# Power System Quality and Reliability

# ECEg-6312

## WEEK 9

## Introduction to Power System Reliability

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**May 2026**

# Topic Overview

- This lecture introduces the fundamental concepts and analytical tools used in power system reliability assessment for evaluation of **generation**, **transmission**, and **distribution** systems:
  1. Fundamental Definition
  2. Probabilistic Modeling of Failures/Exponential distributions
  3. Reliability Metrics: Mean Time To Failure (MTTF), Mean Time To Repair (MTTR), Availability and Unavailability
  4. System Reliability Structures: Series and parallel system
  5. MARKOV process modeling and recursive technique.

# Learning Outcomes

**By the end of this lesson, students will be able to:**

- Define power system reliability and its components
- Model failure behavior using exponential distribution
- Compute Mean Time To Failure (MTTF) and Mean Time To Repair (MTTR)
- Analyze series and parallel system reliability
- Understand Markov processes in reliability modeling
- Apply recursive techniques for system evaluation

# 1. Introduction

- Electrical power systems are trandformed from conventional centralized structures to complex, distributed, and intelligent networks, characterized by:
  - **Highly interconnected grids** (regional and cross-border interconnections) to enable:
    - Power exchange between countries/regions
    - Load sharing and generation balancing
    - Improved utilization of energy resources
  - **Dynamic behavior** due to fluctuating operating conditions driven by continuous changes in both generation and load.
  - **Stochastic nature** arising from uncertain generation and load patterns

# Key Drivers of System Transformation

- **Integration of Renewable Energy Sources (RES):**
  - Intermittent and weather-dependent renewable energy sources (solar, wind)
  - Introduces uncertainty and variability
- **Growth of Microgrid and Distributed Generation (DG):**
  - Decentralized generation near load centers
  - Bidirectional power flow and operational complexity
- **Increased Load Demand and Sensitivity:**
  - Rapid urbanization and industrialization
  - Emergence of sensitive loads (data centers, EV charging, automation systems)

# 2. What is Power System Reliability?

- **Power System Reliability is the:**
  - Probability that a power system performs its intended function adequately, supplying continuous electrical power, for a specified period under stated operating conditions.
- **Key Elements of the Definition:**
  - **Probability-based** → inherently stochastic (uncertainty in failures)
  - **Intended function** → generation, transmission, and distribution of power
  - **Specified time** → reliability is time-dependent
  - **Operating conditions** → normal and contingency states

# 3. Reliability Components

- Power system reliability is broadly divided into two key components:

**1. Adequacy**

- The ability of the power system to supply the **aggregate** electrical **demand** and **energy** requirements at all times.
- **Key Characteristics:**
  - Focuses on steady-state conditions
  - Concerned with capacity sufficiency
- **Evaluated using:** Generation availability, Transmission capability, and Load demand profile

# Cont’d...

- **Typical Issues:**
  - Generation **shortages**, Transmission **congestion**, and Long-term **planning deficiencies**

**2. Security**

- The ability of the power system to **withstand sudden disturbances** without **loss of service** or instability.
- **Key Characteristics:**
  - Focuses on dynamic and transient conditions
  - Concerned with system response to disturbances
- **Examples of Disturbances:**
  - Short-circuit faults, Sudden loss of generation, and Line outages

# 4. Probabilistic Modeling of Failures

- Failures of power system components (transformers, lines, breakers) are random in nature and no clear aging trend is dominant due to:
  - Variable loading and switching operation
  - Environmental effects (temperature, lightning, wind)
  - Material imperfections
- This behavior is well approximated by a constant failure rate, which directly leads to the exponential model.
- Therefore, model time-to-failure ***T*** as a random variable to quantify risk and reliability.

# Cont'd...

- For an exponential probablistic model of risk and reliability, the failure rate is assumed to be constant.

$$h(t) = \lambda$$

- Then the reliability function becomes:

$$R(t) = e^{-\lambda . t}$$

- Failures occur independently and randomly over time.
- Valid for the useful life period (no infant mortality or wear-out).

