Electro-Magnetic Waves: EM Optics I

Overview

- In this lecture you will learn
- EM relations
- Maxwell’s equations and EM waves
- Polarization

- Keywords: electromagnetic theory of light, maxwell’s equations, polarization

Electromagnetic Theory

- In the previous lectures we discussed the wave nature of light. In the later part of 19th century, James Clark Maxwell, unified electricity and magnetism and showed that a coupled electromagnetic wave travels through vacuum with the same speed as that of light. Incidentally, a few year before Maxwell, Fizeau and Foucault had been able to measure the speed of light using rotating mirrors. The fact that EM waves travel with the same speed as light led to the understanding that light along with radio waves, X-rays and so on are all EM waves with different frequencies and therefore different sets of interactions with matter.

Speed of Light

- In 1849 French Physicist Fizeau measured the speed of light by reflecting light from a mirror kept at a distance and passing the light through a cogwheel as shown in the diagram. When the speed of the cogwheel is too slow the reflected light get blocked by the cog. When it is increased at a stage, the reflected light will come through the adjacent opening. By knowing the distances between the source and the mirror and the distance between the cogs on the cogwheel, one can estimate the speed of light. Using this technique Fizeau estimated the speed of light within 5% of the modern value which is about 3×10^8 m/s

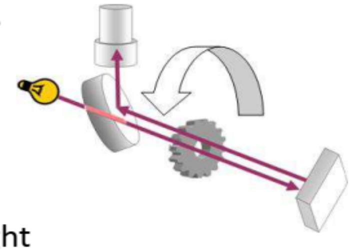


Image courtesy: Wikipedia Commons

Fizeau's setup for measuring speed of light

Maxwell's Equations

- The following observations regarding electricity and magnetism were available before Maxwell.
- Gauss's law, which states that electric field of a point charge decreases as $1/r^2$
- Gauss's law of magnetism which states that magnetic monopoles can not exist.
- Ampere'r right hand rule for relating current flow and magnetic field direction
- Faraday's observation of induced electric field due to a varying magnetic field.
- Maxwell put these known laws in a mathematically consistent framework as shown in the next slide.

Maxwell's Equations

- For a source-free region, i.e. no charges and currents, Maxwell wrote the interdependence of electric field, E, and magnetic field B as follows,

$$\nabla \cdot \vec{E} = 0$$

$$\nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \times \vec{B} = -\mu\varepsilon \frac{\partial \vec{E}}{\partial t}$$

- Here μ is called the magnetic permeability of the medium and ε is called the dielectric susceptibility of the medium. The vacuum values of permeability and permittivity are denoted by μ_o and ε_o. In any medium other than vacuum, we write $\mu = \mu_r\mu_o$ and $\varepsilon = \varepsilon_r\varepsilon_o$, where μ_r is the relative permeability and ε_r is the relative permittivity also called dielectric constant.

EM Wave Equation

- Using the relationships described by Maxwell's equations, it is possible to eliminate one variable, say B, and write an equation that contains only E as the variable. When we do this, one obtains the following equation for E, $$\nabla^2 \vec{E} = \mu\varepsilon \frac{\partial^2 \vec{E}}{\partial t^2}$$

- Exactly same equation is obtained for B as well. From comparing this equation to the wave equation described in previous lectures, we see that they are identical. The quantity $\mu\varepsilon$ serves the role of $1/c_2$. In vacuum one can find $c^2 = 1/\mu_o\varepsilon_o$. This value matches with the speed of light which is around $3x10^8$ m/s implying the light is electromagnetic radiation.

EM Wave Equation

- In the original wave equation, we dealt with a scalar quantity u(r,t), here we see that we are dealing with a vector quantity which must have a direction associated with it. The direction of the E field (or B as they are coupled) is specified by the polarization of the E field.

