1 Plate tectonics: the volcanic source

Earth's biological carbon cycle derives its energy from the sun, which fuels photosynthesis.

A small portion of the organic carbon that is fixed is buried as rock. Likewise, some inorganic carbon is buried as carbonate.

If this burial were to continue without replacement, all the carbon in the atmosphere and oceans would be gone within about 10^6 yr.

There must therefore be a resupply of CO_2 from geologic sources.

This resupply of CO_2 comes via volcanism. The relative magnitude of its flux is small

source	flux (Gt C/yr)
volcanism	0.1
other degassing	0.1
combustion	8.7
respiration	100

Volcanism is nevertheless hugely significant at evolutionary time scales: without it, the carbon cycle would have long ago grinded to a halt, and life as we know it would not exist!

In this way the biological carbon cycle is inextricably tied to the geological carbon cycle, or "rock cycle."

The rock cycle is fueled by the heat flux that comes out of the Earth. We proceed to describe an important manifestation of this heat flux: *plate tectonics*.

We then use mathematical and physical reasoning to quantitatively evaluate predictions of plate tectonics.

1.1 Thermal convection and plate tectonics

There are two internal sources of heat:

- Heat accumulated during the process of planetary accretion, i.e., the gravitational energy dissipated by the formation of the Earth 4.5 Ga.
- Heat generated by the radioactive decay of uranium, thorium, and potassium.

To first approximation, then, we can think of the Earth as being heated from within and cooled at its surface.

We seek an understanding of how the resulting heat flux generates volcanism and crustal motion at the surface.

1.2 Conductive vs. convective heat flow

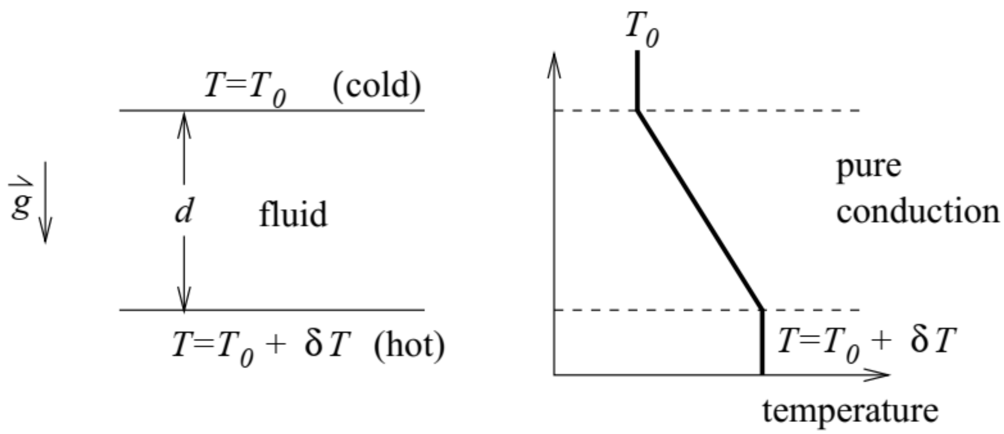
Measurements of temperature within the Earth's crust show that heat "flows" upward out of the Earth.

In general, there are two types of heat flux: conductive and convective.

- Conductive heat flux: only heat is transported, but not the material being heated.
- Convective heat flux: material is transported along with heat.

Which one characterizes the geophysical heat flux?

To investigate this problem, we consider a simpler problem in which a fluid confined between two parallel heat-conducting plates is heated from below.



In the absence of convection—the transport of hot fluid *up* and cold fluid *down*—the temperature gradient is constant.

Recall, however, that hot fluid rises in a thermally expansive fluid.

There are thus two cases of interest:

- δT small: no convective motion, due to stabilizing effects of viscous friction.
- δT large: convective motion occurs.

The convective case corresponds to the plate tectonic motions that interest us.

1.3 Rayleigh number

How large, then, is a "large δT" ? We seek a non-dimensional formulation.

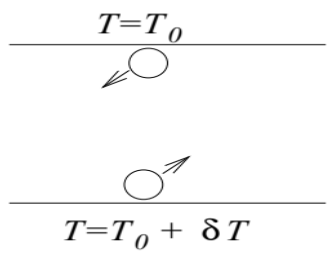
The following fluid properties are important:

- viscosity
- density
- thermal expansivity
- thermal diffusivity (heat conductivity)

Convection is also determined by

- d, the box size
- δT (of course)

Consider a small displacement of a cold blob downwards and a hot blob upwards:



Left undisturbed, buoyancy forces would allow the hot blob to continue rising and the cold blob to continue falling.

