Optical Phase Difference

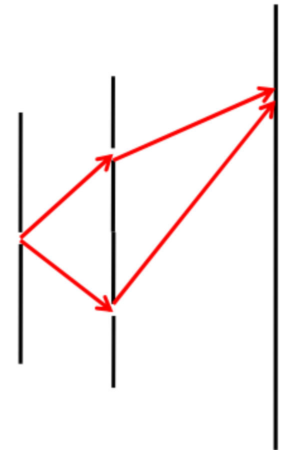
- Having two waves travel different distances is a way to create a phase difference between two waves. Another way is for the two waves to go through media with different refractive indices. As the speed of propagation is different in different media, the two waves will acquire a phase difference as they propagate through the media. The phase difference in this case can be written as $\phi = k_0(n_2 - n_1)d$. Here k_0 is the wave vector in vacuum, n_2 and n_1 are the refractive indices of the media and d is the distance of propagation.

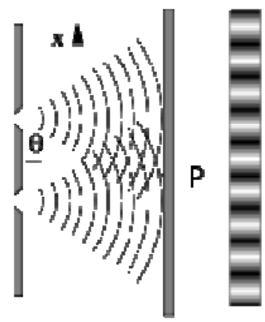
Optical Phase Contrast

- The phase difference produced as mentioned above is useful in a form of imaging called phase contrast or interference contrast. When object of interest is not absorbing, there is no contrast from the surrounding. But the phase could be different creating a contrast.

Young’s Double Slit Experiment

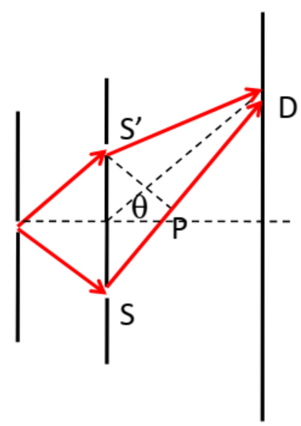
- In the early 19th century, Thomas Young conducted a famous experiment that conclusively demonstrated the wave nature of light. He created a setup as shown in the diagram where a pinhole acts as a point source of light and this is divided by two pinholes or double slits. As shown by the rays, different points along the screen have different phase differences between the waves because they are travelling different distances. Young was able to obtain an interference pattern using this experiment which confirmed the wave picture. In some areas where the phase difference was odd multiples of π, there were dark fringes due to destructive interference.





Analysis of Double Slit Interference Pattern

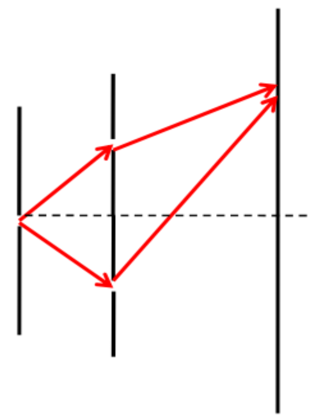
- The path difference between the two waves as represented by the lines is approximately equal to SP where the dashed line S’P is normal to the light path SD. From geometry, SP = d$\sin\theta$, where d is the distance between the slits (pinholes)



Analysis of Double Slit Interference

- Therefore, the phase difference between S'D and SD is $kd\sin\theta$. As we saw before, when the phase difference is an even multiple of π, there is constructive interference and therefore that spot is bright. When $kd\sin\theta$ is an odd multiple of π, then there is destructive interference and therefore that spot is dark. For intermediate phase differences the intensity gradually varies from bright to dark. By using $k = 2\pi/\lambda$, we can write the angular positions of minima θ_{min} as,

$$\sin\theta_{min} = (m + ½)(\lambda/d)$$



The Idea of Coherence

- Why is it that it took till the early part of 19th century for the observation of interference effect?
- The answer lies in the idea of coherence. When we superposed two waves with a phase difference f, we treated the wavevector k as being the same at all points in space and time, similarly we treated f also to be the same at all points in space and time. What is k was a random function of space and time? Then we wouldn't be able to get a nice relation for the superposed wave. The phase difference between the waves would then be also a random function of space or time and the interference effect would get averaged out.

Coherence

- So to observe, interference effect there should be a fixed relationship between the phases of the interfering waves. This uniformity in phases, which is required to observe interference is called coherence. If there is no fixed phase relationship between co-propagating waves, such a radiation of light is called incoherent radiation. In other words, the wavefronts which are regular spheres or planes or other shapes change randomly.

