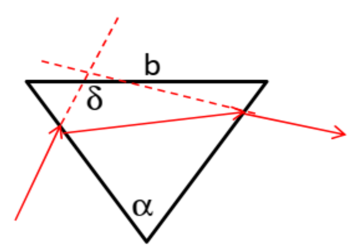
Dispersion

- In the previous lecture we learnt about the wave description of light and the idea of coherent superposition of waves leading to interference phenomena. In this chapter we will focus on another important attribute, namely dispersion.

- As we saw in the double slit interference, the position of the phenomena are dependent on the wavelength of the light. A prism splits white light into different colors which emerge at different refracted angles. We know from the previous lectures that the refracted angle is simply a function of the refractive index. If the different colors are emerging at different angles, it must mean that the refractive index or the velocity for each color must be different. In the wave picture different colors have different wavelengths. The frequency dependent speed of waves is called dispersion. It allows one to disperse the constituents of light using a dispersion element such as a prism or a diffraction grating described in this lecture.

Dispersion from a Prism

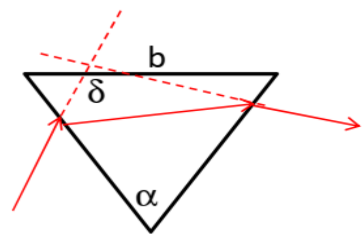


- For a thin prism with wedge angle α, the deviation angle $\delta = (n - 1)\alpha$.

- As n is a function of frequency (or wavelength), we can write
  $$\delta(\lambda) = (n(\lambda) - 1)\alpha$$

Dispersion from a Prism

- The deviation angle depends on the wavelength causing the prism to disperse white light. A quantity called dispersive power is used to characterize the efficiency of dispersion. It is defined as the ratio of bandwidth of the incident light to the smallest resolvable wavelength difference (smallest resolvable angle. $R = \lambda/d\lambda$. Using Rayleigh's criteria for angle resolvability, to be discussed later, one can show that $R = b(dn/d\lambda)$, where b is the base of the prism as shown in the diagram.



Diffraction

- Another phenomena that illustrates the wave nature of light is diffraction. It was observed that the shadow of small objects, for e.g. a small square aperture on an opaque screen is a rather complicated periodic pattern and no a single sharp image of the square aperture as one would expect from geometric optics. The reason for the existence of this pattern, called the diffraction pattern, is the wave nature of light where each point within the aperture acts as a wavefront producing secondary waves which interfere to create the observed pattern as shown in the diagram.

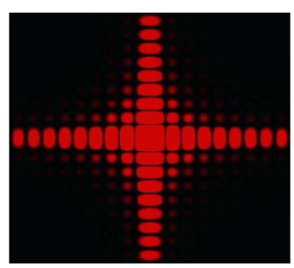


Image courtesy: Wikipedia Commons

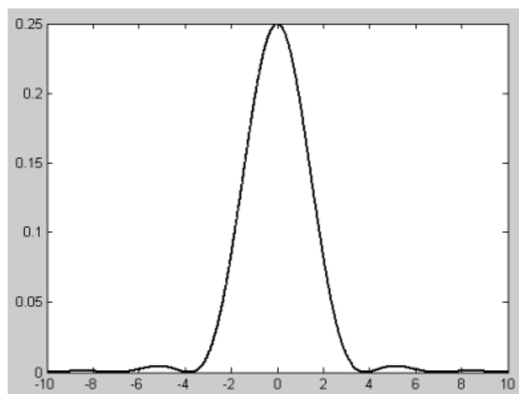
Diffraction Patterns

- It can be shown using the wave description that the diffraction pattern from any arbitrary shaped aperture at a sufficiently far-off observation point is the fourier transform of the aperture. The diffraction pattern at distances far away from the aperture is called far-field diffraction.

