PERFORMANCE MEASURES

INTRODUCTION

The outputs from simulation analysis need to be compared to select the best alternative. Multiple time series data can be summarized using either weighted arithmetic mean values or geometric mean values. Another way to summarize and compare time-series data is to calculate and compare the variance of the data. In this lecture we will discuss about the need for performance measures and some of the performance measures used in simulation studies.

NEED FOR PERFORMANCE CRITERIA

In water resources planning and operation, the alternatives are compared based on their risks. Risk is usually a measure of the (i) probability of occurrence of a specified undesirable outcome or (ii) number of undesirable occurrences over a specified time period or (iii) expected number of undesirable occurrences during a specified time period. While analyzing the performance of hydrosystems, it is common that all undesirable outcomes (or failures) are given equal importance. The magnitude and consequence of failure are not taken into account. The effects of failure are often treated independent to each other. These assumptions are not realistic, since the failures vary in magnitude and hence its significance. For example, consider a release $\leq$ 40% of the demand i.e., deficit of 60% as a failure. However, a deficit of 50% may not as damaging as that of 20%. Also, the occurrence of large deficits over consecutive periods will cause a greater damage than deficits for short period with adequate releases in between. The effects of failures are not independent.

As mentioned above the time series can be compared using mean and variance of the data. Consider two time series with same mean and variance as shown in figure 1.

x = [10, 40, 0, 50, 80, 85, 90, 60, 30, 10] and y = [80, 50, 90, 40, 10, 10, 0, 30, 60, 85].

Let the threshold be at 20. The occurrences below threshold are considered as failures. Even though the two series have same mean and variance, the impacts are different as can be inferred from the figure. For the second time series, the failures occur for a continuous long period.

Performance Criteria are used to evaluate the system performance in simulation. They will also be useful to evaluate and rank different alternative plans or policies using simulation. The commonly used performance measures are reliability, resilience and vulnerability.

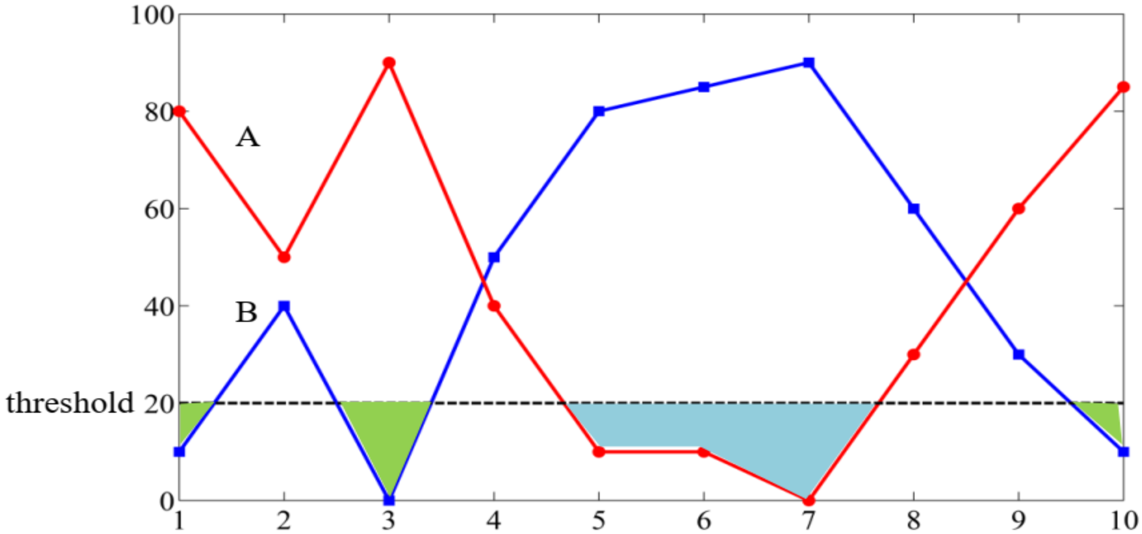

Fig. 1 Two time series with same mean and variance

Reliability

Reliability is a measure of how often the system is in satisfactory state. It is the probability of the system being in satisfactory state. It can also be defined as the number of data/periods in a satisfactory state divided by the total number of data/ periods in the simulation. Hence,

$$\text{Reliability}[X] = [\text{number of time periods t such that } X_t \geq X^T]/n$$

where X^T is the threshold value which determines the failures.

