Power System Quality and Reliability

ECEg-6312

WEEK 12

Multi-Plant Multi-Load System

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June 2026

Topic Overview

- This topic focuses on how multiple generating plants interact with multiple loads under uncertainty, and how interconnections improve system reliability and operational flexibility.
- This lecture covers:
    1. Two plant single load system,
    2. Two plant two load system,
    3. Load forecasting uncertainty, and
    4. Interconnections benefits.

Learning Outcomes

By the end of this lecture, students will be able to:

- Model multi-plant single-load systems
- Analyze multi-plant multi-load reliability structures
- Evaluate system adequacy using probabilistic methods
- Model load forecasting uncertainty
- Quantify benefits of interconnections

1. Introduction

- A **Multi-Plant Multi-Load System** is a generalized representation of modern power systems where multiple generators supplying multiple load centers through an interconnected network.
- From a **reliability analysis** perspective, this structure reflects the realistic behavior of power systems in which both generation availability and load demand are uncertain and spatially distributed [1].
- The **main objective of analyzing such systems** is to evaluate the ability of the network to maintain adequate and continuous power supply under stochastic conditions, while accounting for both generation and transmission limitations.

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- Unlike simple single-plant or single-load models, multi-plant multi-load systems capture the combined effects of:
  - Component outages (random failures and repairs), Load variability, and forecasting errors
  - Power transfer constraints within the network
  - Operational strategies such as load sharing and reserve support
- In reliability terms, the system is typically assessed using:
  - Probabilistic adequacy indices such as LOLP, LOLE, and EENS
  - State-space methods (e.g., Markov modeling)
  - Series–parallel reliability structures and load–generation balance under uncertainty

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- Within this framework, four fundamental cases are commonly studied:
  - **Two-plant single-load systems:** representing basic redundancy and reserve sharing
  - **Two-plant two-load systems:** introducing spatial load distribution and power routing constraints
  - **Load forecasting uncertainty:** which affects system adequacy through demand prediction errors
  - **Interconnection benefits:** where multiple areas are linked to improve reliability through mutual support and diversity effects
- The multi-plant multi-load framework provides a foundational tool for understanding and designing modern interconnected power systems, especially in environments with high renewable penetration, distributed generation, and dynamic load behavior.

2. Two-Plant Single-Load Systems

- A Two-Plant Single-Load System is one of the most fundamental reliability configurations in power system analysis [2].
- It consists of two independent generating plants supplying a single aggregated load demand.
- The main objective of this configuration is to evaluate how redundancy in generation improves the adequacy and continuity of supply.
- From a reliability perspective, this system represents a parallel structure, where the load is successfully supplied as long as at least one generating plant is available.

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- The two-plant single-load system is a simple reliability configuration used to model redundant generation supplying one demand point.
- **System Structure:**
  - **Plant 1:** G_1 with reliability $R_1(t)$
  - **Plant 2:** G_2 with reliability $R_2(t)$
  - **Single load:** L
- The system fails only when both plants are unavailable simultaneously.

