{dx}(k) \ ; \ 0 \leq k < \infty \quad (30)$$



Wiener-Hopf Equations for Causal IIR Wiener Filter

- In eq (30), the constraint we put on k , i.e. $k \geq 0$, will hinder us to write $r_{dx}(k)$ in terms of the convolution of $h(k)$ and $x(k)$
- Therefore, solving eq(30) is not straight forward in contrary to what we have seen for the case non causal IIR filter
- To solve eq(30), let's consider a special case that the input of the Wiener filter is a unit variance white noise, $\varepsilon(n)$ [2]

Causal IIR Wiener Filters

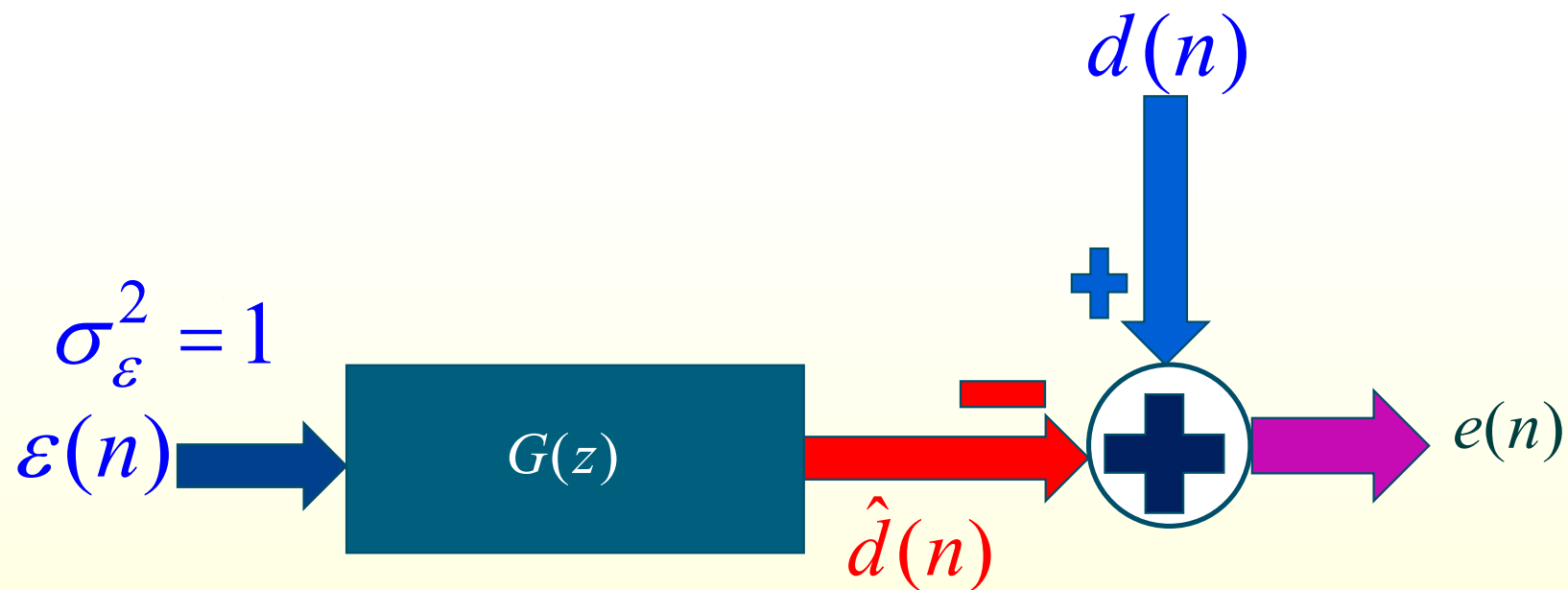


Figure 2: Unit variance white noise input to the Wiener filter

Causal IIR Wiener Filters

- Representing the coefficient of the causal IIR Wiener filter by $g(n)$, the Wiener-Hopf equation when the input is $\varepsilon(n)$ becomes:

$$\sum_{l=0}^{\infty} g(l)r_{\varepsilon}(k-l) = r_{d\varepsilon}(k) ; 0 \leq k < \infty \quad (31)$$

