Power System Quality and Reliability

ECEg-6312

WEEK 11

Transmission System Reliability Analysis

Course Instructor: Demsew Mitiku (PhD)

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Topic Overview

- This lecture introduces analytical approaches used to evaluate the reliability performance of transmission systems and the topics Covered:

    1. Introduction to transmission system reliability analysis
    2. Reliability concepts and performance indices
    3. Average interruption rate method
    4. Loss of Load Probability (LOLP) method
    5. Frequency and duration method
    6. Numerical examples and calculations

Learning Outcomes

By the end of this lecture, students will be able to:

- Explain fundamental concepts of transmission system reliability analysis.
- Describe the importance of reliability assessment in transmission networks.
- Calculate the average interruption rate of a transmission system.
- Apply the Loss of Load Probability (LOLP) method for reliability evaluation.
- Analyze system reliability using frequency and duration techniques.
- Apply reliability analysis results in planning, operation, and maintenance of power systems.

1. Introduction

- Transmission systems are a critical part of electric power systems because they transport electrical energy from generating stations to distribution networks and consumers.
- The reliability of a transmission system determines its ability to continuously deliver electrical power with acceptable quality and minimum interruption [1].
- Transmission system failures may occur due to:
  - Equipment failures and Weather conditions (lightning, wind, storms)
  - Insulation breakdown and Protection system malfunctions
  - Human errors and Maintenance-related outage

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- **Reliability analysis of transmission systems helps engineers to**:
  - Predict system performance under normal and contingency conditions
  - Assess interruption risks and Improve system planning and operation
  - Reduce outage frequency and duration
  - Enhance system security and customer service quality
- **Various techniques are used to evaluate transmission reliability, including**:
  - Average interruption rate method
  - Loss of Load Probability (LOLP) method
  - Frequency and duration method

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Key transmission system reliability parameters include:

- **Reliability:** Probability that a system performs its required function without failure.
- **Adequacy:** Ability of the transmission system to supply customer demand and energy requirements.
- **Security:** Ability of the power system to withstand sudden disturbances.
- **Availability:** Probability that the system is operational and capable of performing its intended function.
- **Failure Rate (λ):** Average number of failures occurring within a specified period.
- **Repair Rate (μ):** Average rate at which failed components are restored to service.
- **Mean Time To Failure (MTTF):** Average operating time before system failure occurs.
- **Mean Time To Repair (MTTR):** Average time required to restore a failed component or system.

2. Reliability Performance Indices

- Reliability performance indices are quantitative measures used to assess the reliability and operational performance of a transmission system.
- These indices help evaluate system adequacy, interruption characteristics, and overall service continuity.
- The three most commonly used reliability performance indices in transmission system reliability assessment are [2]:
  - Loss of Load Probability (LOLP), Interruption Frequency, and Interruption Duration.
- These indices provide quantitative information about system adequacy and outage characteristics.

3. Loss of Load Probability (LOLP) Method

- Loss of Load Probability (LOLP) is a reliability index that measures the probability that the system load demand exceeds the available transmission capacity or generation capability.
- It is used to evaluate the adequacy of a power system and estimate the risk of supply interruptions.
- LOLP represents the likelihood that the available system capacity is insufficient to meet customer demand during a specified period.
- LOLP expression:

$$LOLP = P(Load > Avaliable\ Capacity)$$

$$LOLP = \sum_{i=1}^{n} P_i$$

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- **Where:**
  - P_i = probability of outage states causing load loss and n = number of deficient system states
- **Steps for LOLP Calculation**
  - **Step 1:** Determine transmission system components and capacities
  - **Step 2:** Identify possible outage states
  - **Step 3:** Calculate the probability of each outage state
  - **Step 4:** Compare available capacity with load demand
  - **Step 5:** Identify deficient states where: Load > Available Capacity
  - **Step 6:** Sum the probabilities of all deficient states

Example

- **Problem Statement:** Consider a transmission system supplying a load center through two transmission lines and one transformer substation.
- **System Data:** Load demand P_L= 150 MW
- Transmission components:

Table 1: Transmission Components Data.

State	Capacity (MW)	Failure rate, λ (failures/year)	Repair rate, μ (repairs/year)
Transmission Line 1	100	0.15	30
Transmission Line 2	80	0.10	25
Transformer	180	0.05	20

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- Assume independent outages.
- Determine the Loss of Load Probability (LOLP).