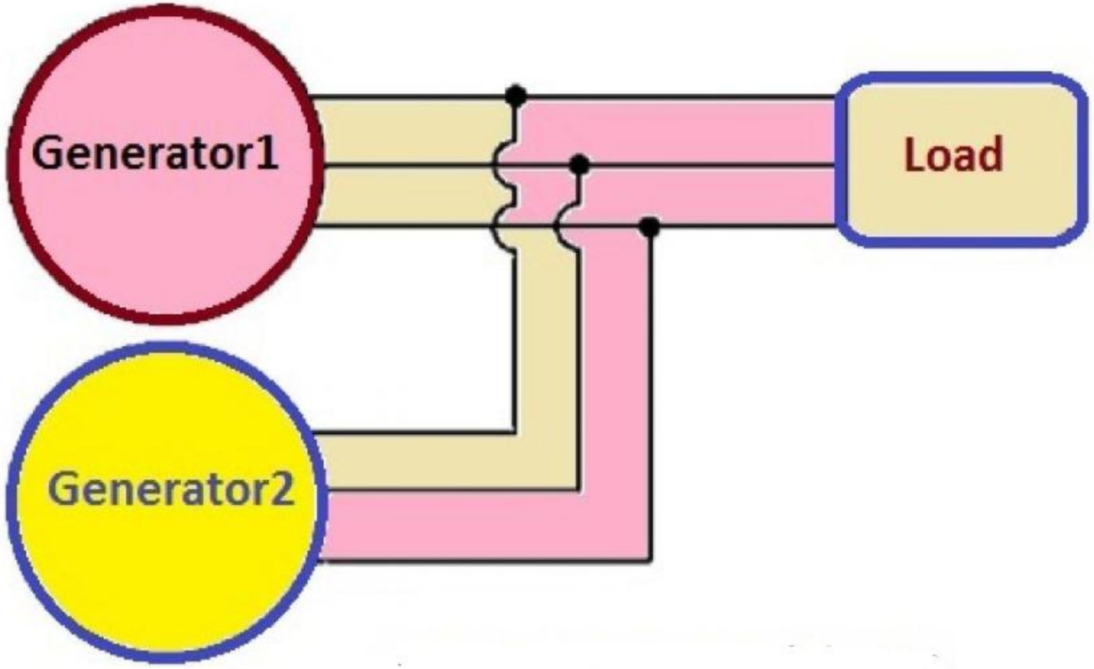

Figure 1: Two-Plant Single-Load Systems

A. Reliability Model

- The reliability model of a two-plant single-load system is developed using a parallel reliability structure, where the system is considered successful if at least one generating plant is operational at any given time.

1. Basic Assumption

- Assume:
  - Failures are statistically independent
  - Load is always connected (no network constraint)
  - Binary state model (UP/DOWN) for each plant

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2. System Success Condition:

- The system is successful if **G1** is **UP** OR **G2** is **UP** and this defines a parallel reliability configuration.

3. System Reliability Expression

- The probability that the system operates successfully is:

$$R_{sys}(t) = 1 - P(G1\ fails\ and\ G2\ fails)$$

- Since failures are independent and generation is redundant:

$$R_{sys}(t) = 1 - (1 - R_1(t))(1 - R_2(t))$$

- System failure occurs only when both plants are down.

$$Q_{sys}(t) = Q_1(t)Q_2(t)$$

- *Where:* $Q_i(t) = 1 - R_i(t)$

B. System Failure Model

- **Exponential Failure–Repair Model (Markov Form)**
  - If each plant follows a two-state Markov process with Failure rate, λ_{i} and Repair rate, μ_{i}, then steady-state availability becomes:

$$A_i = \frac{\mu_i}{\lambda_i + \mu_i}$$

  - System availability becomes:

$$A_{sys} = 1 - (1 - A_1)(1 - A_2)$$

  - The reliability model of a **two-plant single-load system** is a parallel probabilistic structure, where system adequacy depends on the probability that at least one generating unit remains operational.
  - The system exhibits redundancy gain and reliability increases nonlinearly with additional plants

C. State Space Representation

- The state space representation describes all possible operating conditions of the generating plants and their effect on the system status.
- For a two-plant single-load system, each plant has two possible conditions: UP (operational state) and DOWN (failed state)
- Since there are two plants, the total number of states is: $N = 2^n$, where n =2 (the number of plants).

Table 1: State Space Representation.

State	G1	G2	System Status
S1	UP	UP	Operating
S2	UP	DOWN	Operating
S3	DOWN	UP	Operating
S4	DOWN	DOWN	Failure

3. Two-Plant Two-Load System

- A **Two-Plant Two-Load System** is a power system configuration in which two generating plants supply two separate load centers through an interconnected network [2].
- Compared with the two-plant single-load model, this system introduces multiple demand points and power transfer interactions, making it a more realistic representation of practical power systems.
- From a reliability perspective, the system adequacy depends not only on the availability of generating units but also on the ability of the network to transfer power between generation and load locations.

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- **System Structure:** The system consists of
  - **Plant 1:** G_1 with capacity P_1
  - **Plant 2:** G_2 with capacity P_2
  - **Load 1:** L_1 with demand P_{L1}
  - **Load 2:** L_2 with demand P_{L2}
- The interconnections in **Figure 2** allows for:
  - Load sharing
  - Power exchange
  - Reserve support
  - Contingency assistance