Diffraction from a Circular Aperture

- The fourier transform of a circular aperture is given in terms of Bessel functions. The angular distribution of intensity in the far-field (far-field diffraction pattern) I(θ) is given as,

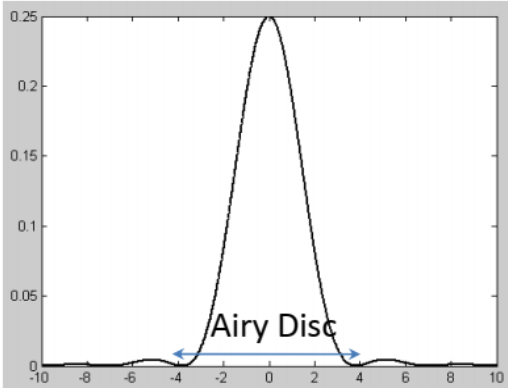
$$I(\theta) = I(0)\left[\frac{2J_1(ka\sin\theta)}{ka\sin\theta}\right]^2$$



where J_1 is the bessel funciton of first kind
with order 1. The plot of 1D bessel function is as shown

Airy Disc

- The diffraction pattern, as shown in the diagram in the previous slide, consists of a bright disk and a dark ring surrounding it. This disc is called Airy disc. The radius of the Airy disc is determined the location of the first zero of the Bessel function which happens when $ka\sin\theta = 3.83$, if r is the radial distance at the observation plane and D is the distance from the aperture to the observation plane, $\sin\theta \sim r/D$. This means that the radius of the airy disc is $r = 0.61\lambda D/a$



Diffraction Limit

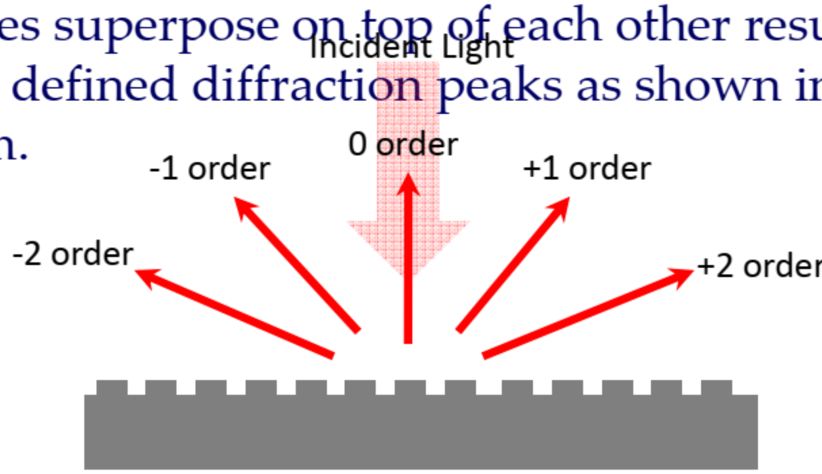
- Consider a beam focused by a lens with focal length f and aperture a. According to the wave picture a point source will produce a diffraction pattern with an airy disc. Thus two points in the object plane can only be resolved (distinguished) if their Airy discs do not overlap. According to the previous slide the airy disc radius at a distance D from the aperture is $r = 0.61\lambda D/a$. In this case D will be equal to the focal length, f, of the lens. So $r = 0.61\lambda f/a$.

Diffraction Limit

- This sets a limit of resolution called the diffraction limit. If two points in the object plane are closer than this distance, their airy discs will overlap and they can not be resolved. Alternately if we try to focus a beam, we can not focus it to a point source. The best we can do is the size of the airy disc. The a/f ratio is called the numerical aperture. So the resolution of a lens or the minimum spot size that is possible with a lens is roughly about $0.61\lambda/NA$

Diffraction Gratings

- We have seen how transmission of light through an aperture creates a diffraction pattern. A diffraction grating is nothing but a periodic arrangement of several apertures. The diffraction patterns from each individual apertures superpose on top of each other resulting in sharply defined diffraction peaks as shown in the diagram.