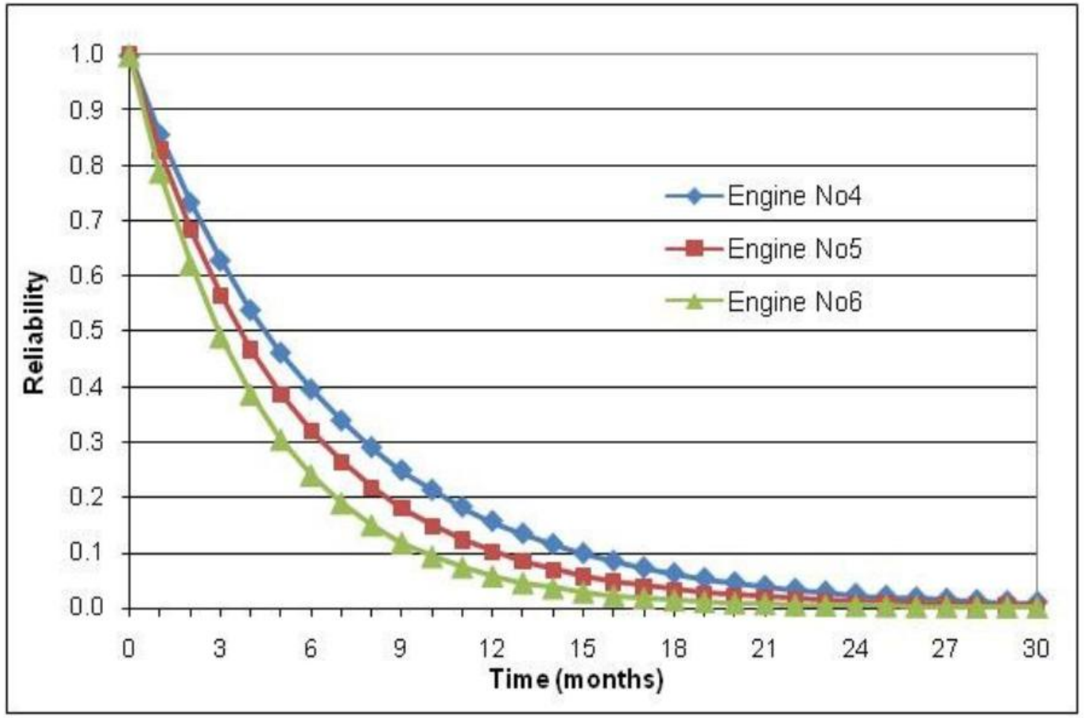


**Figure 1:** Component Reliability Curve [1].

# Probability Density Function (PDF)

- The Probability Density Function (PDF) describes the likelihood that a continuous random variable (time-to-failure T) takes a specific value at time t.
- In reliability analysis, it represents the rate at which failures occur over time.
- The Exponential Distribution PDF:

$$f(t) = \lambda e^{-\lambda . t}, \; t > 0$$

- $f(t)$ have Key Properties like $f(t) \geq 0$ for all *t≥0 and the t*otal probability is unity:

$$Total\ PDF = \int_0^{\infty} f(t)dt = 1$$

- High value of f(t) → higher likelihood of failure at time t

# Cont'd...

- The Relationship of Exponential PDF $f(t)$ with Reliability Function, $R(t)$ for a time-to-failure random variable T:
  - PDF: f(t) → likelihood of failure at time t
  - Reliability: R(t)=P(T>t) → survival probability up to time t
- Reliability is the complement of the cumulative failure probability:

$$R(t) = 1 - F(t)$$

$$F(t) = \int_0^t f(\tau)d\tau$$

- **PDF** → describes when failures occur while **Reliability** → describes how long the system survives.

# Cont'd…

- Therefore:

$$R(t) = \int_t^{\infty} f(\tau)d\tau$$

- Substitute Exponential PDF to the above expression:

$$R(t) = \int_t^{\infty} \lambda e^{-\lambda.\tau} d\tau$$

$$R(t) = e^{-\lambda.t}$$

- The reliability function is directly derived from the PDF
- The exponential PDF leads to an exponentially decaying survival probability

# 5. Reliability Terminologies

- **Reliable system assessment** is built on a set of fundamental probabilistic and statistical terms that describe component behavior over time.
- **Includes:**
  - **Failure Rate (λ):** Number of failures per unit time
  - **Repair Rate (μ):** Number of repairs per unit time
  - **Reliability Function R(t):** Probability system survives up to time t
  - **Unreliability Q(t):** Probability of failure
  - **Hazard Rate h(t):** Failure Intensity

# A. Failure Rate (λ)

- **Failure rate (λ)** is the expected number of failures per unit time for a component or system under specified operating conditions.
- It quantifies how frequently a component fails during its **useful operating life**.
- **Mathematical Expression:**

$$\lambda = \frac{Number\ of\ Failures}{Total\ Operating\ Time}$$

- **High λ** → less reliable component (frequent failures)
- **Low λ** → more reliable component (rare failures)
- It is **measured** using **failures/hour** or **failures/year** (commonly used in power systems studies).