For the first time series, A (shown in red), there are three failures out of 10 occurences. Therefore, the reliability is 0.7. For the second series, B also (shown in blue), number of failures is three and reliability is 0.7.

However, a more reliable system may not necessarily be better than a less reliable system. Reliability neither tells anything about how quickly a system recovers and returns to a satisfactory value, nor does it indicate how bad an unsatisfactory value might be should one occur. Even though a system fails relatively often, the failures may be insignificant and for short durations. Such a system with low reliability will be much preferable to one whose reliability is much higher but where, when a failure does occur, it is likely to be much more severe. This necessitates the need for measures that can quantify these system characteristics.

Resiliency

Resiliency measures the capacity of the system to return to a satisfactory state after failure. It can be expressed as the probability that if a system is in an unsatisfactory state, the next state will be satisfactory. Resilience is an indicator of '**How quickly a system recovers from failure**'. It is the conditional probability of the system being in satisfactory state in period *t+1* when it was not in a satisfactory state in period *t*.

Resiliency [X] = [number of times a satisfactory value follows an unsatisfactory value]/ [number of times an unsatisfactory value occurred]

For the first time series (shown in red), the resilience is 1/3. For the second series (shown in blue), the resilience is 2/2 = 1. This is calculated by avoiding the failure at period 10 since there is no observation at period 11.

Resilience is not defined if there are no unsatisfactory values.

Vulnerability

It is necessary to examine the damage caused by a failure, even though there are a few failures. Vulnerability measures the severity of failure or it is is an indicator of '**How serious is the failure**'. It is a measure of the extent of the differences between the threshold value and the unsatisfactory time series values. The common measures used to calculate vulnerability are expected values, maximum observed values, and probability of exceedance.

Assuming an expected value measure of vulnerability is to be used:

Vulnerability [X] = [sum of positive values of $(X^T - X_t)$]/ [number of times an unsatisfactory value occurred]

For the first time series (shown in red), the expected vulnerability is [(20-10) + (20-10) + (20-0)]/3 =13.33. For the second series also (shown in blue), the expected vulnerability of the blue line is [(20-10) + (20-0) + (20-10)]/3 =13.33.

The reliability and vulnerability of both series are equal. However, the resilience of the second series is more than that of the first. This shows the typical tradeoffs one can define using these three measures of system performance.

COMBINATION OF SIMULATION AND OPTIMIZATION

Often, use of a single algorithm may not be sufficient to model large and complex water resource systems. A combination of Simulation and Optimization (S-O models) is quite often used. A major advantage of the S-O methodology in most situations is that the physical processes such as the mass, energy, and temperature balance are accounted through simulation outside the optimization model, thus reducing the size and complexity of the optimization model itself. Such modeling situations arise especially in management of water quality where the transport of pollutants across a stream is modeled by a simulation model reproducing the physical processes, and the result from such a simulation model is used in the optimization model to evaluate the objective function value. An example of S-O model for water quality management is shown in figure 2.