- But from the property of white noise, we have:

$$r_{\varepsilon}(k-l) = \begin{cases} \sigma_{\varepsilon}^2 = 1 & ; \text{for } l = k \\ 0 & ; \text{otherwise} \end{cases} \quad (32)$$

- Using eq(32), we will obtain the following from eq(31):

$$g(k) = r_{d\varepsilon}(k) ; 0 \leq k < \infty \quad (33)$$


- Since $g(k) = 0$ for $k < 0$, eq(31) can be written in the following form

$$g(k) = r_{d\varepsilon}(k)u(k) \quad (34)$$

 Unit step function

Causal IIR Wiener Filters

- Taking the z transform of eq(34), we have :

$$\begin{aligned} G(z) &= \sum_{k=-\infty}^{\infty} g(k)z^{-k} = \sum_{k=-\infty}^{\infty} r_{d\varepsilon}(k)u(k)z^{-k} \\ &= \sum_{k=0}^{\infty} r_{d\varepsilon}(k)z^{-k} \\ &= [P_{d\varepsilon}(z)]_+ \end{aligned} \quad (35)$$


The causal part of the cross power spectrum

- However, It is unlikely that the input of the Causal IIR Wiener filters be a white noise
- Therefore, let's assume $x(n)$ is a random process with rational power spectrum that has no poles or zeros on the unit circle

Causal IIR Wiener Filters

- Then, the power spectrum, $P_x(z)$, can be written as follows using spectral factorization :

$$P_x(z) = \sigma_0^2 Q(z) Q^*(1/z^*) \quad (36)$$

Where: $Q(z)$ is **minimum phase monic polynomial** and given by

$$Q(z) = 1 + q(1)z^{-1} + q(2)z^{-2} + \dots = \frac{N(z)}{D(z)} \quad (37)$$

- In eq(37), $N(z)$ and $D(z)$, are minimum phase monic polynomials
- If $x(n)$ is filtered by a filter having a system function, $F(z)$, the output signal will be $\varepsilon(n)$

$$F(z) = \frac{1}{\sigma_0 Q(z)} \quad (38)$$

Causal IIR Wiener Filters

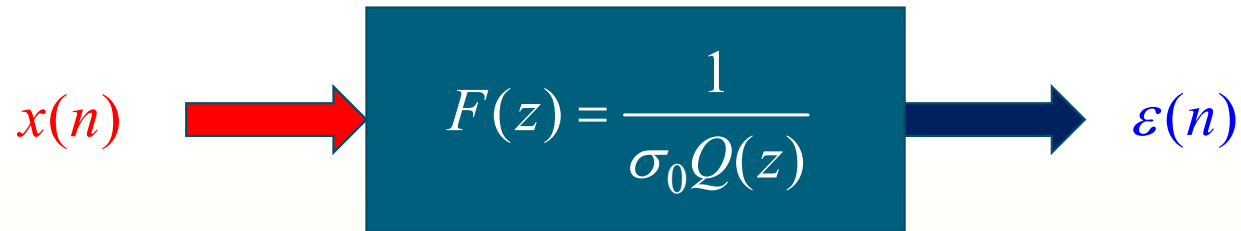


Figure 3: Whitening Filter

- The power spectrum of the out put signal can be calculated as:

$$\begin{aligned} P_{\varepsilon}(z) &= P_x(z)F(z)F^*(1/z^*) \\ &= \left[\sigma_0^2 Q(z)Q^*(1/z^*) \right] \left[\frac{1}{\sigma_0 Q(z)} \right] \left[\frac{1}{\sigma_0 Q^*(1/z^*)} \right] = 1 \end{aligned} \quad (39)$$

- The filter $F(z)$ changes $x(n)$ to unity variance white noise and the filter is called a **Whitening Filter** [2]

Causal IIR Wiener Filters

- It is also noted that $x(n)$ can be recovered by filtering $\varepsilon(n)$ using a filter having a system function of $F^{-1}(z)$