Step 1: Calculate Component Unavailability

- For a two-state model:
- Unavailability relation:

$$U = \frac{\lambda}{\mu + \lambda}$$

- For Line 1: $U = \frac{0.15}{0.15+30} = 0.004975,\ \ A = 0.995025$
- For Line 2: $U = \frac{0.10}{0.10+25} = 0.003984, A = 0.996016$

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- For Transformer:

$$U = \frac{0.05}{0.05+20} = 0.002494,\ A = 0.997506$$

Step 2: Determine Deficient States

- Normal transmission capability:

$$P_C = P_{line-1} + P_{line-2}$$

$$P_C = 100\ MW\ + 80\ MW = 180\ MW$$

- *Transformer rating:* $P_{tr} = 180\ MW$ *and Load demand:* $P_{load} = 150\ MW$
- *A loss-of-load occurs whenever: Available Capacity < 150 MW*

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- Outage assessment:

Table 2: Transmission Outage Assesment.

State	Line 1	Line 2	Transformer	Avaliable Capacity (MW)	State Probability
S1	Ok	Ok	Ok	180	$A1^{*}A2^{*}A3 = 0.9886$
S2	Ok	OK	Outage	0	$A1 * A2^{*}U3$=0.002472
S3	Ok	Outage	Ok	100	$A1^{*}U2^{*}A3$=0.003954
S4	Ok	Outage	Outage	0	$A1^{*}U2^{*}U3$=0.0000099
S5	Outage	Ok	Ok	80	$U1 * A2 * A3 = 0.004943$
S6	Outage	Ok	Outage	0	$U1^{*}A2^{*}U3$=0.00001236
S7	Outage	Outage	OK	0	$U1 * \mathrm{U2} * A3$ =0.0000198
S8	Outage	Outage	Outage	0	$U1 * \mathrm{U2} * \mathrm{U3}$ =0.00000005

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Step 3: Compute Deficient-State Probabilities

- From S2 to S8 there is a deficient power supply and the probabilities of state deficiency includes: P(S2), P(S3), P(S4), P(S5), P(S6), P(S7), and P(S8)

Step 4: Calculate LOLP

- LOLP is obtained by summing the probabilities of deficient states:

$$LOLP = \sum_{i=2}^{8} P_i = 0.002472 + 0.003954 + 0.0000099 + 0.004943 + 0.00001236 + 0.0000198 + 0.00000005 = 0.01141$$

$$LOLP = 1.14\%$$

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- Final Remark and Result Interpretation
  - The transmission system has approximately a 1.141% probability of being unable to satisfy the load demand because of transmission component outages.
  - A lower LOLP indicates a more reliable system, while a larger value suggests that reliability improvements such as redundancy, parallel transmission paths, or faster repair strategies may be needed.
  - For utility-grade studies, deficient-state probabilities are commonly computed using state enumeration, Capacity Outage Probability Tables (COPT), or Monte Carlo simulation. Here, a two-state reliability model is used for instructional purposes.

4. Average Interruption Rate Method

- The "average interruption rate method" primarily refers to the calculation of standardized reliability indices used by electric utilities to measure [3]:
  - How often customers are without power and also,
  - How long customers are without power, as defined in IEEE Standard 1366.
- Commonly known as:
  - **Average Interruption Rate (λ_L):** How many times per year the load point is expected to lose power.
  - **Average Outage Duration (r_L):** The average time required to isolate the failure and restore power to that load point (hours/interruption).
  - **Annual Outage Time (U_L):** The total expected downtime at that load point per year (hours/year).

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- If a load point relies on **a series of critical transmission components** where the failure of any single component cuts off the power, its reliability is measured using the metrics [3]:

  A. **Average Interruption Rate or Frequency (λ_L):** The total average interruption rate is the direct sum of the individual annual failure rates of all components in that series path:

$$\lambda_L = \sum_{i=1}^{n} \lambda_i$$

  B. **Total Annual Outage Time (U_L):** The total expected downtime per year at that load point is the sum of the individual component downtimes:

$$U_L = \sum_{i=1}^{n} \lambda_i r_i$$

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C. Average Outage Duration (r_L): The average time the load point stays dark per outage event is derived by dividing total annual downtime by the overall failure rate:

$$r_L = \frac{U_L}{\lambda_L}$$

- Where:
  - λ_i is interruption/year for each transmission system component.
  - r_i is duration of each interruption for each system components (hours/interruption)
  - λ_L is average interruption frequency (outages/year)
  - r_L is the average interruption duration (hours/interruption)
  - U_L is the annual system unavaliability duration (time)

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- **General Mathematical Formulas for n Parallel Components:** To solve for the three indices, we first define the Unavailability (U_i) of each individual component (i) based on a standard calendar year (8760 hours):

$$U_i = \frac{\lambda_i r_i}{8760}$$

A. Annual Outage Time (U_L): For independent parallel components, the load point is only down when every single component is down.