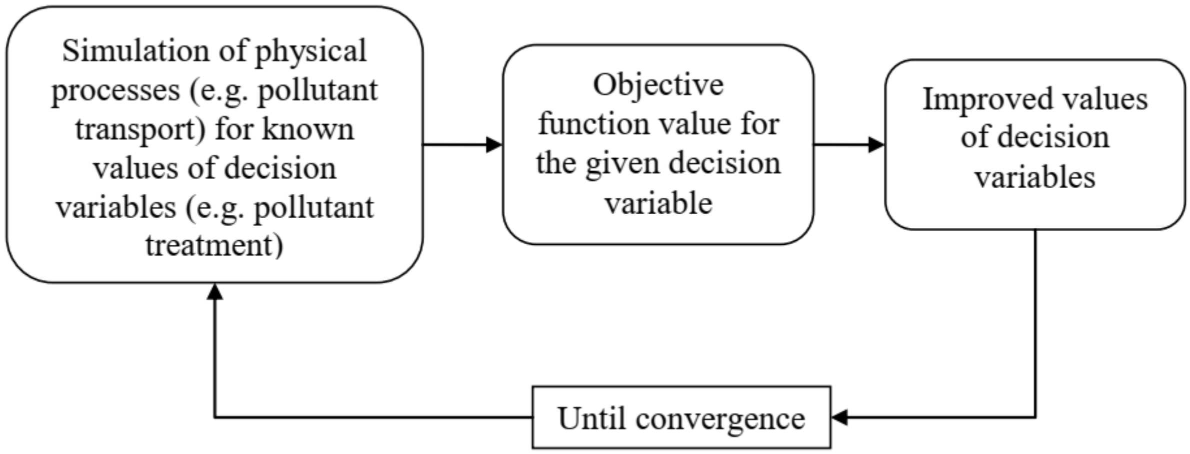

Fig. 2 Water quality management

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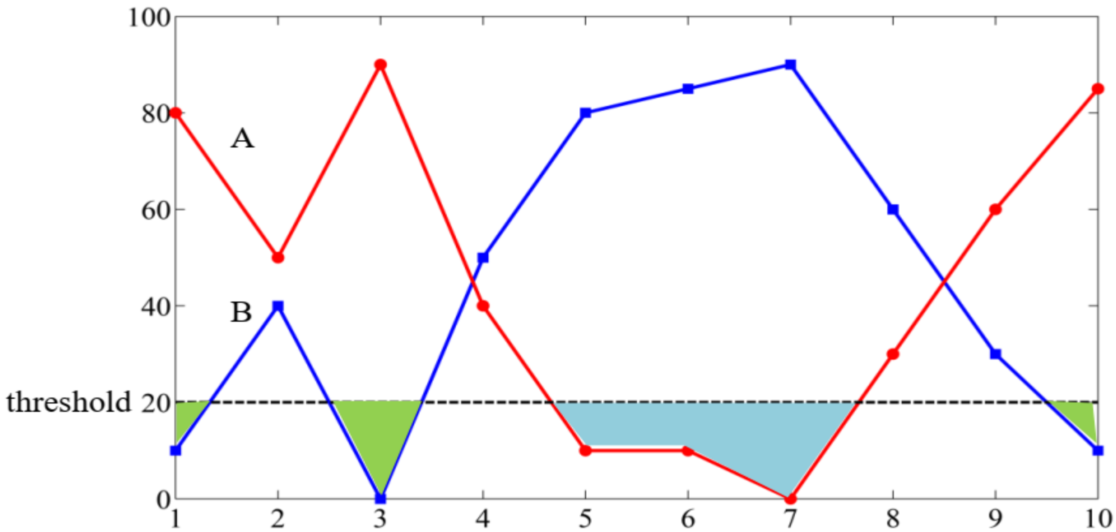

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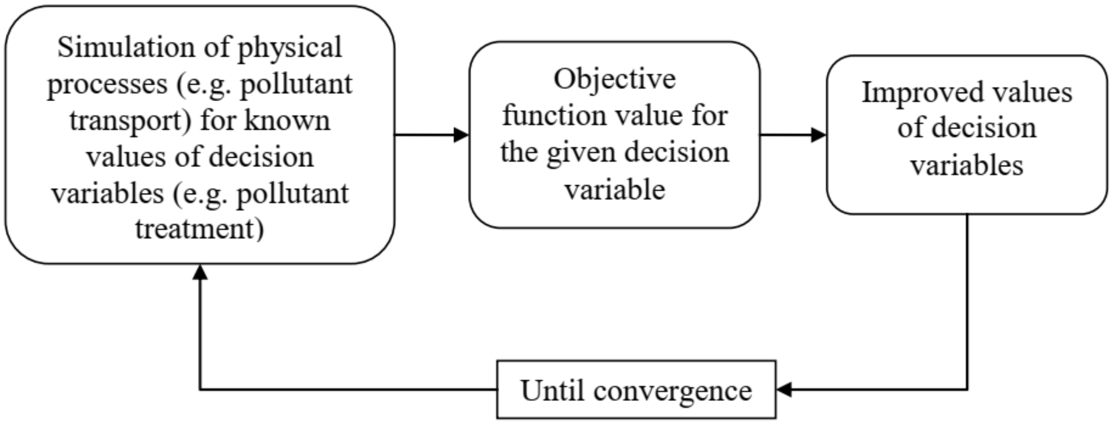