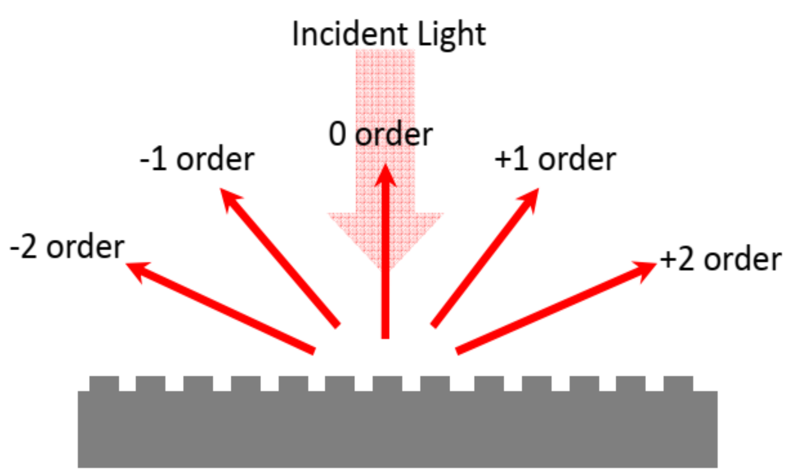
Diffraction Grating in Reflection

Diffraction Gratings

- The corrugations in the diagram shown in the previous slide provide the role of multiple slits. Gratings can operate in reflection mode, as shown in the diagram, or in transmission mode. There may be amplitude gratings, such as periodic slits cut on a opaque material, or phase grating, where a periodic thickness difference creates a phase difference between beams emerging from the thicker portion compared to the thinner portion (such as the grating in the diagram). If the spacing between the corrugations (apertures) is d, then the angular position of maxima, θ_m, are given by

$$d\sin\theta_m = m\lambda,$$

- The different positions correspond to different values of m are called as diffraction orders, so one can have 0^{th} order, +1 order, +2 order, -2 order and so on as shown in the diagram.



Diffraction Grating in Reflection

Dispersive Power of Gratings

- From the grating equation given in the previous slide, we see that a small wavelength difference $\delta\lambda$ will cause a change in the angular position of the mth order by $\delta\theta_m$ given by,

  $$\delta\theta_m = m\delta\lambda / d\cos\theta_m$$

- If this change is larger than the width of the mth order diffraction peak then the wavelength change is resolvable. From the fourier transform relation, it can be shown that the width of the mth order diffraction peak with N corrugations is $\lambda / Nd\cos\theta_m$

Resolving Power of Gratings

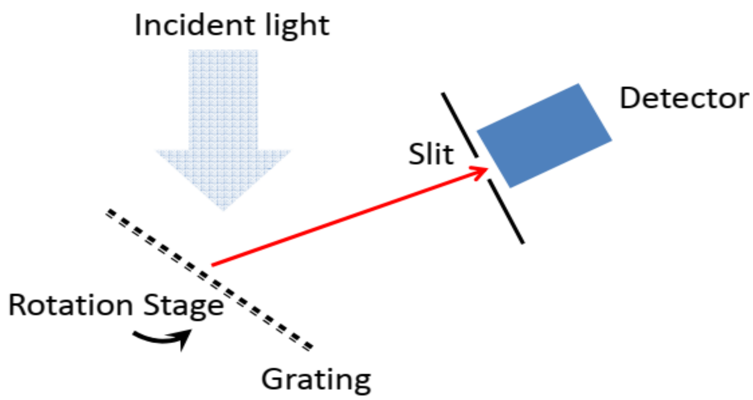
- The resolving power R, defined as $\lambda/\delta\lambda$, is then given by,

  $$R = mN$$

- The resolving power of good commercial gratings can approach 10^6 enabling high resolution spectroscopy for various applications

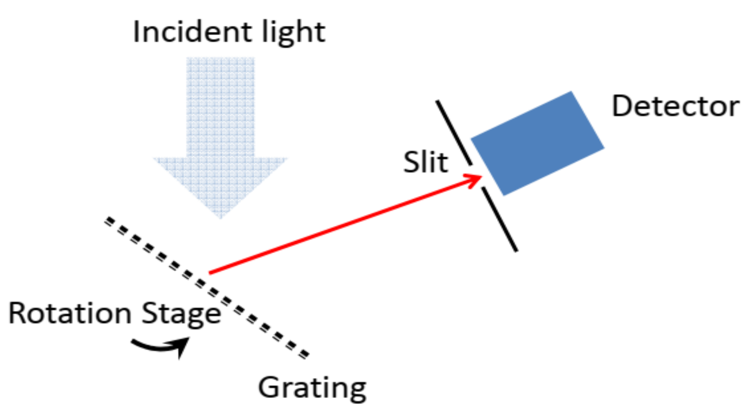
Spectrometer

- The main use of a diffraction grating in spectroscopy. In a spectrometer, as shown in the diagram, the spectrum of the incident light is measured by using a diffraction grating to disperse the incident light. An exit slit filters a small angular field which determines the wavelength resolution of the instrument.



Grating Spectrometer

- The smaller the slit width, the better the resolution. However, a small slit results in less power reaching the detector. As the grating rotates a different wavelength is sampled at the slit. In this manner, the composition of incident radiation can be determined.



Summary

- In the last two lectures we saw the wave picture of light.

- The wave description is useful in analyzing situations involving interference or diffraction of waves.