# Cont'd

- For constant failure rate, the reliability of the system is expressed using an Exponential Model:

$$R(t) = e^{-\lambda . t}$$

- This shows that **reliability** decreases↓ exponentially as **time** increases↑.
- From Engineering Insight, **Failure rate** is assumed constant in the **useful life period** of many power system components such as:
  - Transmission lines, Transformers, and Circuit breakers
- **It is a key input for:**
  - Reliability indices (SAIFI, SAIDI), Markov models, and System availability studies

# B. Repair Rate (μ)

- **Repair rate (μ)** is the expected number of successful repairs completed per unit time after a system or component failure.
- It represents the **speed of system restoration** to an operational state.
- **Mathematical Expression:**

$$\mu = \frac{Number\ of\ Repairs}{Total\ Repair\ Time}$$

- **High μ** → fast restoration (high maintainability)
- **Low μ** → slow repair process (poor maintainability)

# Cont'd..

- It is quantified using repairs/hour, repairs/day or repairs/year (depending on system scale).
- Reliability Relationship:
  - Repair rate is directly linked to Mean Time To Repair (MTTR):

$$MTTR = \frac{1}{\mu}$$

- μ captures the maintenance efficiency of the power system which depends on:
  - Availability of spare parts, Skilled maintenance crew,
  - Accessibility of equipment, and Fault diagnosis time

# Cont'd

- **Repare rate has a significant role** in overall system performance and it strongly influences:
  - System availability
  - Outage duration
  - Service restoration speed
- **In Power Systems Context Critical for:**
  - Distribution network restoration
  - Substation maintenance planning
  - Reliability-centered maintenance (RCM)

# C. Unreliability Function Q(t)

- **The unreliability function Q(t)** represents the probability that a component or system fails before or at time t and measures **risk of system failure** over time.
- It is also referred to as the failure probability function.
- **Mathematical Formulation:**

$$Q(t) = P(T < t)$$

- **Reliability and unreliability** are complementary events:

$$Q(t) = 1 - R(t)$$

$$Q(t) = 1 - e^{-\lambda . t}$$

# D. Hazard Rate (Failure Intensity)

- **The hazard rate h(t)**, also called **failure intensity**, is the instantaneous failure rate of a component at time t, given that it has survived up to that time.
- It represents the conditional **probability of failure** in the **next instant**.
- Mathematical Formulation:

$$h(t) = \lim_{\Delta t \to 0} \frac{P(t \le T < t + \Delta t | T \ge t)}{\Delta t}$$

- Using probability density function (PDF) and reliability function:

$$h(t) = \frac{f(t)}{R(t)}$$

# Example

1. A distribution transformer supplying a critical load has the following reliability parameters:
   - Failure rate: **$\lambda$=0.02** failures/year
   - Repair rate: **$\mu$=2** repairs/year
   - Assume **exponential** failure distribution
   - Mission time: **t =1** year
2. Determine the following:
   - Reliability of the transformer at t =1 year and Unreliability of the transformer
   - Mean Time To Failure (MTTF) and Mean Time To Repair (MTTR)
   - Availability of the transformer

# Solution

**(a). Reliability of the transformer at t=1 year:** Using exponential reliability model

$$R(t) = e^{-\lambda . t},\ R(1) = e^{-0.02*1} = 0.9802$$

- Probability transformer survives 1 year ≈ 98.02%.
- Failure probability ≈ 1.98%.

**(b). Unreliability of the transformer:**

$$Q(t) = 1 - R(t) = 1 - 0.9802 = 0.0198$$

- 1.98% chance of failure in one year.

# Cont'd...

## (c) Mean Time To Failure (MTTF):

$$\text{MTTF} = \frac{1}{\lambda} = \frac{1}{0.02} = 50\ years$$

- On average, transformer fails every 50 years

## (d). Mean Time To Repair (MTTR):

$$\text{MTTR} = \frac{1}{\mu} = \frac{1}{2} = 0.5 years = 6\ months$$

## (e). Availability of Transformer (A):

$$A = \frac{MTTF}{MTTF + MTTR} = \frac{50}{50 + 0.5} = 0.9901$$

- Transformer is available 99.01% of the time

# 6. Analyze Series and Parallel System Reliability

- Power systems are composed of multiple interconnected components such as:
  - Generators
  - Transmission lines
  - Transformers
  - Protection and switching devices
- The overall system reliability depends not only on individual components but also on how they are structurally connected.