Fig. 2 Water quality management

FUZZY OPTIMIZATION

INTRODUCTION

The models discussed so far are crisp and precise in nature. The term crisp means dichotonomous i.e., yes-or-no type (or true or false) and not more-or-less type. This indicates that the model is unequivocal or it contains no ambiguities. Often most of the real situations are not crisp; but are vague. Fuzziness is the vagueness in the events, phenomena or statements like "*tall* men", "*beautiful* flower", "*profitable* deal" etc. In planning, plan A is *better* than plan B or plan A is *more acceptable* to some and *less acceptable* to others. In this lecture, we will illustrate how fuzziness can be incorporated in water resources optimization models.

FUZZY SET THEORY AND MEMBERSHIP FUNCTIONS

Let X be a crisp set of integers, whose elements are denoted by x. Consider a set of integer numbers ranging from 20 - 30. Hence, set A = [20, 30]. In classical or crisp set theory, any number, say x either exists in A or not, i.e., set A is crisp. Hence membership in a classical subset A of X can be expressed as a characteristic function

$$\mu_A(x) = \begin{cases} 1 & if\ x \in A \\ 0 & if\ x \notin A \end{cases} \tag{1}$$

The set [0, 1] is called the valuation set. Suppose when it is not certain about the existence of x in A, then set A is fuzzy. The degree of truth attached to that statement is defined by a membership function. The fuzzy set A is characterized by the set of all pairs of points denoted as

$$A = \{x, \mu_A(x)\} \quad x \in X \tag{2}$$

where $\mu_A(x)$ is the membership function of x in A. The closer the value of $\mu_A(x)$ is to 1, the more x belongs to A. For example, let the possible releases X from a reservoir be X = {25 30 35 40 45 50} and the irrigation demand be 40, then the fuzzy set A of "satisfiable releases without causing crop damage" may be

$$A = \{(25,0.25), (30,0.5), (35,0.75), (40,1), (45,0.75), (50,0.5)\}$$

This function is normally represented by a geometric shape which maps each point x to a membership value between 0 and 1, i.e., membership function ranges from 0 (completely false) to 1 (completely true). Commonly used membership function shapes are triangular, trapezoidal and bell shape (Gaussian). For the above example, the variation of membership function is assumed a triangular one as shown in figure 1.

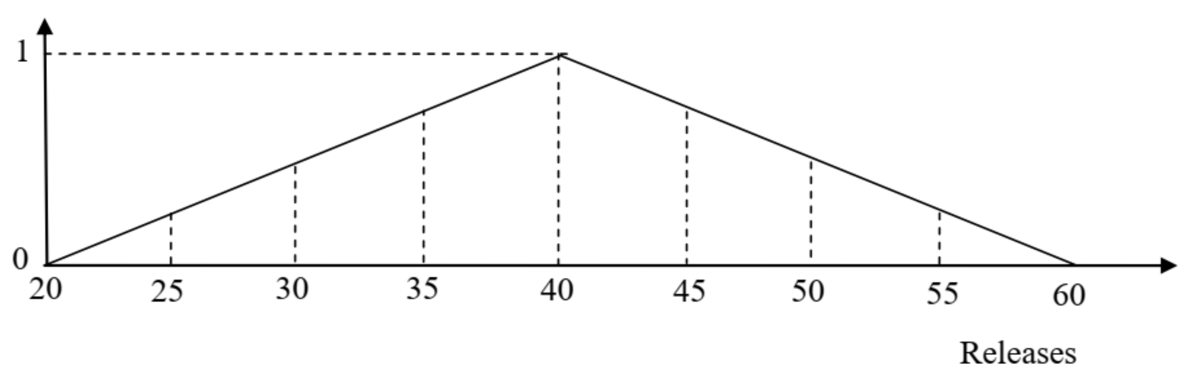

Fig. 1 Triangular shaped membership function

If X is a finite set $\{x_1, x_2, x_3, \ldots, x_n\}$ then the fuzzy set can be expressed as

$$A = \mu_A(x_1)/x_1 + \mu_A(x_2)/x_2 + \ldots + \mu_A(x_n)/x_n = \sum_{i=1}^{n} \mu_A(x_i)/x_i \qquad (3)$$

If X is infinite

$$A = \int_x \mu_A(x)/x \qquad (4)$$

The basic set theory operations like union, intersection and compliment can be defined for fuzzy sets also. Let A and B be two fuzzy sets and μ_A and μ_B be their membership functions as shown in figure 2.