There are however damping (dissipation) mechanisms:

- diffusion of heat
- viscous friction

Let D = thermal diffusivity, which has units

$$[D] = \frac{\text{length}^2}{\text{time}}$$

The temperature difference between the two blobs can therefore be maintained at a characteristic time scale

$$\tau_{\text{th}} \sim \frac{d^2}{D}$$

We also seek a characteristic time scale for buoyant displacement over the length scale d.

Let

$$\begin{aligned} \rho_0 &= \text{mean density} \\ \Delta\rho &= -\alpha\rho_0\Delta T, \qquad \alpha = \text{expansion coefficient} \end{aligned}$$

Setting $\Delta T = \delta T$,

$$\begin{aligned} \text{buoyancy force density} &= |\vec{g}\Delta\rho| \\ &= g\alpha\rho_0\,\delta T. \end{aligned}$$

Note units:

$$[g\alpha\rho_0\delta T] = \frac{\text{mass}}{(\text{length})^2(\text{time})^2}$$

The buoyancy force is resisted by viscous friction between the two blobs separated by $\sim d$.

Viscosity is analogous to heat diffusivity, but it instead refers to the diffusion of momentum, so that fast-moving fluid causes nearby slow-moving fluid to to move faster, and vice-versa.

Because viscous stresses $\propto$ velocity gradients, the viscous friction between the two blobs diminishes like $1/d$.

We represent the viscosity with the coefficient μ, which has dimensions

$$[\mu] = \left(\frac{\text{mass}}{\text{volume}}\right) \times \text{diffusivity} = \frac{M}{L^3}\frac{L^2}{T} = \frac{M}{LT}.$$

The rescaled viscosity has units

$$\left[\frac{\mu}{d}\right] = \frac{\text{mass}}{(\text{length})^2(\text{time})}.$$

Dividing the rescaled viscosity by the buoyancy force, we obtain the characteristic time τ_m for convective motion:

$$\tau_m \sim \frac{\mu/d}{\text{buoyancy force}} = \frac{\mu}{g\alpha\rho_0\, d\, \delta T}.$$

Convection (sustained motion) occurs if

$$\begin{aligned} \text{time for motion} &< \text{diffusion time for temperature difference} \\ \tau_{\text{m}} &< \tau_{\text{th}} \end{aligned}$$

Thus convection requires

$$\frac{\tau_{\text{th}}}{\tau_{\text{m}}} > \text{constant}$$

or

$$\frac{\rho_0 g\alpha d^3}{\mu D}\delta T \equiv \text{Ra} > \text{constant}$$

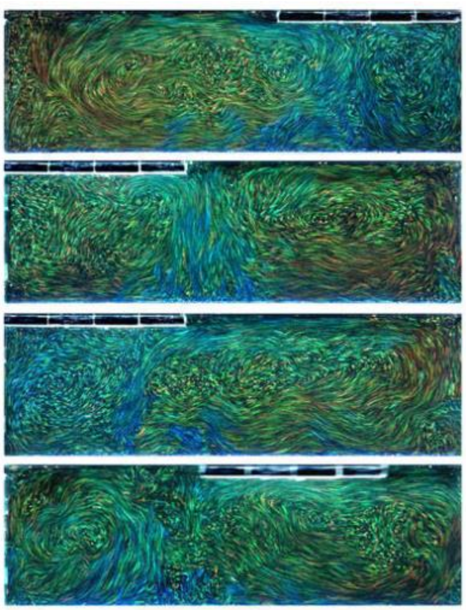
Ra is called the *Rayleigh number*. (A detailed stability calculation reveals that the critical constant is 1708.)

Our derivation of the Rayleigh number shows that the convective instability is favored by

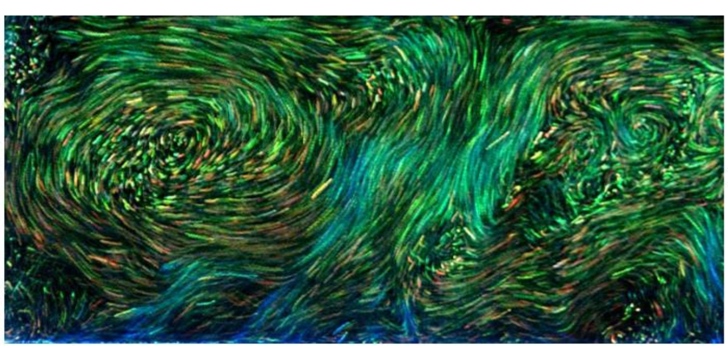
- large δT, α, d, ρ_0.
- small μ, D.

