

**STOCHASTIC SIMULATION OPTIMIZATION OF WATER RESOURCES &  
MANAGEMENT**

FINAL EXAMINATION

TIME ALLOCATION: 3 HOURS

INSTRUCTION TO STUDENTS: ATTEMPT **ALL** QUESTIONS

**QUESTION 1 (20 Marks)**

A reservoir is planned both for gravity and lift irrigation through withdrawals from its storage. The total storage available for both the uses is limited to 5 units each year. It is decided to limit the gravity irrigation withdrawals in a year to 4 units. If  $X_1$  is the allocation of water for gravity irrigation and  $X_2$  is the allocation for lift irrigation, the two objectives planned to be maximized are expressed as

$$\text{Maximize } Z_1(X) = 3x_1 - 2x_2 \quad \text{and} \quad Z_2(X) = -x_1 + 4x_2$$

For above problem, do the following

- (i) Generate a Pareto Front of non-inferior (efficient) solutions by plotting Decision space and Objective space.
- (ii) Formulate multi objective optimization model using weighting approach with  $w_1$  and  $w_2$  as weights for gravity and lift irrigation respectively.
- (iii) Solve it, for (i)  $w_1=1$  and  $w_2=2$  (ii)  $w_1=2$  and  $w_2=1$
- (iv) Formulate the problem using constraint method

### QUESTION 2 (20 Marks)

Probability density function (PDF) of a random variable  $X$  is

$$f(x) = 6x^2 \quad 0 \leq x \leq 1$$
$$= 0 \quad \text{else where}$$

Determine (1) Cumulative distribution function (cdf); (2) Expected value,  $E(X)$ ; (3) Variance,  $\text{Var}(X)$ ; (4)  $P[X \geq 0.6]$ ; and (5)  $P[0.4 \leq X \leq 0.7]$

### QUESTION 3 (20 Marks)

The monthly streamflow at a reservoir site is represented by a random variable  $X$  which follows normal distribution with a mean of 100 units and a standard deviation of 50 units. Find (1)  $P[X > 150]$ ; (2)  $P[X \leq 40]$  and (3) The flow value which will be exceeded with a probability of 0.8.

### QUESTION 4 (20 Marks)

The time to failure of a pump in a water distribution system is assumed to follow an exponential distribution with the parameter  $\lambda = 0.0137/\text{day}$  (5 failures per year). Compute the reliability for 10 days operation.

### QUESTION 5 (20 Marks)

A farming company wanted to decide on the size and number of pumps required for lift irrigation. Four differently sized pumps ( $x_1$  through  $x_4$ ) were considered. The objective was to minimize cost and the constraints were to supply water to all fields (who have a strong seasonally fluctuating demand). That meant certain quantities had to be supplied (quantity

constraint) and a minimum number of fields per day had to be supplied (routing constraint). For other reasons, it was required that at least 6 of the smallest pumps should be included. The management wanted to use quantitative analysis and agreed to the following suggested linear programming approach. The available budget is Rs. 42 lakhs.

The optimization problem is

$$\begin{array}{ll}
 \text{Minimize} & 41,400 x_1 + 44,300 x_2 + 48,100 x_3 + 49,100 x_4 \\
 \text{Subject to} & 0.84 x_1 + 1.44 x_2 + 2.16 x_3 + 2.4 x_4 \geq 170 \\
 & 16x_1 + 16 x_2 + 16 x_3 + 16 x_4 \geq 1300 \\
 & x_1 \geq 6 \\
 & x_2, x_3, x_4 \geq 0
 \end{array}$$