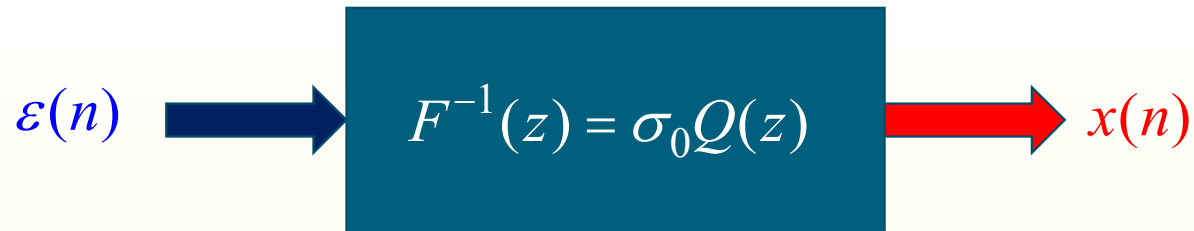


Figure 4: Recovering $x(n)$ by filtering $\varepsilon(n)$

- The power spectrum of the output signal can be calculated

$$\begin{aligned} P_x(z) &= P_\varepsilon(z) F^{-1}(z) \left[F^{-1}(1/z^*) \right]^* \\ &= 1^* \left[\sigma_0 Q(z) \right] \left[\sigma_0 Q^*(1/z^*) \right] = \boxed{\sigma_0^2 Q(z) Q^*(1/z^*)} \quad (40) \end{aligned}$$

- It is noted that the result of eq(40) is similar to eq(36)

Causal IIR Wiener Filters

- Now let's find the optimum causal IIR Wiener filter, $H(z)$, which estimates $d(n)$ from the observation $x(n)$
- Let $x(n)$ is filtered by three cascaded filters having system function of $F(z)$, $F^{-1}(z)$, and $H(z)$ as shown in the figure below

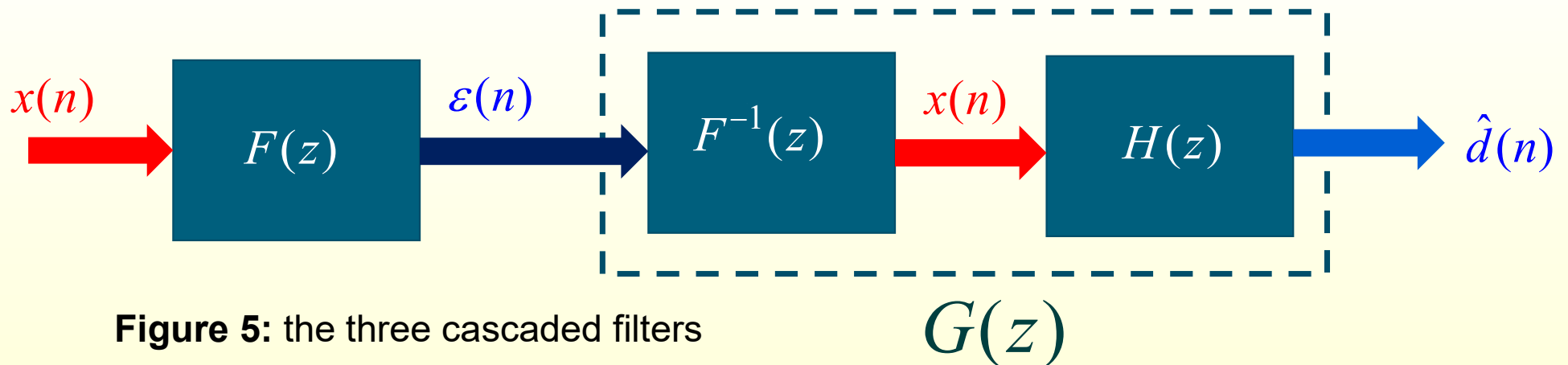


Figure 5: the three cascaded filters

Where:

$$G(z) = F^{-1}(z)H(z) \quad (41)$$

Causal IIR Wiener Filters

- From eq(35), we have already proved that, when the input of the causal IIR filter having unit sample response of $g(n)$ is $\varepsilon(n)$, the system function of the filter $G(z)$ becomes:

$$G(z) = [P_{d\varepsilon}(z)]_+ \quad (42)$$

- To find $P_{d\varepsilon}(z)$, first lets calculate $r_{d\varepsilon}(k)$ as follows:

$$r_{d\varepsilon}(k) = E \left\{ d(n) \varepsilon^*(n-k) \right\} \quad (43)$$

- From input-output relation of the first filter we have:

$$\begin{aligned} \varepsilon(n) &= f(n) * x(n) \\ &= \sum_{l=-\infty}^{\infty} f(l)x(n-l) \end{aligned} \quad (44)$$

