

VARIABILITY

The three measures of central tendency just discussed - the mean, median and mode - have limited meaning as typical or representative values if we are not aware of the spread or variation in the scores. For example, look at the following scores. The scores represent (hypothetical) marks out of 30 in an examination for two samples of students.

Sample A	10	12	15	18	20
Sample B	2	8	15	22	28

If we calculate the mean of each sample, we find they both have the same mean, namely 15. Simple inspection indicates, however, that the measurements in Sample B are more variable, or dispersed, than those in Sample A - that is, the scores in Sample B differ more from each other. Our measures of central tendency will have more meaning if we can describe this variation in some way. In other words, indicators of central tendency, such as the mean, or the median, tell us what performance is typical for a group. However, this is not enough information to describe a distribution of observations. We also need to know how widely the scores are dispersed around this central point.

The Range

The simplest and most readily calculated way to establish the amount of variability in a set of scores is the range. In any sample of measurements, the range is taken as the number of scale points from the lowest to the highest value, inclusive. The range for the measures 10, 12, 15, 18 and 20 is 11. The range for the measurements 2, 8, 15, 22 and 28 is 27. As we noted earlier, the measurements from the second set quite clearly exhibit greater variation than those in the first set, and this is reflected in a much greater range.

The range, however, has a major disadvantage. This is because its value is determined by two and only two scores in a distribution - the most extreme scores. These two values may or may not be representative of the range of the bulk of the scores. A wide gap may exist between the lowest score and the next lowest one, in which case the lowest score would not reflect the spread of the greater portion of the data. An example is shown in Table 6.1.

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Table 6.1: Data illustrating problems in use of the range

X	
21	3
20	4
19	
18	9
17	8
16	5
15	0
14	0
13	0
12	1

In this case, the scores progress continuously from 16 to 21, a range of 6, however the actual range for the distribution is 10. The one person who got a score of 12 determines this broader range compared with the narrower range within which the bulk of the data is dispersed. For this reason, we should use another method of describing variability whenever possible.

Percentiles

Sometimes it is informative to divide the range into one hundredth. Suppose that 500 students obtained scores on a maths test, on which the scores ranged from 17 to 90. One hundredth of the number of students is 5 ($500 \div 100 = 5$). What score was obtained by the person 5th from the bottom? This person would be one hundredth (1%) of the way from the bottom. Suppose that this score was 19. We can say that the first percentile (sometimes written %ile) was 19. Eight percent of the 500 students would encompass the lowest 40 (8% of 500 is 40). Accordingly, the score of the 40th person would define the 8th percentile. At the higher end of the distribution, student number 450 from the bottom would be 90% of the way to the top of the group (90% of 500 is 450). Suppose that this person obtained a score on the maths test of 82. This score (82) marks the 90th percentile in the distribution of maths scores. The person at the very top is beyond the 99th percentile, the person at the bottom is below the 1st percentile. In calculating percentiles, it is customary to start at the lowest score, so that a low percentile, such as let us say, the 9th, indicates that a particular score was near the bottom. A high percentile, such as the 96th, indicates that the score was near the top of the distribution.

Since percentiles divide a distribution into 100 units, a description of the variability or dispersion of a distribution in terms of percentiles is still very unwieldy: in theory you would need to know 100 values in order to describe the distribution. For this reason, it is common to calculate quartiles. The quartiles are the percentiles that divide a distribution into four equal segments; the 25th, 50th and 75th percentiles. A score which is one quarter of the way from the lowest score in a distribution is the 1st quartile, the point half way from the bottom (two quarters) is the 2nd quartile, and the point three quarters of the way is the 3rd quartile. These are abbreviated respectively Q1, Q2 and Q3.

Interquartile range and semi-interquartile range

A special form of the range statistic is calculated not by subtracting the lowest score from the highest, but by subtracting the score standing at the 1st quartile from the one marking the 3rd quartile. This difference is referred to as the **interquartile** range. As a formula, it is calculated in the following way:

$$\text{Interquartile range} = Q_3 - Q_1$$

Because the point of quartiles is to divide the distribution into four regions, whereas the interquartile range covers the middle 50% (i.e., half the scores), the interquartile range is frequently divided by two to yield the semi-interquartile range.

The Standard Deviation

The best indicator of dispersion of scores would be one which takes into account the location of every score in the distribution. Since variability indicates spread of scores and since scores in distributions tend to be distributed around a central point (the mean or median) a very reliable indicator of variability would begin at the mean (or average) and calculate the distances of scores above-and below* that central point. We could then compute the average distance of all the scores from the mean. The distance from each observation to the mean is called deviation.

For example, imagine the following five numbers stand for the number of statistics exercises correctly answered by 5 students:

3, 4, 5, 6, 7

Here the mean is 5. How widely dispersed are the scores around the mean? Our deviations are as follows:

below the mean	above the mean
3-5 = -2	6-5 = +1
4-5 = -1	7-5 = +2
-3	+3

As you can see from this, some of the deviations about the mean are positive, some are negative. In the above example, if we added all the deviations together (as we would have to do as the first step in our calculation of the mean deviation) we would get a value of zero. To avoid this (since a value of zero tells us very little about the spread of scores around the mean) a method of eliminating the minus signs must be used. We find that method in squaring the deviations so that all values will be positive numbers, averaging them and then "un-squaring" them. The resulting value is called a **standard deviation (SD)**.

$$\text{Thus } SD = \sqrt{\frac{1}{N} \sum (X_i - \bar{X})^2}$$

That is:

1. Subtract the mean (\bar{X}) from each individual score (X_i) and square each of these differences, thus giving a set of "squared deviations".
2. Add together all these squared deviations.
3. Divide by the number of subjects (N) - this gives us what is called the mean-square deviation, or variance.
4. Take the square root of this value, to give SD.