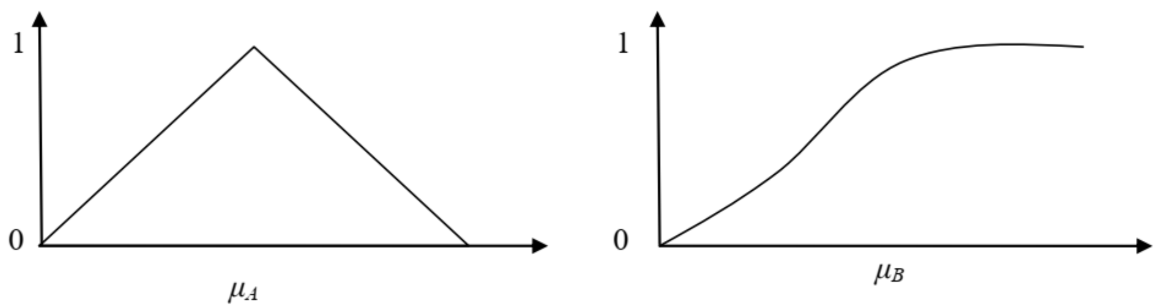

Fig. 2 Membership functions of A and B

The union of fuzzy sets A and B can be defined as

$$\mu_{A\cup B}(x) = \max\left[\mu_A(x), \mu_B(x)\right] = \begin{cases} \mu_A(x) & if \;\; \mu_A > \mu_B \\ \mu_B(x) & if \;\; \mu_A < \mu_B \end{cases} \qquad (5)$$

The intersection of fuzzy sets A and B can be defined as

$$\mu_{A\cap B}(x) = \min\left[\mu_A(x), \mu_B(x)\right] = \begin{cases} \mu_A(x) & if \;\; \mu_A < \mu_B \\ \mu_B(x) & if \;\; \mu_A > \mu_B \end{cases} \qquad (6)$$

The complement of fuzzy sets A is $\mu_{\bar{A}}(x) = 1 - \mu_A(x)$

The three operations are shown in figure 3.