Causal IIR Wiener Filters

- Using eq(44), we will have

$$\varepsilon^*(n-k) = \sum_{l=-\infty}^{\infty} f^*(l)x^*(n-k-l) \quad (45)$$

- Then, using eq(43 & 45), we will have

$$\begin{aligned} r_{d\varepsilon}(k) &= E\left\{d(n)\varepsilon^*(n-k)\right\} \\ &= E\left\{d(n)\left[\sum_{l=-\infty}^{\infty} f^*(l)x^*(n-k-l)\right]\right\} = \sum_{l=-\infty}^{\infty} f^*(l)E\left\{d(n)x^*(n-k-l)\right\} \\ &= \sum_{l=-\infty}^{\infty} f^*(l)r_{dx}(k+l) \\ &= f^*(-k) * r_{dx}(k) \end{aligned} \quad (46)$$

Causal IIR Wiener Filters

- Then we can find $P_{d\varepsilon}(z)$ by taking the z transform of eq(46):

$$\begin{aligned}
 P_{d\varepsilon}(z) &= \mathbf{z} \left\{ f^*(-k) * r_{dx}(k) \right\} = F^*(1/z^*) P_{dx}(z) = \left[\frac{1}{\sigma_0 Q^*(1/z^*)} \right] P_{dx}(z) \\
 &= \frac{P_{dx}(z)}{\sigma_0 Q^*(1/z^*)} \quad (47)
 \end{aligned}$$

- Then from (42 & 47), the causal IIR Wiener filter estimating $d(n)$ from $\varepsilon(n)$ becomes

$$G(z) = [P_{d\varepsilon}(z)]_+ = \frac{1}{\sigma_0} \left[\frac{P_{dx}(z)}{Q^*(1/z^*)} \right]_+ \quad (48)$$

- Then from (41 & 48), we have:

$$H(z) = F(z)G(z) = \left[\frac{1}{\sigma_0 Q(z)} \right] \left[\frac{1}{\sigma_0} \left[\frac{P_{dx}(z)}{Q^*(1/z^*)} \right]_+ \right] = \boxed{\frac{1}{\sigma_0^2 Q(z)} \left[\frac{P_{dx}(z)}{Q^*(1/z^*)} \right]_+} \quad (49)$$

Causal IIR Wiener Filters: Mean Square Error

- ξ_{\min} can be written as follows:

$$\xi_{\min} = \frac{1}{2\pi} \int_{-\pi}^{\pi} \left[P_d(e^{j\omega}) - H(e^{j\omega})P_{dx}^*(e^{j\omega}) \right] d\omega \quad (50)$$

- Or equivalently,

$$\xi_{\min} = \frac{1}{2\pi j} \oint_C \left[P_d(z) - H(z)P_{dx}^*(1/z^*) \right] z^{-1} dz \quad (51)$$

Causal Linear Prediction

- So far we have seen linear estimation problem using causal IIR Wiener filter, Now we will proceed to the causal linear prediction analysis

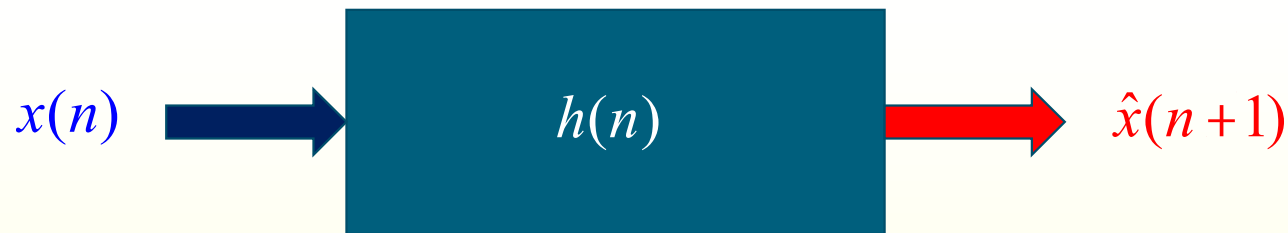


Figure 6: Linear prediction using causal IIR wiener filter

- The output of the causal linear predictor is given by:

$$\hat{x}(n+1) = \sum_{k=0}^{\infty} h(k)x(n-k) \quad ; \quad k \leq n \quad (52)$$

