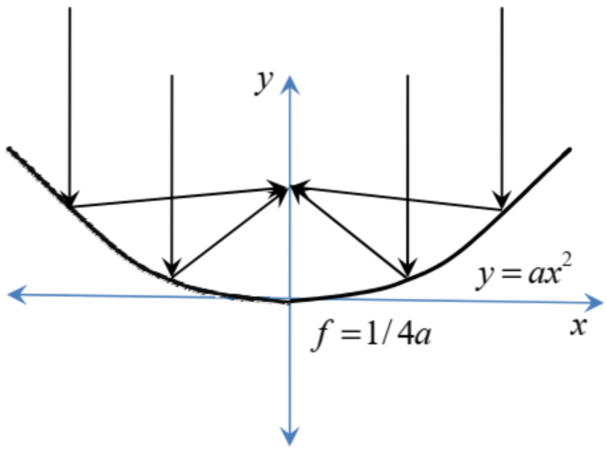
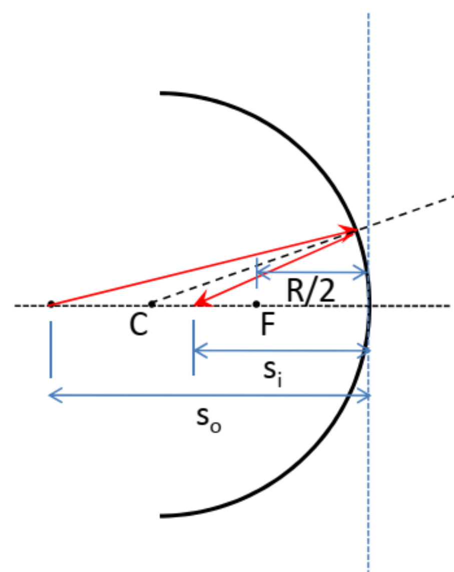
Non-Planar Reflectors



- Parabolic mirrors: Using the law of reflection at various points where the incident light ray strikes the parabolic surface, one can show that all rays will go through the focus shown in the diagram. So the parabolic surface acts as a perfect focusing mirror

Spherical Mirrors

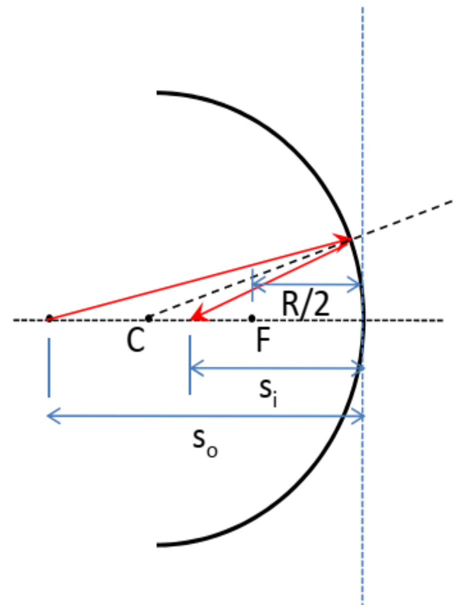
- However, spherical mirrors are easier to manufacture. In the paraxial regime a spherical mirror can be assumed to have a focus of f = R/2. Paraxial approximation considers only rays that are very close to the normal, i.e. small angles where sinθ = θ



Mirror Equation

- Using the geometrical construction shown in the diagram, tt can be shown that

$$\frac{1}{s_o} + \frac{1}{s_i} = \frac{2}{R}$$

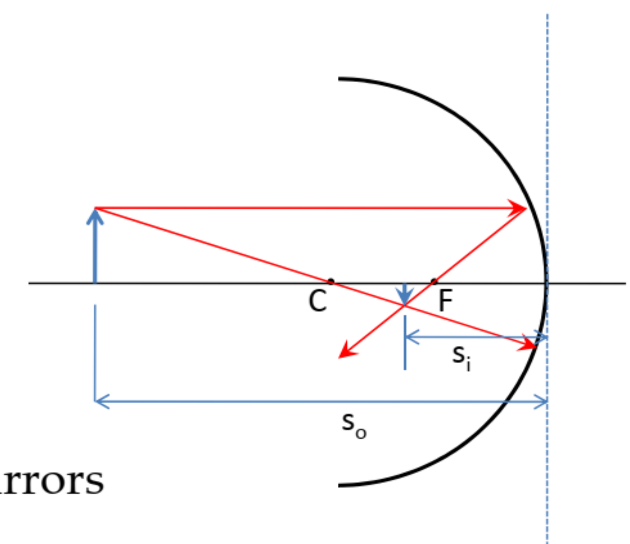


- This is the mirror equation (Gauss)