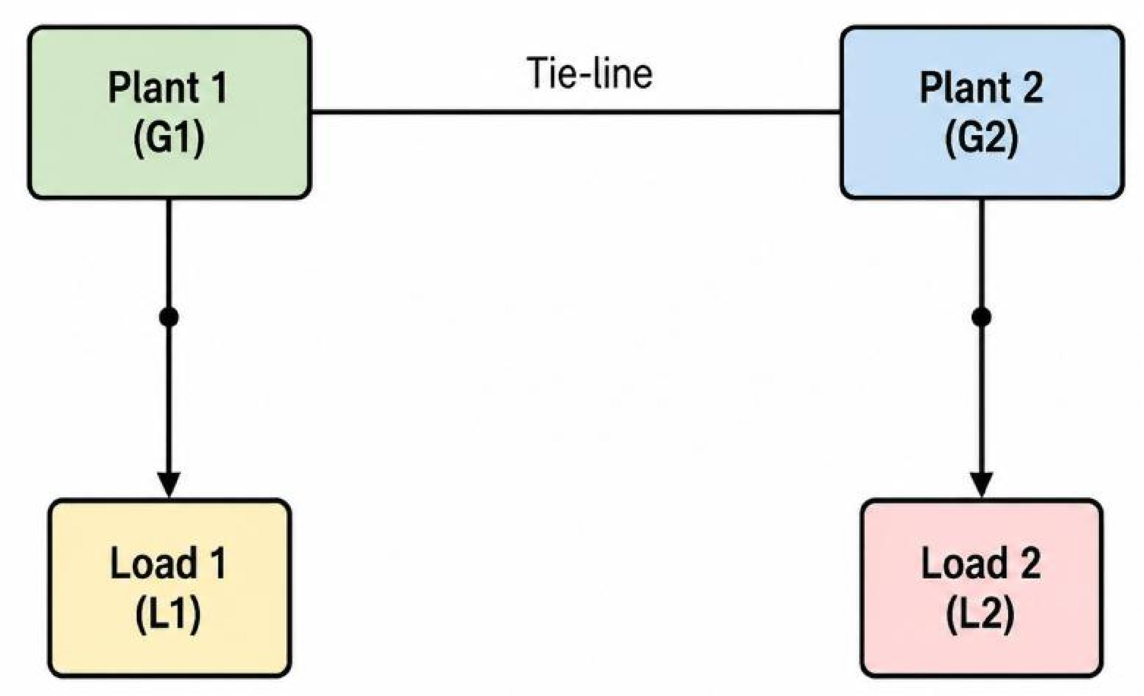

Figure 2: Two-Plant Two-Load Systems

A. Operating Conditions

- The operating conditions of a two-plant two-load system describe the different possible system states based on the availability of generating plants and the ability to supply the connected loads.

1. Normal Operating Condition

  - Both generating plants are available and supply their corresponding loads.
  - **Condition:** G1 is UP and G2 is UP and the power balance equation is $P_1 + P_2 \geq P_{L1} + P_{L2}$
  - **Characteristics:**
    - Both loads receive full power demand and reserve capacity may be available
    - Tie-line power transfer is minimal or optimized
    - Highest system reliability

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2. **Partial Outage Condition:** One generating plant becomes unavailable while the remaining plant continues operating.

- **Case A:** Plant 1 operating (G1=UP), Plant 2 failed (G2=Down)
  - **Available power:** $P_{available}=P_1$ and System remains operational if: $P_1 \geq P_{L1} + P_{L2}$
- **Case B:** Plant 2 operating (G2=UP), Plant 1 failed (G1=Down)
  - **Available power:** $P_{available}=P_2$ and System remains operational if: $P_2 \geq P_{L1} + P_{L2}$
- **Characteristics:** Load sharing through interconnections becomes important,
  - Tie-line transfers increase
  - Some loads may experience curtailment if capacity is insufficient

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3. **Overload or Load Curtailment Condition:** Available generation exists but is insufficient to meet total demand.

- **Condition:**

$$P_1 + P_2 \leq P_{L1} + P_{L2}$$

- **Characteristics:**
  - Partial load shedding may occur
  - Priority loads are supplied first
  - Reliability indices such as LOLP increase

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4. **Complete System Failure Condition:** Both generating plants are unavailable.
   - **Condition:** G_1=DOWN, and G_2=DOWN
   - **Available generation:**

$$P_{available} = 0$$

   - **Characteristics:**
     - Total interruption of load supply
     - Maximum loss-of-load state
     - System reliability becomes zero for that state

B. State Space Representation

- The state space representation of a two-plant two-load system is summarized in **Table 2**:

Table 2: Operating States of Two-Plant Two-Load System.