- By setting $d(n) = x(n+1)$, the cross correlation, $r_{dx}(k)$, can be:

$$r_{dx}(k) = E \left\{ d(n)x^*(n-k) \right\} = E \left\{ x(n+1)x^*(n-k) \right\} = r_x(k+1) \quad (53)$$

Causal Linear Prediction

- Taking the z transform of eq(51) $P_{dx}(z)$ can be found as:

$$\begin{aligned} P_{dx}(z) &= \mathbf{z}\{r_{dx}(k)\} = \mathbf{z}\{r_x(k+1)\} \\ &= zP_x(z) \end{aligned} \quad (54)$$

- Using eq(49 & 54), the causal linear predictor is given by:

$$H(z) = \frac{1}{\sigma_0^2 Q(z)} \left[\frac{P_{dx}(z)}{Q^*(1/z^*)} \right]_+ = \frac{1}{\sigma_0^2 Q(z)} \left[\frac{zP_x(z)}{Q^*(1/z^*)} \right]_+ \quad (55)$$

- Recalling $P_x(z) = \sigma_0^2 Q(z) Q^*(1/z^*)$, eq(55) will be simplified to:

$$H(z) = \frac{1}{\sigma_0^2 Q(z)} \left[z \left(\frac{\sigma_0^2 Q(z) Q^*(1/z^*)}{Q^*(1/z^*)} \right) \right]_+ = \frac{1}{Q(z)} [zQ(z)]_+ \quad (56)$$

Causal Linear Prediction

- Recalling that $Q(z)$ is a monic polynomial having the form:

$$Q(z) = 1 + q(1)z^{-1} + q(2)z^{-2} + q(3)z^{-3} + \dots \quad (57)$$

- Then the term $[zQ(z)]_+$ in eq(56) simplified to:

$$\begin{aligned} [zQ(z)]_+ &= [z + q(1) + q(2)z^{-1} + q(3)z^{-2} + q(4)z^{-3} + \dots]_+ \\ &= q(1) + q(2)z^{-1} + q(3)z^{-2} + q(4)z^{-3} + \dots \\ &= z[Q(z) - 1] \end{aligned} \quad (58)$$

- Hence, from eq(56 & 58), we have:

$$H(z) = \frac{1}{Q(z)} [zQ(z)]_+ = \frac{1}{Q(z)} z[Q(z) - 1] = z \left[1 - \frac{1}{Q(z)} \right] \quad (59)$$

Causal Linear Prediction: Mean square Error

- For the minimum mean square error, ξ_{\min} , we have had:

$$\xi_{\min} = \frac{1}{2\pi j} \oint_C \left[P_d(z) - H(z)P_{dx}^*(1/z^*) \right] z^{-1} dz \quad (60)$$

- But it is noted that $P_d(z) = P_x(z)$ and $P_{dx}(z) = zP_x(z)$, eq(60) simplified to:

$$\xi_{\min} = \frac{1}{2\pi j} \oint_C \left[P_x(z) - H(z)z^{-1}P_x^*(1/z^*) \right] z^{-1} dz \quad (61)$$

- Using the symmetry property of power spectrum, $P_x(z) = P_x^*(1/z^*)$:

$$\xi_{\min} = \frac{1}{2\pi j} \oint_C P_x(z) \left[1 - H(z)z^{-1} \right] z^{-1} dz \quad (62)$$

Summary

- **IIR Wiener Filter:**
 - ✓ The filter's coefficient to be determined are infinite
 - ✓ The design of IIR filter has fewer parameters compared to FIR filters
- **Non Causal IIR Weiner Filter**
 - ✓ No constraint imposed on the filter
 - ✓ **Application:** smoothing using Wiener smoothing filter
- **Causal IIR Weiner Filter**
 - ✓ Constraint imposed on the filter to be causal
 - ✓ **Applications:** Linear Causal Prediction and Estimation/Filtering

References

- [1] Charles W. Therrien, "*Discrete Random Signals and Statistical Signal Processing*", Prentice Hall, Pp.356, 1992.
- [2] Monson H. Hayes, "*Statistical Digital Signal Processing and Modeling*", John Wiley and sons, Pp.358-359, 1996.

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Thank You!