SIGNIFICANCE OF DIFFERENCES BETWEEN DISTRIBUTIONS.

It has already been pointed out that the usual application of inferential statistics involves observing differences between samples and asking if these are so large as to require rejection of the null hypothesis: If they are, the difference is said to be significant. Inferential statistics provide criteria for deciding whether the difference is significant or not.

One of the simplest inferential statistics is called Chi-Square (x^2). This statistic is appropriate for use with nominal data. Basically, it can be used for testing for significance of differences (a) between a distribution actually obtained and a theoretically expected one: (b) between distributions of two empirically measured variables. Both these cases will be discussed in following sections.

Difference between an empirical and a theoretical distribution.

Suppose that you threw a dice 300 times and obtained the following distribution of is, 2s, 3s, 4s, 5s, and 6s:

Table 13.1: Frequency of scores on throwing a dice.

Score	f
1	40
2	58
3	34
4	61
5	66
6	41

Theoretically, the chances of throwing a 1 or a 6 or any other score are all equal: a 1 should occur one sixth of the time, a 2 one sixth of the time, and so on. Thus, when the dice was thrown 300 times, you would expect fifty is, fifty 2s, fifty 3s, and so on —The question now arises of whether or not this numerical difference is statistically significant. The statistic x^2 can be used to test for statistical significance. The null hypothesis is, of course:

H_o: There is no difference between the distributions.

H_i: There is a difference between the distributions.

PARAMETRIC STATISTICS

Inferential statistics can be divided into two classes: parametric statistics on the one hand, nonparametric, on the other. The crucial difference is that parametric statistics involve assumptions about the distributions of the variables in question, whereas nonparametric tests make few or only simple assumptions which are easily met by many different forms of data. It is common to refer to nonparametric tests as "distribution free"; even though this may not be completely accurate, it leads to a convenient and simple way of distinguishing between the two categories of statistical test; parametric tests assume certain properties of the data, nonparametric do not. Nonparametric procedures will not be dealt with in this unit, and little more will be said about them here.

Assumptions and Prerequisites

The tables of critical values for parametric statistics make certain assumptions about

(a) the type of data (nominal, ordinal, interval, ratio);

(b) the distributions of the variables in the population;

(c) the independence of samples;

(d) the sample variances.

If these assumptions are not met, at the very least the critical values in the appropriate tables will not be correct for the stated Alpha level. If the table shows that with p = .05 the critical value of the statistic t is 2.20 with df = 11, this is true only if the assumptions applying to t are met. If they are not, the real critical value for 11 df is different from 2.20 or, to put it another way, the probability of incorrectly rejecting the null hypothesis is not equal to exactly **0.05,** but is an unknown amount larger or smaller. Which it is, and by how much, depends upon the exact case in question. In psychological statistics we are usually anxious to keep 0^4--error small; for this reason, the following material will concentrate on increased probability of - error. Because the assumptions determine whether or not the statistical test can validly be applied or not, they are often referred to as prerequisites: if the prerequisites are met, the statistical test may be used, if not, it may not be used.

Although special statistics may have their own special prerequisites (or assumptions, to put it another way), it is possible to summarize typical prerequisites for applying the most common parametric statistics. These are:

1. Data must be interval or ratio in nature.

In the case of nominal or ordinal data, nonparametric tests are used. (Since Chi-square is used with nominal data, you might conclude that it is a nonparametric test— you would be right.)

2. The variables must be normally distributed in the population.
3. Samples must be independent, i.e., not correlated with each other.
4. Sample variances must be equal.

Robust tests

A failure of data to satisfy the prerequisites results in the Alpha levels stated in the tables of critical values being incorrect. This means that the probability of falsely rejecting the null hypothesis (i.e. of making an 0^4 - error) is frequently greater than the level stated in the table of critical values. The risk of error associated with rejecting the null hypothesis is greater than .05 or .01.

Some statistical tests are not much affected by failure to meet the prerequisites; i.e., they continue to reject the null hypothesis with a likelihood of error which remains close to .05, even when the prerequisites are not met. Such tests are said to be robust. In fact, despite the emphasis many applied statisticians place on fulfilling prerequisites, many parametric tests are very robust.

A Priori testing of prerequisites

Because of the importance of using parametric tests where possible, it is desirable to know how to check whether or not a particular set of data is suitable for parametric statistical analysis. Tests exist with which this can be done, and these are often referred to as a priori tests. Whether data are interval or not can be determined by considering the nature of the scale with which they are obtained. If the characteristics of the scale yield numbers with equal distances between equidistant scores, then the data are interva. In a number of cases, the convention in psychology is to assume interval data, even though this has not been proved (e.g., data from many psychological rating scales). Otherwise, when in doubt you should assume non-interval data and apply nonparametric statistics. Since the population data are not known, normality of the population distribution cannot be tested directly. However, it is often possible to infer normality of the population distribution by thinking about the nature of the variable in question. It is also possible to test statistically whether sample data are normally distributed; this can be done by applying Chi-square, as explained in Section 13.1— the distribution in your sample is the obtained distribution (C), the normal distribution is the expected distribution (E). If samples are normally distributed, this adds to the plausibility of assuming that the populations are normally distributed. Independence of samples can usually be ascertained by considering the sampling procedure: If membership in the first sample influences membership in the second, then the samples are not independent. Finally, equality of sample variances can be tested statistically in a number of ways, for instance by applying the statistic F_{in},, This simply involves dividing the largest sample variance by the smallest.