- Therefore, the system unavailability is the product of all individual unavailabilities:

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$$U_L = \prod_{i=1}^{n} U_i \times 8760 \text{ [hours/year]}$$

B. Average Outage Duration (r_L): When all parallel components are down, the system recovers as soon as any single component is repaired.

- Therefore, the average restoration time follows a parallel rate addition rule:

$$\frac{1}{r_L} = \sum_{i=1}^{n} \frac{1}{r_i}, \quad r_L = \frac{1}{\sum_{i=1}^{n} \frac{1}{r_i}} \quad \text{[hours]}$$

C. Average Interruption Rate (λ_L): Using the fundamental reliability relationship ($U_L = r_L \times \lambda_L$), the system failure rate is found by dividing total annual downtime by the average outage duration: $\boldsymbol{\lambda_L = U_L / r_L}$

Example

- **A remote mining facility** is connected to a regional high-voltage grid substation. Power must travel from the grid through three transmission components in series to reach the load point:
    - Component 1: A 50-mile overhead transmission line (L_1).
    - Component 2: A 132kV/33kV step-down power transformer (T_1).
    - Component 3: A main 33kV vacuum circuit breaker (B_1).
- **Component Reliability Data:**
    - Overhead Line L1: $\lambda_1 = 0.4$ interruption/year, $r_1 = 5$ hours/interruption
    - Transformer T_1: $\lambda_2 = 0.05$ interruption/year, $r_2 = 48$ hours/interruption
    - Circuit Breaker B1: $\lambda_3 = 0.15$ interruption/year, $r_3 = 2$ hours/interruption

Solution

- **Calculate the three load-point transmission reliability indices:** Average Interruption Rate (λ_L), Annual Outage Time (U_L), and Average Outage Duration (r_L).

Step 1: Calculate System Average Interruption Rate (λ_L):

- For a series system, the total interruption rate is simply the sum of the individual component failure rates. Any single failure causes a system failure.

$$\lambda_L = \lambda_1 + \lambda_2 + \lambda_3 = 0.40 + 0.05 + 0.15 = 0.60\ interuptions/year$$

- This relatively high interruption rate highlights the vulnerability of series-connected transmission components, where a single component failure can interrupt the entire supply.

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Step 2: Calculate Total Annual Outage Time (U_L):

- The total expected downtime per year at the load point is the sum of the individual annual downtimes for all components.

$$U_L = \sum_{i=1}^{n} \lambda_i r_i = \lambda_1 r_1 + \lambda_2 r_2 + \lambda_3 r_3$$

$$U_L = 0.4 \times 5 + 0.05 \times 48 + 0.15 \times 2 = 4.70\ hours/year$$

Step 3: Calculate Average Outage Duration (r_L):

$$r_L = \frac{U_L}{\lambda_L} = \frac{4.70}{0.6} = 7.833\ hours/interruption$$

Example

- An industrial manufacturing plant is fed by a 132 kV substation. The substation is connected to the main transmission grid via two parallel high-voltage transmission lines (Line A and Line B). Power can flow fully through either line.
- A total loss of power to the plant only occurs if both lines are out of service at the same time (overlapping outage).
- **Component Reliability Data:**
  - Transmission Line A: $\lambda_1 = 2.0$ interruption/year, $r_1 = 6$ hours/interruption
  - Transmission Line B: $\lambda_2 = 1.5$ interruption/year, $r_2 = 8$ hours/interruption

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- **Calculate the three transmission reliability indices (**Average Interruption Rate (λ_L), Annual Outage Time (U_L), and Average Outage Duration (r_L)) at the plant's substation load point.

Solution:

Step 1: Calculate Component Unavailabilities

$$U_i = \frac{\lambda_i r_i}{8760}$$

- For Line A (U_1):

$$U_1 = \frac{\lambda_1 r_1}{8760} = \frac{2\times 6}{8760} = 0.00136986$$

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- For Line B (U_2):

$$U_2 = \frac{\lambda_2 r_2}{8760} = \frac{1.5 \times 8}{8760} = 0.00136986$$

$$U_L = U_1 \times U_2 = 0.00136986 \times 0.00136986 \times 8760\ hours/year = \mathbf{0.01644}\ hours/year$$

Step 2: Calculate Load Point Average Outage Duration (r_L)**:**

$$r_L = \frac{r_1 r_2}{r_1 + r_2} = \frac{6 \times 8}{6 + 8} = \mathbf{3.43}\ hours$$