State	Plant 1	Plant 2	Load Status	System Condition
Normal operation S1	UP	UP	All loads supplied	Fully operational
Partial outage (Case A) S2	UP	DOWN	Limited or full support	Degraded operation
Partial outage (Case B), S3	DOWN	UP	Limited or full support	Degraded operation
Overload condition, S4	UP/DOWN	UP/DOWN	Partial load supplied	Load curtailment
Complete outage, S5	DOWN	DOWN	No load supplied	System failure

C. Reliability Model

- The reliability model of a two-plant two-load system evaluates the probability that the system can continuously supply both loads while considering the availability of generating plants and the power transfer capability of the network.
- Unlike the two-plant single-load model, the reliability assessment here depends on both generation adequacy and load distribution constraints.
- For successful operation:

$$P_1 + P_2 \geq P_{L1} + P_{L2}$$

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Reliability Expression

- Let: R_1 =reliability of Plant 1 and R_2 = reliability of Plant 2
- The probability that both plants fail:

$$Q_{sys} = (1 - R_1)(1 - R_2)$$

- Hence system reliability becomes:

$$R_{sys} = 1 - Q_{sys}$$

- The previous expression assumes unlimited power transfer but in practical systems: $|P_{tie}| \leq P_{max}$
- The success criterion becomes: $P_{available} \geq P_{demand}$ and $|P_{tie}| \leq P_{max}$

D. Reliability Indices

- Common adequacy indices include:

1. **Loss of Load Probability (LOLP):**

$$LOLP = P(P_{available} < P_{demand})$$

2. **Expected Energy Not Supplied (EENS):**

$$EENS = \sum_{i=1}^{n} (P_{demand} - P_{available})P_i$$

3. **Loss of Load Expectation (LOLE):** Measures the expected duration during which the available generation capacity is insufficient to meet the system load demand over a specified period, T.

$$LOLE = \sum_{i=1}^{n} P_i T_i$$

Example

- Consider a small interconnected regional power system supplying two towns:
  - Plant 1 (Hydro + Wind hybrid plant): Capacity=150 MW, λ=0.02 failures/year, and μ=2 repairs/year
  - Plant 2 (Thermal diesel/gas plant): Capacity=120 MW, λ=0.03 failures/year, and μ=3 repairs/year
  - Load 1 (Industrial city load): L_1= 80MW
  - Load 2 (Residential town load): L_2= 90MW
- This represents a simplified microgrid / distribution-level interconnected system.
- Total demand: P_D= 170 MW
- Total installed capacity: P_G=150+120MW= 270 MW
- Tie-line transfer limit: $P_{tie}^{max} = 50$ MW

Solution

Step 1: Individual Plant Availability

- Using Markov steady-state availability:

$$A_i = \frac{\mu_i}{\lambda_i + \mu_i}$$

- Plant 1: $A_1 = \frac{2}{0.02+2} = 0.9901$
- Plant 2: $A_2 = \frac{3}{0.03+3} = 0.9901$

Step 2: System State Probabilities

Table 3: State Probability for Reliability Analysis.

States	Plant 1	Plant 2	Probability
S1	UP	UP	$A_1A_2=0.9803$
S2	UP	DOWN	$A_1(1-A_2)=0.0098$
S3	DOWN	UP	$(1-A_1)A_2=0.0098$
S4	DOWN	DOWN	$(1-A_1)(1-A_2)=0.0001$

Step 3: Capacity Adequacy Check per State

- **State S1:** Both plants available

$(P_G=270 \text{ MW}) \geq (P_D= 170 \text{ MW}) \leftrightarrow \text{OK}$

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- **State S2:** Only G1 available

$$(P_G=150 \text{ MW}) \geq (P_D= 170 \text{ MW}) \leftrightarrow \text{Deficit}=20 \text{ MW}$$

- Check transfer capability:
  - G1 can supply L1 fully = 80 MW and Remaining capacity = 70 MW
  - But L2 needs 90 MW → deficit remains even with tie-line $P_{tie}^{max} = 50$ MW, Still **Unserved load=40 MW**.
- **State S3:** Only G2 available

$$(P_G=120 \text{ MW}) \geq (P_D= 170 \text{ MW}) \leftrightarrow \text{Deficit}=50 \text{ MW}$$

  - L2 needs 90 MW → deficit remains even with tie-line $P_{tie}^{max} = 50$ MW, **Unserved load=50 MW**

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- **State S4:** Both failed, $P_G=0 \Rightarrow$Total blackout