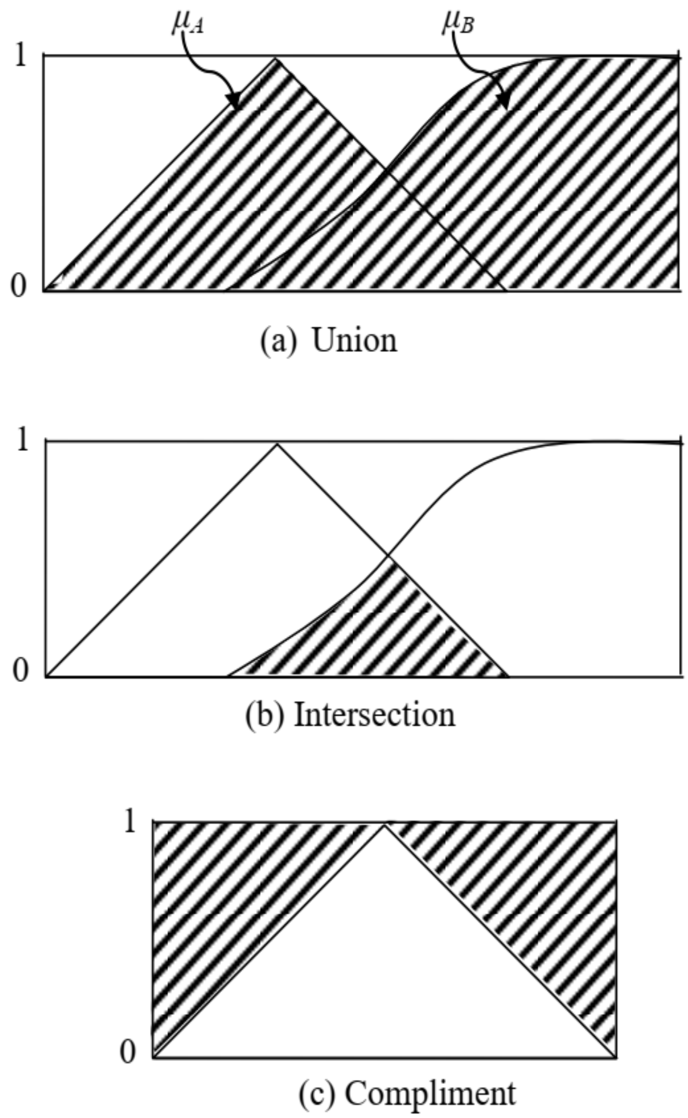

Fig. 3 Fuzzy set operations (a) Union (b) Intersection and (c) Compliment

FUZZY OPTIMIZATION

Conventional optimization models find the optimum value of design variables which optimizes the objective function subject to satisfying the stated constraints. If the system is fuzzy, then this optimization problem needs to be revised. In a fuzzy system, the objective and constraints are expressed by the membership functions. Hence a decision is the intersection of the fuzzy objective and constraint functions. For example, consider the water allocation problem in which the objective function is "The water allocated for irrigation should be substantially greater than 10". The membership function for objective function f is

$$\mu_f(x) = \begin{cases} 0 & \text{if } x < 10 \\ \left[1 + (x-10)^{-2}\right]^{-1} & \end{cases} \tag{7}$$

Let the constraint be "The amount of water allocated should be around 11.5". The membership for this constraint is $\mu_g(x) = \left[1 + (x-11.5)^4\right]^{-1}$. Then the decision can be described by the membership function, $\mu_D(x)$ as

$$\begin{aligned} \mu_D(x) &= \mu_f(x) \wedge \mu_g(x) \\ &= \begin{cases} 0 & \text{if } x < 10 \\ \min\left\{\left[1 + (x-10)^{-2}\right]^{-1}, \left[1 + (x-11.5)^4\right]^{-1}\right\} & \text{if } x \geq 10 \end{cases} \end{aligned} \tag{8}$$

This is shown in figure 4.

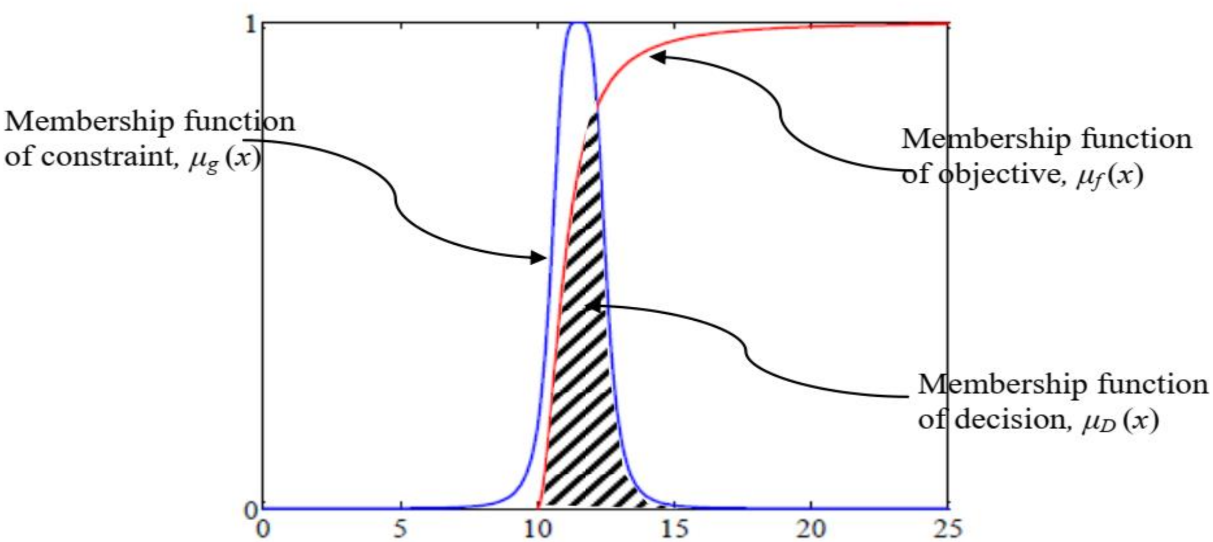

Fig.4 Fuzzy decision

Formulation:

Let the conventional optimization problem be

Minimize $f(X)$

Subject to m constraints

$ll_j \leq g_j(X) \leq ul_j \qquad \text{for } j = 1,2,\ldots,m$

where ll_j is the lower bound and ul_j is the upper bound of the jth constraint.