Finite Objects: Magnification

- It can be shown that image magnification is given by,

$$M = -\frac{R}{R + 2s_o}$$



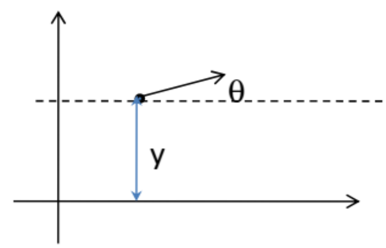
- Ray Diagrams
- Virtual image from convex mirrors
  - Use ray diagrams
- Sign convention:
  - Convex: f is negative
  - Concave: f is positive

Matrix Method for Geometric Optics Analysis

- Optical systems with several components are easily analyzed by a matrix method. Here, a ray is characterized by the position and the direction as shown in the diagram.

- Transmission through any component is described by a matrix multiplication as follows

$$\begin{pmatrix} y_2 \\ \theta_2 \end{pmatrix} = \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix} \begin{pmatrix} y_1 \\ \theta_1 \end{pmatrix}$$



Matrix Method for Geometric Optics Analysis

- Transmission through a number of components, e.g a lens with focal length f1, free space passage through distance d1, reflection at a mirror followed by free space passage through distance d2 and finally transmission through a lens with focal length f2 can be analyzed by simply multiplying the appropriate matrices in the right order. For the train of optical elements given below, the equivalent matrix will be

$$M_{net} = M_n{*}M_{n-1}{*}\dots{*}M_2{*}M_1$$

Some Matrix Representations

- Consider translation in free space through distance d,
- Then $y_2 = y_1 + d^*\theta_1$ and $\theta_2 = \theta_1$
- Therefore, the matrix representing translation will be

$$M = \begin{pmatrix} 1 & d \\ 0 & 1 \end{pmatrix}$$

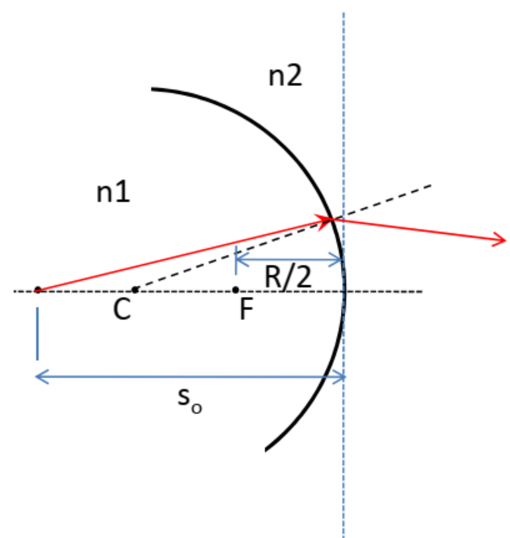
- Similarly the matrix representing plane reflection will be

$$M = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Spherical Refraction

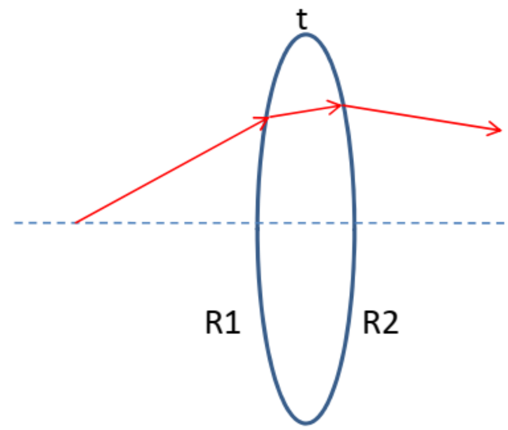
- The matrix representing spherical refraction as shown in diagram can be found using simple geometry to be,

$$M = \begin{pmatrix} 1 & 0 \\ \frac{n_1 - n_2}{n_2 R} & \frac{n_1}{n_2} \end{pmatrix}$$