# Cont'd...

**Why System-Level Reliability Analysis?**

- Individual component reliability is not sufficient because:
  - Even highly reliable components can form an unreliable system in series
  - Redundancy can significantly improve system performance
- Two Fundamental System Structures

  **1. Series Configuration:** All components must operate for system success

  - Failure of any component → system failure

  **2. Parallel Configuration:** System operates if at least one component works

  - Failure occurs only if all components fail

# Cont'd...

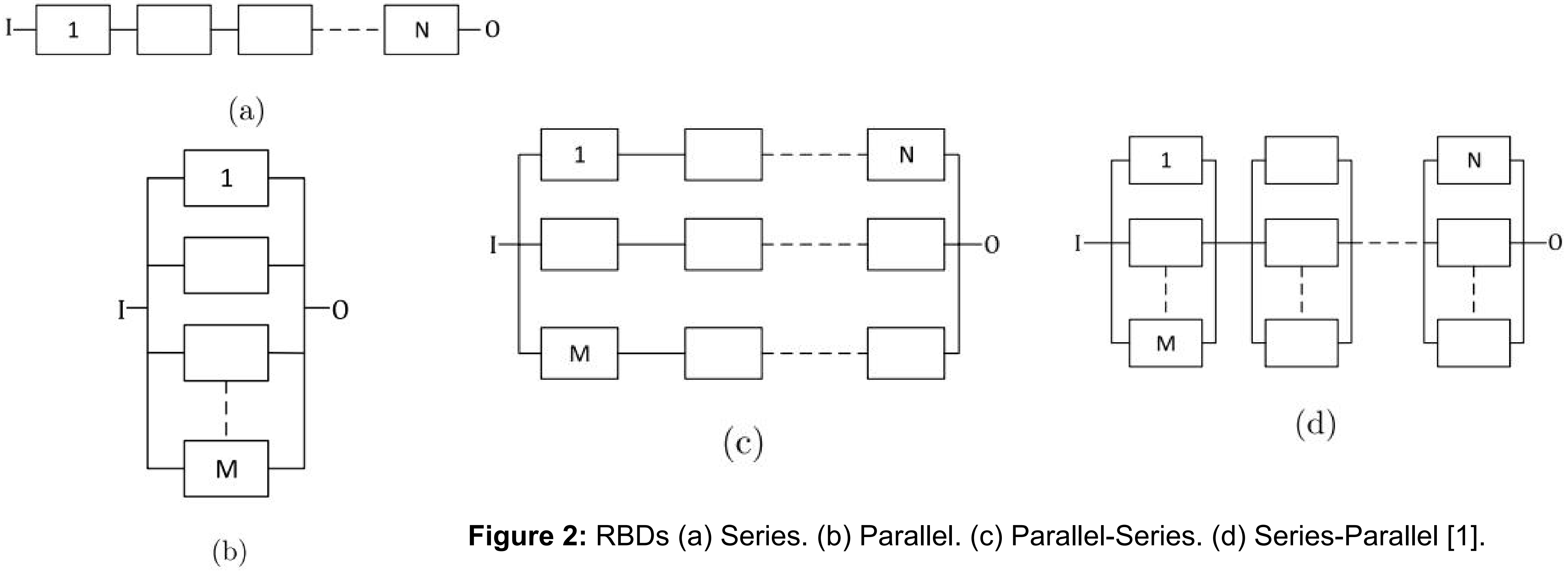


**Figure 2:** RBDs (a) Series. (b) Parallel. (c) Parallel-Series. (d) Series-Parallel [1].

# A. Series Configuration

- A system is in series configuration when:
  - All components must operate successfully for the system to function
  - Failure of any single component causes system failure
- Reliability Expression:

$$R_s = \prod_{i=1}^{n} R_i$$

- For components modeled with exponential distribution:

$$R_s = e^{-(\lambda_1+\lambda_2+\lambda_3+\ldots.+\lambda_n)t}$$

$$\lambda_{eq} = \lambda_1 + \lambda_2 + \lambda_3 + \ldots. + \lambda_n$$

# Cont'd...

- Series systems **accumulate** failure rates.
- Characteristics of Series Configuration:
  - Reliability **decreases** as **number of components** increases
  - Dominated by the **weakest component** (highest failure rate)
  - No redundancy → high vulnerability