Coherence

- Coherence can be spatial as well as temporal. An incident radiation can be said to be spatially and temporally coherent when the phase difference between two points at any given time is a function of their relative separation and time. When the phase difference between these points at any given time can be expresses as a function of their relative separation but changes randomly at different time while maintaining the spatial relationship, the radiation is spatially coherent but temporally incoherent. Similarly one could have situations where an incident radiation is temporally coherent but spatially incoherent.

Coherence

- In practical situations, there is a coherence length within which we can well approximate the radiation to be spatially coherent and a coherence time within which we can assume temporal coherence.

- Most natural light sources such as lamps and sunlight produce incoherent light which makes it difficult to observe interference effects requiring large coherence lengths of incident radiation. Young solved this problem by having slits in his experimental setup.

Coherence in Double Slit Experiment

- Young used an incoherent light source for his experiment. We know this because there were no coherent light sources (lasers) at his time. Then how did he get the necessary coherence to observe the interference fringes? He passed the incident radiation through a small pinhole, this collecting a very small area of the radiating wavefront within which the waves are almost coherent and then he passed this nearly coherent light through the two pinholes which act as two spatially coherent light sources producing a fixed phase difference at the observation screen causing fringes to appear.

Interferometers

- The phenomena of interference can be used as an image contrast mechanism as described earlier and discussed in detail later. In addition to that it can be used as a metrological tool, i.e in the measurement of small distances and refractive index changes.

Interferometers

- Recall that the intensity changes from maximum to minimum when phase difference changes by π. This is equivalent to a displacement of $\lambda/2$ and as λ is ~ 500 nm, interferometry by measuring fringe shifts can be used to measure sub-nm displacements and fine changes in refractive index (because it contributes to phase difference). Therefore it serves as an excellent optical measurement tool for applications such as characterizing the thickness of thin films (where the wave reflected from the top surface of the film interferes with the wave reflected from the substrate on which the film is deposited)

Beam Solutions

- We write down the simplest wave equation which is also called a plane wave solution because all points in a plane have the same phase in the 3D version of the solution we wrote down. However other solutions are possible and are more applicable to practical situations. In practice light beams are finite.

- The wave equation in 3 dimensions can be written as

$$\nabla^2 u = \frac{1}{c^2}\frac{\partial^2 u}{\partial t^2} \qquad \nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

where the left hand side is called the laplacian operator give by,

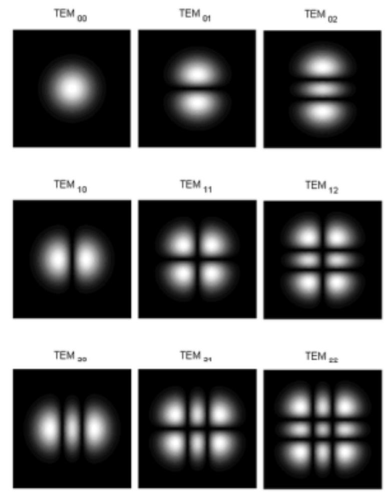
- In cylindrical coordinates, under the paraxial approximation (i.e. propagation direction close to the optical axis), one obtains a paraxial helmholtz equation which admits 'beam-like' solutions. These are solutions that describe waves which propagate along a given direction, say z-axis, and have a certain intensity and phase profile across the radial direction (i.e. across the beam)

Some Types of Beams

- The simplest beam solution is a gaussian beam as shown in the diagram. In a gaussian beam the intensity is maximum at the center of the beam and then decreases according to a gaussian function given by

$$I(r) = I_0 \exp\left(-\frac{2r^2}{\sigma^2}\right)$$

- Here σ corresponds to the beam's 'spot size'.



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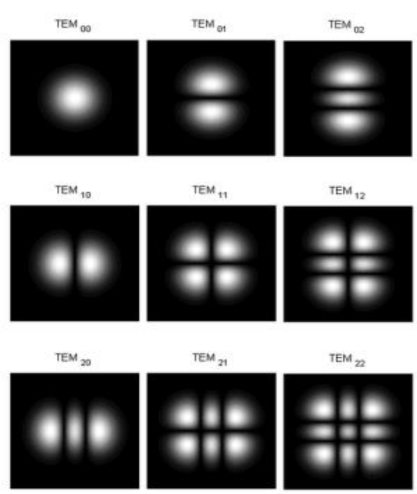
Higher Order Beams

- Gaussian beams are normally encountered with laser outputs. It is also possible to have higher order beams which describe several lobes as we see in the adjoining diagram.



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