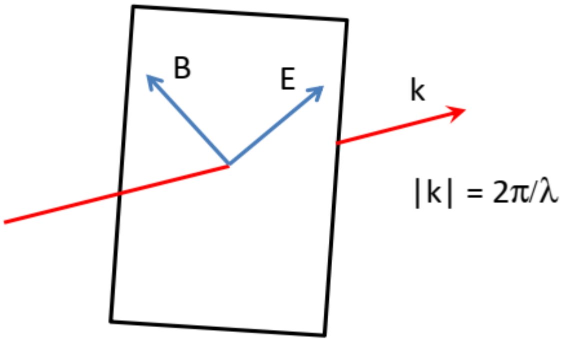
- We will discuss the aspect of polarization later in this lecture

Refractive Index

- Previously, we simply defined refractive index as the ratio of speed of light in vacuum to that in a medium without specifying the origin of this quantity. Now, in EM theory using the relationship between ε, μ and speed of light, one can show that the refractive index of any medium can be written as $n^2 = \mu_r\varepsilon_{r,}$ One can develop an atomistic model of ε and μ by considering the electronic interactions in a material and this relation links the refractive index to the basic molecular configuration of a given material. This relationship explains why metals have high reflectivity and why some materials are colored and so on. Most materials we deal with in optics are non-magnetic, which have $\mu_r = 1$. In this case the refractive index is given by $n^2 = \varepsilon_r$

EM Waves are Transverse Oscillations

- A property of the EM waves is that they are transverse oscillations. This means that the oscillation of the Electric or Magnetic fields occur perpendicular (transverse) to the direction of propagation. For the light propagation direction as shown in the diagram below, the E vector will be in the plane perpendicular to the propagation vector, denoted a k vector. The B field will be perpendicular to the E vector and the k vector. The magnitude of k vector is 2π/λ



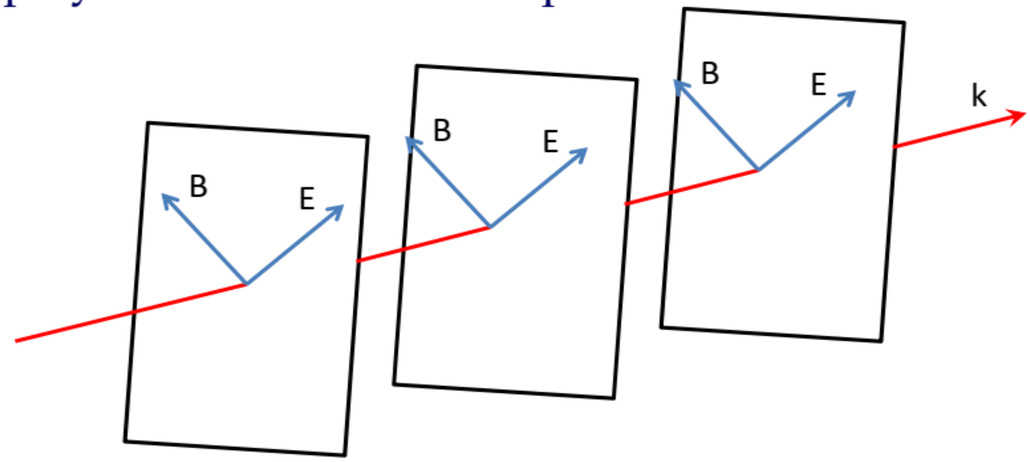
E and B vectors lie in a plane perpendicular to the direction of propagation

Polarization

- We saw in the previous slide that E and B vectors of the propagating light wave lie in a plane normal to the direction of propagation specified by a k-vector. But this offers an infinite number of choices. The specific arrangement of E and B vectors as a wave propagates is denoted by its polarization state. Under plane wave assumption, three polarization classes emerge. These are,
  - Linear Polarization
  - Circular Polarization, and
  - Elliptical polarization
- We will discuss these in the next few slides