In other words, convection occurs when the buoyancy force $\rho_0 g \alpha d^3 \, \delta T$ exceeds the dissipative effects of viscous drag and heat diffusion.

Note that box height enters Ra as d^3. This means that small increases in box size can have a dramatic effect on Ra.



Here the viscous flow moves the floating boundary and the the boundary affects the flow, an interplay roughly analogous to fluid motions beneath tectonic plates.



A close-up (red is cool, blue is warm).

1.4 Convection in the Earth

The Earth's radius is about 6378 km. It is layered, with the main divisions being the inner core, outer core, mantle, and crust.

The Earth's crust—the outermost layer—is about 30 km thick.

The mantle ranges from about 30–2900 km.

The mantle is widely thought to be in a state of thermal convection. The source of heat is thought to be the radioactive decay of isotopes of uranium,

thorium, and potassium. Another heat source is related to the heat deriving from the gravitational energy dissipated by the formation of the Earth roughly 4.5 Ga.

At long time scales mantle rock is thought to flow like a fluid. However its effective viscosity is the subject of much debate.

One might naively think that the huge viscosity would make the Rayleigh number quite small. Recall, however, that Ra scales like d^3, where d is the "box size". For the mantle, d is nearly 3000 km!!!

Consequently Ra is probably quite high. Estimates suggest that

$$3 \times 10^6 \lesssim \mathrm{Ra}_{\mathrm{mantle}} \lesssim 10^9$$

which corresponds to roughly

$$10^3 \times \mathrm{Ra}_c \lesssim \mathrm{Ra}_{\mathrm{mantle}} \lesssim 10^6 \mathrm{Ra}_c$$

The uncertainty derives principally from the viscosity, and its presumed variation by a factor of about 300 with depth.

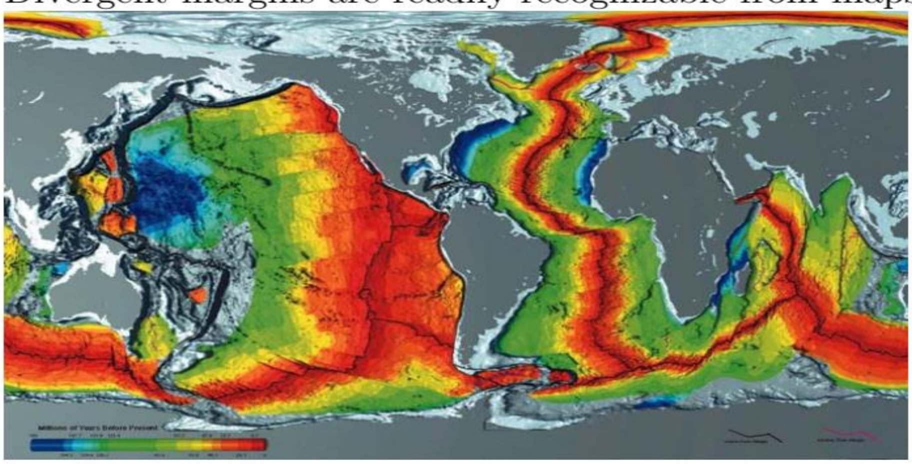
Regardless of the uncertainty, we can conclude that Ra for the mantle is more than sufficient for convection, and therefore that convection is likely the driving force of plate tectonics.

1.5 Mid-ocean ridges, seafloor age, and volcanism

Two kinds of plate motion caused by plate tectonics are of interest to the carbon cycle:

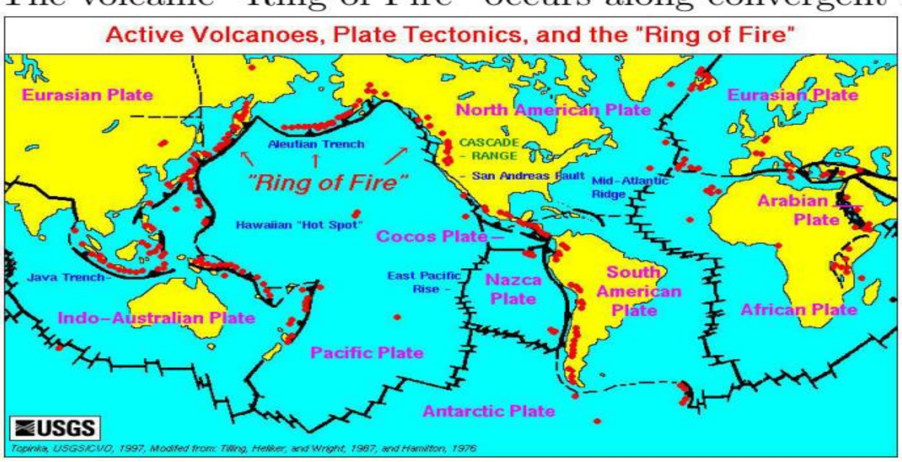
- Divergent (constructive) margins, typically at mid-ocean ridges, where new seafloor is created by volcanic processes.
- Convergent (destructive) margins, where plates collide and, typically, one plate is subducted beneath the other. The process forms island chains and mountain ranges, and results in volcanism about 250 km into the over-riding plate.