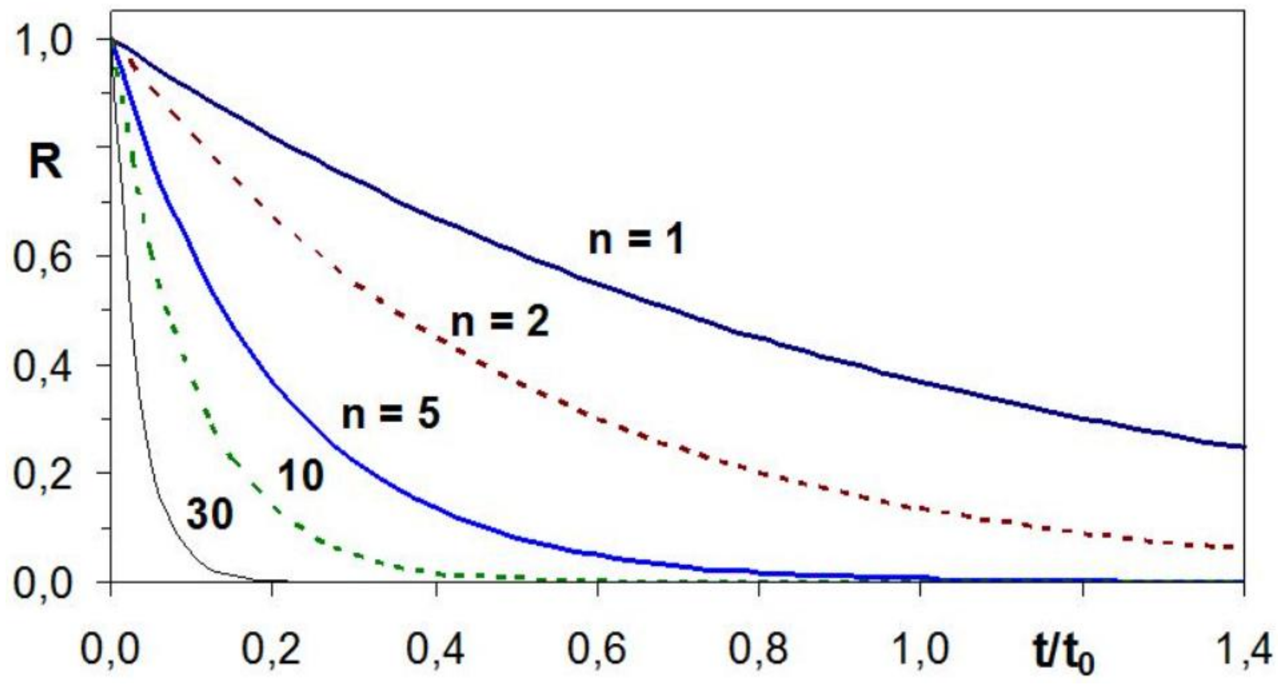


**Figure 3:** Series system reliability curve for various number of elements [3].

# Example

- A radial distribution feeder supplying a critical load consists of two components connected in series:
    1. **Line section:** Failure rate **$\lambda 1=0.1$** failures/year and Repair rate **$\mu 2=5$** repairs/year
    2. **Transformer**: Failure rate **$\lambda 2=0.02$** failures/year and Repair rate **$\mu 2=2$** repairs/year
- **Assume:** Exponential failure and repair distributions and Mission time t=1 year.
    1. Write the probability density function (PDF) for: a). the line b). the transformer.
    2. Determine the reliability function of: a). The line b). The transformer. Evaluate both at t=1 year.
    3. Determine the equivalent system failure rate $\lambda eq$ for the series system.
    4. Compute the system reliability at t=1 year.