Spherical Lenses

- Using the previous result for a single spherical interface we can analyze the transmission through a spherical lens as shown in the diagram



- One has to multiply the matrix for the spherical interface with radius R2, followed by translation through thickness t, followed by spherical interface with radius R1

Spherical Lens: Focal length

- Assuming refractive index to be same on both sides of the lens, one can show that the equivalent matrix for the lens is $M = \begin{pmatrix} 1 & 0 \\ \frac{-1}{f} & 1 \end{pmatrix}$ where, $\frac{1}{f} = (n-1)\left(\frac{1}{R_2} - \frac{1}{R_1}\right)$

- This result is for t = 0 or thin lens

Focusing by Thin Lens

- A consequence of our definition of f in the previous slide is that $f > 0$ for bi-convex or plano-convex lenses and $f < 0$ for bi-concave or plano-concave lenses.

- Using the matrix derived in the previous slide, it is straightforward to show that a lens will focus all parallel rays to a single point at a distance f from the center of the lens. Therefore, f in the previous slide is the focal length of the lens.

Diffraction Limit

- According to geometric optics all parallel rays are focused on to a *single point*. But later on we will see that the wave nature of light implies that this is not possible. There is a limit of how much one can focus light. This limit is called the diffraction limit.

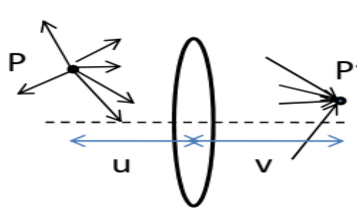
Lens Equation

- Applying the transformation matrix $M = \begin{pmatrix} 1 & 0 \\ \frac{-1}{f} & 1 \end{pmatrix}$

for a lens to the set of rays from P we get,

$y' = (1 - v/f)y + \{u(1 - v/f) + v\}\theta$ and

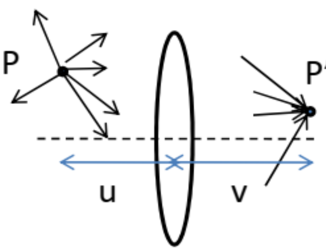
$\theta' = -y/f + (1 - u/f)\theta$



Lens Equation

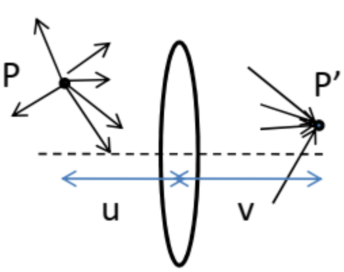
- If point P is to be imaged on to point P′ all rays passing through the lens must pass through P′ irrespective of θ. This means y′ must be independent of q. By rearranging the coefficient of q in the equation for y′, we get the famous lens equation that links the object and image distance with the focal length of the lens

  $$1/f = 1/u + 1/v$$



Lens Magnification

Also, we see that, image magnification = M = y′/y = -v/u. By appropriate choice of f and u, one can create single lens object magnifiers which are the basic stepping stones to optical microscopy.



Lens Systems

- Using the transformation matrix for lens derived in this lecture, one can analyze lens systems consisting of multiple lenses (convex or concave) with different focal distances.

- Modern microscope lenses consists of several lenses arranged to compensate for image aberrations such as spherical aberration where rays striking at different distances from the lens axis (called the optical axis) focus at different points (due to the error in approximating a spherical surface with a parabolic surface); or chromatic aberration where light with different wavelengths (color) gets focused at slightly different focal points.

Summary

- As we saw in this lecture, geometric optics is the simple application of laws of reflection and refraction and the behavior of an optical component is a function of its geometry (e.g. focusing of rays by a parabolic surface.