Divergent margins are readily recognizable from maps of seafloor age:



Seafloor age. Young seafloor is red, old is blue (NOAA, Wikipedia).

The volcanic "Ring of Fire" occurs along convergent margins:



Volcanism • (USGS).

Total CO_2 outgassing at convergent and divergent margins is roughly equal, within a factor of two or so.

1.6 Seafloor heat flux and topography

Aspects of the qualititative discussion above can be quantitatively expressed and compared to observations.

In the following, we calculate the heat flux out of the seafloor as a function of its age, or distance from the ridge, and compare it to measurements.

As the seafloor cools with age, it becomes more dense and effectively sinks more deeply into the underlying mantle. We predict how this change occurs and also compare it to measurements.

All this derives from the diffusive transport of heat, to which we now turn.

1.6.1 Heat equation

To keep things simple, we consider spatial variations of temperature T only with respect to a single spatial coordinate z (i.e., depth beneath the seafloor).

Define

$$j = \text{flux of heat through a unit area},$$

which has units

$$[j] = \frac{\text{watt}}{\text{m}^2} = \frac{\text{joule/s}}{\text{m}^2}.$$

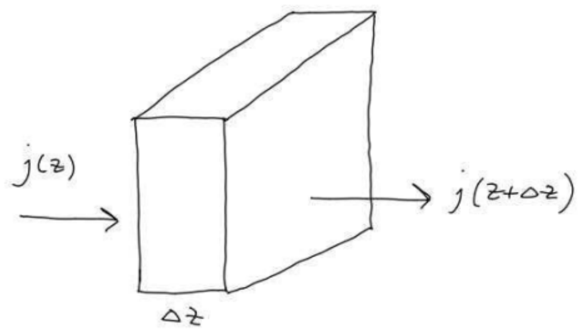
Fourier's law states that the flux j is proportional to the temperature gradient:

$$j = -k\frac{\partial T}{\partial z}, \tag{1}$$

where k is the *thermal conductivity*, with units

$$[k] = \frac{\text{watt}}{{}^\circ\text{K} \cdot \text{m}}.$$

Consider a slab with an incoming heat flux $j(z)$ and an outgoing heat flux $j(z + \Delta z)$:



If there are neither sources nor sinks of heat, then a net heat flow into the slab must change its temperature (i.e., energy is conserved).

Suppose the temperature of the slab increases by an amount ΔT during a short time Δt.

Define

$$\begin{aligned} c &= \text{heat required for a } 1^\circ\text{K change in a unit mass of the slab} \\ &= \textit{specific heat capacity}. \end{aligned}$$

which has units

$$[c] = \frac{\text{joule}}{\text{kg }^\circ\text{K}}.$$

Then

$$\begin{aligned} [j(z) - j(z+\Delta z)]\Delta t &= (\text{heat absorbed by slab over time } \Delta t)/\text{area} \\ &= c\Delta T \left(\frac{\text{slab mass}}{\text{area of slab face}}\right) \\ &= c\Delta T \rho \Delta z \end{aligned}$$

where ρ is the density of the slab.

Rearranging, we have

$$\rho c \frac{\Delta T}{\Delta t} = \frac{j(z) - j(z+\Delta z)}{\Delta z}$$

In the limit as $\Delta t \to 0$ and $\Delta z \to 0$,

$$\rho c \frac{\partial T}{\partial t} = -\frac{\partial j}{\partial z}.$$

Substitution of Fourier's law (1) for j then gives

$$\boxed{\frac{\partial T}{\partial t} = D\frac{\partial^2 T}{\partial z^2}}\,, \tag{2}$$

the one-dimensional *heat equation* or *diffusion equation*, where the *thermal diffusivity*

$$D = \frac{k}{\rho c}, \qquad [D] = \frac{\text{length}^2}{\text{time}}.$$