# Cont'd...

5. Determine the system probability density function (PDF).
6. Calculate the Mean Time To Failure (MTTF) of the system.
7. Estimate the equivalent repair rate $\mu_{eq}$ of the system and compute the Mean Time To Repair (MTTR).
8. Determine the availability of the system.
9. Based on your results:
    - Identify the weakest component
    - Suggest one engineering solution to improve system reliability

# Solution

**Given:**

- **Line**: $\lambda_1$=0.10, $\mu_1$=5;
- **Transformer**:$\lambda_2$=0.02, $\mu_2$=2

**(a). Probability Density Function (PDF)**

- Using: $f(t) = \lambda e^{-\lambda t}$
- Line: $f(t)_{line} = 0.1e^{-0.1t}$
- Transformer: $f(t)_{trafo} = 0.02e^{-0.02t}$

**(b). Reliability Function R(t)** at *t=1year*

- Using: $R(t) = e^{-\lambda t}$
- Line: $R(t)_{line} = e^{-0.1t} = 0.9048$
- Transformer: $R(t)_{trafo} = e^{-0.02t} = 0.9802$

**(c). Equivalent Failure Rate**

*For series system:*

$$\lambda_{eq}=\lambda_1+\lambda_2=0.10+0.02=0.12$$

# Solution

**(d). System Reliability at t=1**

$$R_s(t) = e^{-\lambda_{eq}t} = e^{-0.12t}$$

$$R_s(1) = e^{-0.12*1} = 0.8869$$

**(e). System PDF**

$$f_s(t) = \lambda_{eq}e^{-\lambda_{eq}t} = 0.12e^{-0.12t}$$

**(f). Mean Time To Failure (MTTF)**

$$MTTF = \frac{1}{\lambda_{eq}} = \frac{1}{0.12} = 8.33years$$

**(g). Equivalent Repair Rate and MTTR**

- for a series system

$$\mu_{eq} = \frac{\sum \lambda_i}{\sum \frac{\lambda_i}{\mu_i}} = \frac{0.10+0.02}{\frac{0.1}{5}+\frac{0.02}{2}} = 4$$

$$MTTR = \frac{1}{\mu_{eq}} = \frac{1}{4} = 0.25\ years$$

**(c). Availability:** $A = \frac{MTTF}{MTTF+MTTR} = \frac{8.33}{8.33+0.25} = 0.9709$

# Cont'd...

**(i). Engineering Interpretation:**

- Weakest Component
  - **The line is weaker because:** Higher failure rate $\lambda_1$=0.10 which dominates system reliability
- Improvement Strategy
  - Introduce redundancy, e.g.: Parallel feeder and
  - Distributed generation (DG) backup
- Improve line reliability via:
  - Preventive maintenance
  - Upgrading conductor/insulation

# B. Parallel Configuration

- Derivation of Parallel System Reliability Expression:

**Step 1:** Define the System Behavior

- **For a parallel system:**
  - **System works** if at least **one component works.**
  - **System fails** only if **all components fail.**

**Step 2: Define Failure Probability**

- $R_i$ = reliability of component i and
- $Q_i = 1 - R_i$ = probability of failure

**Step 3:** Compute System Failure Probability

Assuming independent failures:

$$Q_p = P(\text{all components fail}) = \prod_{i=1}^{n} Q_i$$

$$Q_p = \prod_{i=1}^{n} (1 - R_i)$$

**Step 4:** Convert to Reliability

Since: $R_p = 1 - Q_p$

$$R_p = 1 - \prod_{i=1}^{n} (1 - R_i)$$

# Example

- A critical load (a hospital) is supplied by two parallel feeders to improve reliability.
- **Each feeder has the following characteristics:**
  1. **Feeder 1:** Failure rate $\boldsymbol{\lambda_1=0.15}$ failures/year and Repair rate $\boldsymbol{\mu_1= 3}$ repairs/year
  2. **Feeder 2**: Failure rate $\boldsymbol{\lambda_2= 0.10}$ failures/year and Repair rate $\boldsymbol{\mu_2= 4}$ repairs/year
- **Assume:** Exponential failure distributions, independant failures, and Mission time t=1 year.

**Questions:**

1. Determine the reliability of each feeder at t=1 year.
2. Determine the system unreliability

# Cont'd...

**Questions:**

3. Compute the overall system reliability
4. Compute the MTTF of each feeder
5. Compute the MTTR of each feeder
6. Compute the availability of each feeder
7. Evaluate the overall system performance