The fuzzy optimization problem can be stated as

Minimize $f(X)$

Subject to m constraints

$$g_j(X) \leq G_j \qquad \text{for} \quad j = 1,2,\ldots,m$$

where G_j is the fuzzy interval the constraint $g_j(X)$ should belong.

The feasible region of this fuzzy system is the intersection of all these G_j's and is defined by the membership function

$$\mu_S(X) = \min_{j=1,2,\ldots,m} \left[\mu_{G_j}(g_j(X))\right]$$

The optimum value is the maximum value of the intersection of objective function and feasible domain.

$$\mu_D(X^*) = \max \ \mu_D(X)$$

where

$$\mu_D(X) = \min\left[\mu_f(X), \min_{j=1,2,\ldots,m} \mu_{G_j}(g_j(X))\right]$$

FUZZY LINEAR PROGRAMMING

A classical LP model can be defined as

$$\text{Maximize } f(x) = c^T x$$

Subject to $\qquad Ax \leq b; \quad x \geq 0 \qquad (9)$

In this model, the coefficients A, b, and c are crisp numbers; $\leq$ is meant in a crisp sense and "*maximize*" is a strict imperative.

While dealing with real problems, the objective function and constraints themselves may be vague as discussed in the last section. The $\leq$ sign might not be meant in the strictly mathematical sense but smaller violations might well be acceptable. This results in fuzziness in the model.

Fuzziness in LP Model

In an LP model, the coefficients of the vectors b or c or of the matrix A itself can have a fuzzy character. This can happen either because they are fuzzy in nature or because perception of them is fuzzy. In classical LP, the violation of any single constraint by any amount renders the solution infeasible. In real situation, the decision maker might accept small violations of constraints but might also attach different (crisp or fuzzy) degrees of importance to violations of different constraints. Fuzzy LP offers a number of ways to allow for all those types of vaguenesses.

Fuzzy LP

In fuzzy LP, the goal and the constraints are represented by fuzzy sets and then aggregate them in order to derive a maximizing decision (*Bellman-Zadeh's* approach). In contrast to classical LP, FLP is ***NOT*** a uniquely defined type of model but many variations are possible, depending on the assumptions or features of the real situation to be modeled.

Symmetric Fuzzy LP

In this method, decision maker can establish an aspiration level, z, for the value of the objective function he wants to achieve. Each of the constraints is modeled as a fuzzy set. Fuzzy LP can then be formulated as:

$$\begin{aligned} c^T x &\gtrapprox z \\ Ax &\lessapprox b; \quad x \geq 0 \end{aligned} \tag{10}$$

Objective function is converted to a fuzzy goal. Fuzzified version of $\geq$ has the linguistic interpretation "essentially greater than or equal". Fuzzified version of $\leq$ has the linguistic interpretation "essentially smaller than or equal". Each constraint and objective function will be represented by a fuzzy set with a membership function $\mu_i(x)$. Membership function $\mu_i(x)$ increases monotonously from 0 to 1 with a value 0 if the constraints (including objective function) are strongly violated and a value 1 if they are very well satisfied (i.e., satisfied in the crisp sense). Membership function can expressed as

$$\mu_i(x) = \begin{cases} 1 & \text{if } B_i x \leq d_i \\ \in [0,1] & \text{if } d_i < B_i x \leq d_i + p_i \\ 0 & \text{if } B_i x > d_i + p_i \end{cases} \quad i = 1,2,\ldots m+1 \tag{11}$$

where p_i is tolerance interval (subjectively chosen).