Step 4: LOLP Calculation, Loss of Load Probability:

$$LOLP = P(S2) + P(3) + P(4)$$

$$LOLP = 0.0197, LOLP = 1.97\%$$

Step 5: LOLE Calculation: Assume yearly study period, T=8760 hours

$$LOLP = 0.0197 \times 8760 = 172.6\ hours/year$$

Step 6: Expected Energy Not Served (EENS)

$$EENS = \sum_{i=1}^{n} (P_{demand} - P_{available})P_i$$

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$$EENS = [40 \times 0.0098 + 50 \times 0.0098 + 170 \times 0.0001] \times 8760$$

$$\underline{EENS = \mathbf{7875.24}\ MWh/year}$$

- Final Remarks:
  - Even with high generator availability (~99%), reliability issues still exist
  - Tie-line limit (60 MW) is a major bottleneck
  - Most losses come from single-plant operation states (S2, S3) rather than total failure
  - Interconnection improves reliability but does not eliminate risk if transfer capacity is constrained
- This two-plant two-load system demonstrates that:
  - Reliability is not only generation-dependent but also transmission-constrained

4. Load Forecasting Uncertainty

- Load forecasting uncertainty refers to the difference between the predicted electrical demand and the actual demand observed in a power system [3].
- Since load behavior depends on several dynamic and stochastic factors, it cannot be predicted with complete accuracy.
- Therefore, uncertainty in load forecasting must be incorporated into reliability analysis and system planning.
- From a reliability perspective, forecasting uncertainty directly affects generation adequacy, reserve requirements, and the probability of meeting system demand.

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- Forecasted load can be represented as [3]:

$$\mathrm{L(t)} = \hat{L}(t) + \varepsilon(t)$$

- Where:
  - L(t) = actual load demand at time t
  - $\hat{L}(t)$= forecasted load demand
  - ε(t) = forecasting error
- Major factors contributing to uncertainty include:

1. **Weather variations:** Temperature, Humidity, Rainfall...
2. **Economic factors:** Industrial activity, Market changes, and Energy prices
3. **Consumer behavior:** Daily and seasonal usage patterns, Population growth, and Lifestyle changes
4. **Renewable energy penetration:** Solar irradiance variability and Wind intermittency
5. **Unexpected events:** Equipment outages, Public events, and Natural disasters

Probabilistic Representation

- Forecasting error is often modeled as a random variable with a normal distribution:

$$\varepsilon \sim N(\mu, \sigma)$$

- where:
  - $\mu = 0$, is mean
  - σ is Standard deviation
- The probability density function of load uncertainty becomes [2]:

$$f(\varepsilon) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(\varepsilon-\mu)^2}{2\sigma^2}}$$

Impact on Reliability

- Load uncertainty influences:

1. **Loss of Load Probability (LOLP)**
   - Higher uncertainty increases the probability of insufficient generation.
2. **Loss of Load Expectation (LOLE)**
   - Uncertainty generally increases expected load interruption duration.
3. **Reserve Requirement**
   - Additional reserve capacity becomes necessary: $\mathbf{R_{eserve} = P_G - \hat{L}}$
   - Larger uncertainty requires larger reserve margins.

5. Interconnections Benefits

- From a reliability perspective, interconnections improve system adequacy and operational security by enabling reserve sharing, reducing the probability of supply interruption, and increasing system flexibility [4].
- Major Benefits of Interconnections

1. **Reliability Improvement**
    - Interconnected systems provide mutual assistance during generation outages.
2. **Reserve Sharing**
    - Instead of each area maintaining large independent reserves, systems can share reserve capacity.

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3. **Peak Load Sharing**
   - Different areas generally reach peak demand at different times.
   - Total peak requirement becomes:

$$P_{combined} < P_{peak1} + P_{peak2}$$

4. **Economic Operation**
   - Interconnections allow economic dispatch among different regions and power can be imported from lower-cost generating areas.
5. **Emergency and Contingency Support**
   - During major disturbances: Generator failures, Transmission outages, Blackouts
   - Interconnected systems can provide: Emergency power import, Faster service restoration

References

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[4]. K. Z. Akdemir et al., “Evaluating grid stress and reliability in future electricity grids across a range of demand, generation mix, and weather trends,” Advances in Applied Energy, vol. 20, p. 100249, Dec. 2025, doi: 10.1016/j.adapen.2025.100249.

Thank You!