**Solution:**

1. **Reliability of Each Feeder at t=1 year.**

$$R(t) = e^{-\lambda t}$$

**Feeder 1:** $R1(1) = e^{-0.15} = 0.8607$

**Feeder 2:** $R2(1) = e^{-0.1} = 0.9048$

2. **Parallel System Reliability**

$$R_p = 1 - (1 - R_1)(1 - R_2)$$

$$R_p = 1 - (1 - 0.8607)(1 - 0.9048) = 0.9867$$

# Cont'd...

**Solution:**

**3. System Unreliability, $\boldsymbol{Q_p}$**

$$Q_p = 1 - R_p = 1 - 0.9867 = 0.0133$$

➞ *Only **1.33%** chance of failure per year*

**4. Mean Time To Failure (MTTF)**

**Feeder 1:** $MTTF_1 = \frac{1}{0.15} = 6.67 years$

**Feeder 2:** $MTTF_2 = \frac{1}{0.1} = 10 years$

**Solution:**

**5. Mean Time To Repair (MTTF)**

**Feeder 1:** $MTTR_1 = \frac{1}{\mu_1} = \frac{1}{3} = 0.333 years$

**Feeder 2:** $MTTR_2 = \frac{1}{\mu_2} = \frac{1}{4} = 0.25\ years$

**6. Availability of Each Feeder, *A***

$$A = \frac{MTTF}{MTTF+MTTR}$$

# Example

**Solution:**

**6. Availability of Each Feeder, *A***

$$A = \frac{MTTF}{MTTF + MTTR}$$

**Feeder 1:** $A_1 = \frac{6.67}{6.67+0.333} = 0.9524$

**Feeder 2:** $A_2 = \frac{10}{10+0.25} = 0.9756$

**7. Engineering Interpretation**

- Feeder 1 has low reliability ($R_1$=0.8607)
- Feeder 2 has better performance ($R_2$=0.9048)
- The parallel system provides high reliability ($R_p$=0.9867)
- Parallel feeders have very low failure risk ($Q_p$=1.33%)
- Parallel system reliability (98.67%) is much higher than individual feeders
- Failure probability drastically reduced from:~14→ 1.33%

# 7. Marcov Process Modeling

- **Classical reliability models** (series/parallel) assumes **static configuration** and no explicit time-dependent state transitions.
- However, real power systems are dynamic and repairable, requiring:
  - Modeling of failure and repair cycles
  - Time-dependent state probabilities
- This dynamic nature of system characterstic is modeled using **Markov Process**.
  - A Markov process is a stochastic process where future state depends only on the current state:

$$P(X_{t+1}|\, X_t, X_{t-1}, \ldots) = P(X_{t+1}|\, X_t)$$

# Two-State Markov Model

- A two-state Markov model represents a repairable power system component by two mutually exclusive states:
  - **State 0** (Up): Component is **operational**
  - **State 1** (Down): Component is **failed**
- The component continuously transitions between these states due to failure and repair processes.
- State Transition Mechanism:
  - **Failure transition:** $\lambda$ (Up → Down)
  - **Repair transition:** $\mu$ (Down → Up)

# Cont'd...

- **State Probabilities:**
  - ***$P_o(t)$***: Probability the component is operational at time t.
  - ***$P_1(t)$***: Probability the component is failed at time t.
- Markov Differential Equations:

$$\frac{dP_o(t)}{dt} = -\lambda P_o(t) + \mu P_1(t)$$

$$\frac{dP_1(t)}{dt} = \lambda P_o(t) - \mu P_1(t)$$

- Steady-State Solution: $\frac{dP_o}{dt} = \frac{dP_1}{dt} = 0,$

$$P_o = \frac{\mu}{\lambda+\mu}, P_o = \frac{\lambda}{\lambda+\mu}$$

# Cont'd...

- Key Reliability Metrics:
  - Availability (A):

$$A = P_o = \frac{\mu}{\lambda + \mu}$$

  - Unavailability (U):

$$U = P_1 = \frac{\lambda}{\lambda + \mu}$$

- Important Properties:

$$P_o + P_1 = 1$$

  - Captures dynamic behavior (failure + repair)

**Assumes:** Exponential failure and repair times → Constant rates λ, μ

# Exercise

- **A distribution transformer** supplying a critical load operates with the following characteristics:
  - Failure rate: $\lambda=0.04$ failures/year and Repair rate: $\mu=2$ repairs/year
- **Assume:** Two-state Markov model (Up/Down), Continuous-time operation, and Steady-state analysis are required.
- **Questions:**
  1. Formulate the Markov state model
  2. Write the state transition equations
  3. Determine the steady-state probabilities
  4. Compute the availability and unavailability and Interpret the results for power system operation

# References

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# Thank You!