Step 3: Calculate Load Point Interruption Rate (λ_L):

$$\lambda_L = \frac{U_L}{r_L} = \mathbf{0.00479}\ interruptions/year$$

Final Summary

- Based on the calculation, here are the three transmission system reliability indices for the load point:

1. **Average Interruption Rate (λ_L): 0.00479** interruptions per year (The system will experience an outage roughly once every **208.5 years**.
2. **Annual Outage Time (U_L): 0.01644** hours per year (Equivalent to roughly **59.2 seconds** of total expected downtime annually).
3. **Average Outage Duration (r_L): 3.43 hours** (When a rare double-line failure does occur, the plant will be without power for an average of **3** hours and **26** minutes while technicians scramble to fix at least one line).

5. Frequency and Duration Method

- The Frequency and Duration Method is one of the most widely used approaches for evaluating transmission system reliability [4].
- It estimates both the expected frequency of service interruptions and the expected duration of those interruptions due to failures in transmission components.
- Unlike adequacy-based methods such as LOLP, which indicate the probability of load deficiency, the frequency–duration method provides additional information on the occurrence rate and persistence of outages.

Basic Reliability Measures

A. Interruption Frequency (F)

- Interruption frequency represents the expected number of interruptions occurring in a transmission system during a specified period [4].

$$F = \sum_{i=1}^{n} \lambda_i$$

B. Interruption Duration (D)

- Interruption duration represents the average time required to restore the system following a failure.

$$D = \frac{\sum_{i=1}^{n} \lambda_i r_i}{\sum_{i=1}^{n} \lambda_i}$$

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C. Expected Annual Outage Time

- The annual outage time is calculated as:

$$U = F \times D$$

Where:

- U = annual outage time (hours/year).
- r_i = repair time of component i (hours)
- λ_i = failure rate of component i

- The **Frequency–Duration Method** adds **repair-time effects** and therefore can determine:
  - Average outage duration, Annual outage time, and Service continuity impact

Example

- A transmission substation supplies a load center through several transmission components. The utility company wants to evaluate the transmission system reliability using the Frequency and Duration Method. The reliability data for the transmission system components are given below:

Table 3: Transmission System Reliability Assesment Data.

State	Failure rate, λ (failures/year)	Repair time, r (hours)
Transmission Line 1	0.20	5
Transmission Line 2	0.15	4
Transformer	0.08	12
Circuit Breaker	0.05	2
Busbar	0.03	8

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Determine:

- System interruption frequency
- Average interruption duration
- Annual outage duration
- Availability of the transmission system
- Interpret the results.

Solution:

Step 1: Calculate System Interruption Frequency

- The system interruption frequency is obtained by summing all component failure rates
- Interruption frequency equation:

$$F = \sum_{i=1}^{n} \lambda_i = \lambda_1 + \lambda_2 + \lambda_3 + \lambda_i + \lambda_i$$

$$F = 0.2 + 0.15 + 0.08 + 0.05 + 0.03 = 0.51$$

- The system interruption/year is 0.51.

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Step 2: Calculate Average Interruption Duration

- The duration is obtained using a weighted average of repair times.

$$D = \frac{\sum_{i=1}^{n} \lambda_i r_i}{\sum_{i=1}^{n} \lambda_i}$$

$$D = \frac{0.2 \times 5 + 0.15 \times 4 + 0.08 \times 12 + 0.05 \times 2 + 0.03 \times 8}{0.51} = \frac{2.90}{0.51} = \mathbf{5.69\ hours/interruption}$$

Step 3: Calculate Annual Outage Duration

- Annual outage time is: U=F× $D = 0.51 \times 5.69 = \mathbf{2.90\ hours/year}$

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Step 4: Calculate System Availability

- Total yearly hours: 8760 and the availability equation is:

$$A = 1 - \frac{U}{8760} = 1 - \frac{2.90}{8760} = 0.99967$$

- Availability percentage: **99.967%**
- The transmission system experiences: **0.51** interruptions/year which mean Approximately one interruption every **1.96** years.
- Each interruption lasts approximately: 5.69 hours, the transformer contributes **significantly** because of its longer repair time.

References

[1]. R. Billinton and R. N. Allan, Reliability Evaluation of Power Systems, 2nd ed. New York, NY, USA: Plenum Press, 1996.

[2]. R. Billinton and W. Li, Reliability Assessment of Electric Power Systems Using Monte Carlo Methods. New York, NY, USA: Plenum Press, 1994.

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[4]. C. Singh and R. Billinton, System Reliability Modelling and Evaluation. London, U.K.: Hutchinson Educational Publishers, 1977.

Thank You!