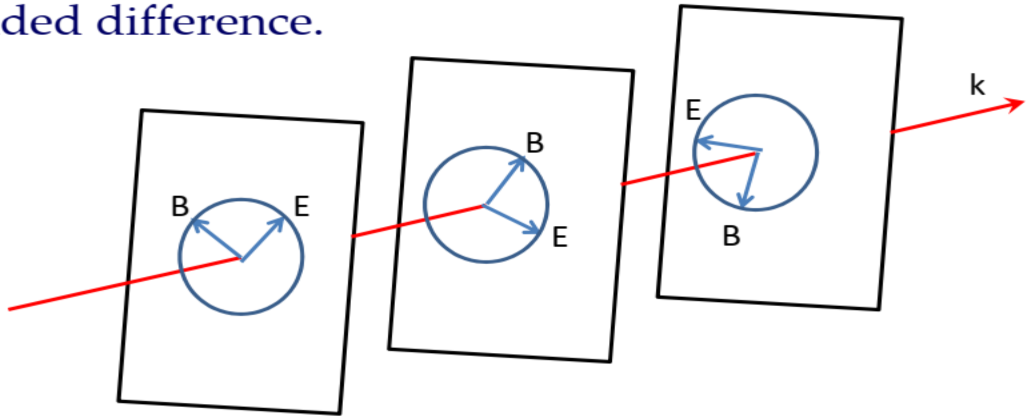
Linear Polarization

- When the direction of E vector remains a constant as the light wave propagates, as shown in the diagram below, we say that the light is linear polarized. Later on we will see that there are two important cases for linear polarizations called Transverse Electric (TE) and Transverse Magnetic (TM) polarizations which often display different behavior upon reflection



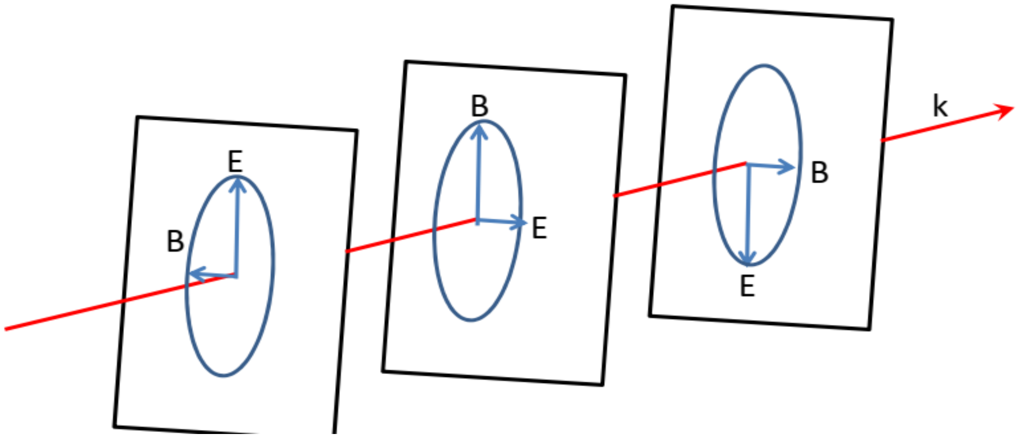
Circular Polarization

- When the E vector moves in a circular path as the light wave propagates, we say that the light is circularly polarized. There can be right circular or left circular polarizations depending on the direction of rotation of the E or B fields, and they lead to different behavior when the medium is chiral, i.e. left-handed and right-handed difference.



Elliptical Polarization

- The third important class of polarization for plane waves is called elliptical polarization where the E and B vector tips describe an ellipse as the light wave propagates. Note that the amplitude of the E and B fields change with propagation length. As with circular polarization, there can be left elliptical and right elliptical polarization.



Polarization State

- The polarization state of the light field can be manipulated by optical elements such as polarizers, wave-plates and so on. These are anisotropic elements where the refractive index is different along different directions.

- The polarization state of light can also change upon reflection. This phenomenon is the basis for a technique called ellipsometry which is used to measure thickness and refractive index of ultra-thin films.