Wave Optics

- We saw in the last lecture that the phenomena of reflection and refraction could be explained by treating light as a ray obeying the laws of reflection and refraction at interfaces. This treatment is referred to as geometrical optics.

- However people discovered phenomena which could not be explained by geometric optics such as interference of light, diffraction patterns and so on which required the treatment of light as a wave.

- This lecture will provide a brief overview of wave mechanics and the wave treatment of light

- Keywords: Wave optics, superposition of waves, interference

Overview

- In this lecture you will learn,
- The wave description of light
- Meaning of quantities such as wavelength, wavenumber, frequency, phase, wavefront etc.
- Superposition of waves
- Interferometers
- Beam solutions of wave equation

Wave Equation and Wave Solutions

- A wave is an oscillatory disturbance propagating through a medium. The simplest case is when a wave propagates through a homogenous and isotropic medium where the wave equation in one dimension can be written as,

$$\frac{\partial^2 u}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 u}{\partial t^2}$$

- The simplest solution of the wave equation can be written as, $u(x,t) = A\sin(kx - \omega t + \phi)$,

Wave Solutions

- Note that the solution is a function of space and time. Oscillations in space and time means there are two kinds of frequencies, spatial frequency, which is related to the wavelength, and the temporal frequency which is related to the period of the oscillations

Wave Solution

- The simple wave solution $u(x,t) = A\sin(kx - \omega t + \phi)$, has the following attributes

- The function $u(x,t)$ describes the amplitude of the wave at all points in space and time.

- The quantity A represents the maximum amplitude possible

- The quantity, k is referred to as the spatial frequency and it is related to the wavelength λ, as $k = 2\pi/\lambda$

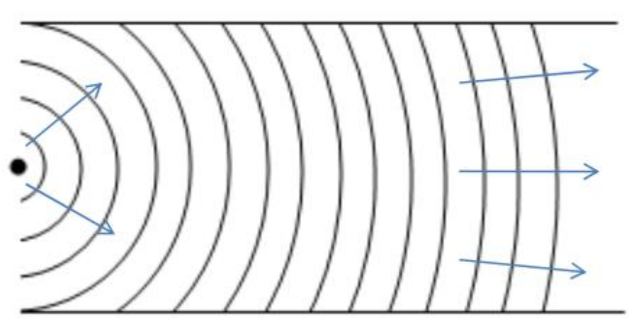
Wave Solutions

- The quantity, w is referred to as the angular frequency and it is related to the (temporal frequency) ν, as $\omega = 2\pi\nu$
- The spatial and angular frequencies are themselves related to each other through the sped of the wave as, $c = \omega/k$

Wave Solution

- The quantity ϕ, is called the phase of the wave. More importantly when one considers two waves u_1 and u_2, the phase difference $\phi_1 - \phi_2$, determines the alignment of the peaks and troughs of the two waves. If they are aligned the amplitudes add up, if they are completely misaligned the amplitudes cancel each other.
- The intensity, which is defined as power per unit area is related to the square of the amplitude of wave function, i.e. $I(x,t) \sim A^2$

Wave-fronts and Propagation



- Wavefronts are surfaces of constant phase in a wave. Point light sources produce an isotropic radiation pattern which can be characterized by spherical wavefronts. This means that along the surface of the sphere the phase remains constant. However, as we see from the diagram, sufficiently far away from the light source, the wave can be assumed to be a plane wave, i.e. all points in a plane normal to the direction of propagation (indicated by the blue arrows) have the same phase.

Propagation in a Medium Other than Air

- The speed of light in vacuum is denoted by c. The wavevector and the frequency are related by the speed of light as $c = \omega/k$. We know that when light propagates through a medium with refractive index n, the speed of light becomes $v = c/n$. Later on we will see that the frequency of the light does not change when it changes the medium of propagation. This means that the wavevector and hence the wavelength should change. In a medium of refractive index n, the wavevector is $k = nk_0$, where k_0 is the wavevector in vacuum. Alternately, one can say that the wavelength is $\lambda = \lambda_0/n$ in a medium of refractive index n. Here λ_0 is the wavelength in vacuum.

Superposition of Waves

- The wave equation written down in the previous slide is a linear equation, which means that is u1 and u2 are solutions, then u1 + u2 is also a solution. This solution is called a superposition of waves u1 and u2 and leads to the phenomena of interference.
- When two or more waves co-propagate, the propagation can be considered to be the superposition of all the co-propagating waves. This phenomena is called wave interference.

Superposition of Waves

- As pointed out in the previous slide superposition of two waves is determined by the phase difference, so let us write $u_1 = A_1 \sin(kx - \omega t)$ and $u_2 = A_2 \sin(kx - \omega t + \phi)$, where ϕ is the phase difference between u_1 and u_2.
The superposition of u_1 and $u_2 = u_{sup}$ can be written as,
$u_{sup} = A_{sup} \sin(kx - \omega t + \phi_s)$ where the new amplitude is,
$A_{sup}^2 = A_1^2 + A_2^2 + 2A_1A_2\cos(\phi)$, in terms of intensity this is

$$I_{sup} = I_1 + I_2 + 2\sqrt{I_1}\sqrt{I_2}\cos(\phi)$$

Interference Signal

- The superposed intensity derived in the previous slide will be a sinusoidal function in terms of the phase difference. It will attain a maximum value of $(a_1 + a_2)^2$ when phase difference is 0 or even multiples of π and minimum of $(a_1 - a_2)^2$ when the phase difference is odd multiples of π. The former case is called constructive interference and the latter is destructive interference. Specifically when $a_1 = a_2$, destructive interference results in total attenuation of the wave, i.e. there is no resultant wave.

Interference Signal

- How can we produce two waves with a phase difference from a single light source? We do this by creating optical delays. Imagine a wave u_2 which has travelled an extra distance of d with respect to the wave u1, then from the wave solution we see that u_2 will have a phase difference of kd with respect to $u_{1,}$ where k is the wave vector.