Now assuming linear increase over the tolerance interval $\mu_i(x)$ will be

$$\mu_i(x) = \begin{cases} 1 & \text{if } B_i x \leq d_i \\ 1 - \dfrac{B_i x - d_i}{p_i} & \text{if } d_i < B_i x \leq d_i + p_i \\ 0 & \text{if } B_i x > d_i + p_i \end{cases} \quad i = 1,2,\ldots m+1 \tag{12}$$

Hence, fuzzy LP model can be defined as

$$\begin{aligned} &\text{Maximize } \lambda \\ \text{Subject to} \quad &\lambda p_i + B_i x \leq d_i + p_i \qquad i = 1,2,\ldots,m+1 \\ &x \geq 0 \end{aligned} \tag{13}$$

where λ is one new variable.

The optimal solution is the vector (λ, x^*). Hence in fuzzy LP model maximizing solution can be obtained by solving one standard (crisp) LP with only one more variable and one more constraint than the original crisp LP model.

Example:

A farming company wanted to decide on the size and number of pumps required for lift irrigation. Four differently sized pumps (x_1 through x_4) were considered. The objective was to minimize cost and the constraints were to supply water to all fields (who have a strong seasonally fluctuating demand). That meant certain quantities had to be supplied (quantity constraint) and a minimum number of fields per day had to be supplied (routing constraint). For other reasons, it was required that at least 6 of the smallest pumps should be included. The management wanted to use quantitative analysis and agreed to the following suggested linear programming approach. The available budget is Rs. 42 lakhs.

The optimization problem is

$$\begin{aligned}
\text{Minimize} \quad & 41{,}400\, x_1 + 44{,}300\, x_2 + 48{,}100\, x_3 + 49{,}100\, x_4 \\
\text{Subject to} \quad & 0.84\, x_1 + 1.44\, x_2 + 2.16\, x_3 + 2.4\, x_4 \geq 170 \\
& 16x_1 + 16\, x_2 + 16\, x_3 + 16\, x_4 \geq 1300 \\
& x_1 \geq 6 \\
& x_2, x_3, x_4 \geq 0
\end{aligned}$$

The solution of this problem using classical LP is

Min Cost = Rs. 38,64,975

$x_1 = 6$, $x_2 = 16.29$, $x_3 = 0$, $x_4 = 58.96$.

Fuzzy LP

As the demand forecasts had been used to formulate the constraints, there was a danger of not being able to meet higher demands. It is safe to stay below the available budget of Rs. 42 lakhs. Therefore, bounds and spread of the tolerance interval are fixed as follows:

Bounds: $d_1 = 37{,}00{,}000$; $d_2 = 170$; $d_3 = 1{,}300$; $d_4 = 6$

Spreads: $p_1 = 5{,}00{,}000$; $p_2 = 10$; $p_3 = 100$; $p_4 = 6$

Objective function in the classical LP problem is transformed as a constraint

$$41{,}400\, x_1 + 44{,}300\, x_2 + 48{,}100\, x_3 + 49{,}100\, x_4 + \lambda \leq 42{,}00{,}000$$

The optimization problem constraints are (acc. to eqn. 13)

Maximize λ

Subject to

$$0.083\, x_1 + 0.089\, x_2 + 0.096\, x_3 + 0.098\, x_4 + \lambda \leq 8.4$$

$$0.084\, x_1 + 0.144\, x_2 + 0.216\, x_3 + 0.240\, x_4 - \lambda \geq 17$$

$$0.16\, x_1 + 0.16\, x_2 + 0.16\, x_3 + 0.16\, x_4 - \lambda \geq 13$$

$$0.167\, x_1 - \lambda \geq 1$$

$$\lambda, x_2, x_3, x_4 \geq 0$$

Solutions obtained using classical and fuzzy LP are given below:

Classical LP	**Fuzzy LP**
Z = 38,64,975	Z = 39,88,250
$x_1 = 6$; $x_2 = 16.29$; $x_4 = 59.96$	$x_1 = 17.41$; $x_2 = 0$; $x_4 = 66.54$
Constraints:	
1. 170	1. 174.33
2. 1300	2. 1343.328
3. 6	3. 17.414

Through Fuzzy LP, a "leeway" has been provided with respect to all constraints and at additional cost of 3.2%. The decision maker is not forced into a precise formulation because of mathematical reasons even though he/she might only be able or willing to describe his/her problem in fuzzy terms.