Electro-Magnetic Waves: EM Optics II

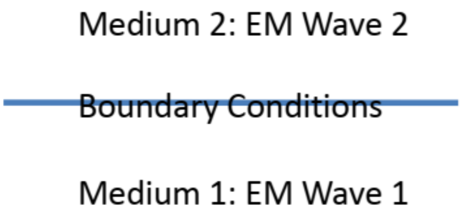
Overview

- In this lecture you will learn,
- EM waves at interfaces: Boundary conditions
- Reflection and transmission of light
- Fresnel coefficients
- Some anisotropic optical elements

- Keywords: electromagnetic boundary conditions, Fresnel coefficients, polarization

Light at Interfaces

- Now we are ready to tackle the problem of what happens when light hits an interface between two media (i.e. materials with different refractive indices). We know that light can either get refracted or reflected. We will see how the refractive index difference affects the amplitude of reflection and transmission of light at interfaces.
- An interface is a discontinuity and in order to handle a discontinuity we need to know the boundary conditions that we must enforce on the EM wave solutions that we write down for the homogenous medium on both sides of the interface.



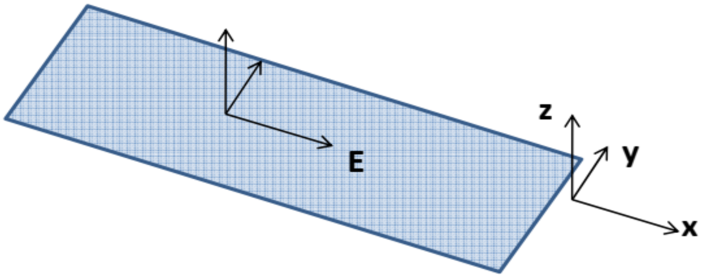
Medium 2: EM Wave 2

Boundary Conditions

Medium 1: EM Wave 1

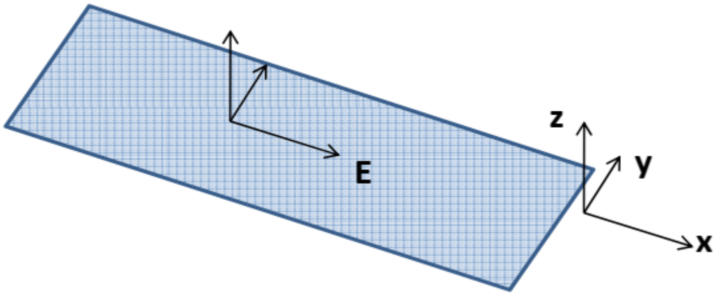
EM Boundary Conditions

- Consider a plane interface between two media as shown in the diagram below. The components of E (or B) fields that are parallel to the interface, in this case Ex and Ey, are called tangential components of the field vector. Ez is called the normal component because it is perpendicular to the interface. The boundary conditions relate the tangential and normal components on one side of the medium to the ones on the other side. These conditions follow from Maxwell's relations although we don't discuss the derivation here.



Boundary Conditions

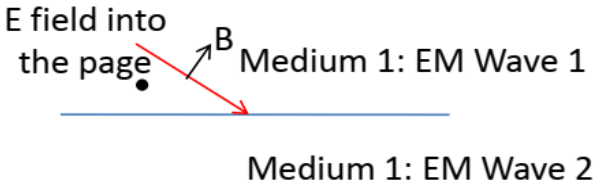
- Tangential components of E field and normal component of B field is continuous, $E_{t1} = E_{t2}$ and $B_{n1} = B_{n2.}$ Where, t represents tangential and n represents normal component



- The normal component of E are related as $\varepsilon_1 E_{n1} = \varepsilon_2 E_{n2}$ and, the tangential components of B are related by $B_{t1}/\mu_1 = B_{t2}/\mu_2$

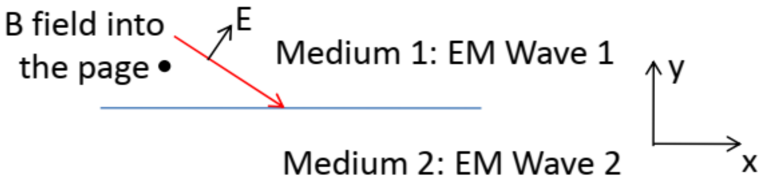
TE Polarization

- In the previous lecture we saw that light can be linearly polarized. Consider light incident on an interface as shown in the diagram below. Imagine that the electric field is perpendicular (transverse) to the incident plane, i.e. the plane containing the incident light direction and the normal to the interface. In that case there is no component of E field which is normal to the interface, i.e. E field is entirely tangential to the interface. This polarization state is called Transverse Electric (TE) polarization. In this case E and B fields are as shown in the left figure.



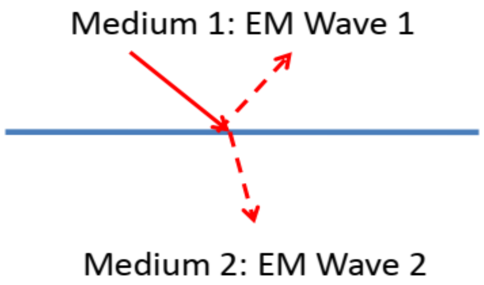
TM Polarization

- The other important case occurs when B field is normal to the incident plane. This state is called Transverse Magnetic (TM) polarization. Here B field doesn't have any normal component to the interface. The polarization state determines which boundary conditions must be applied as we see later.



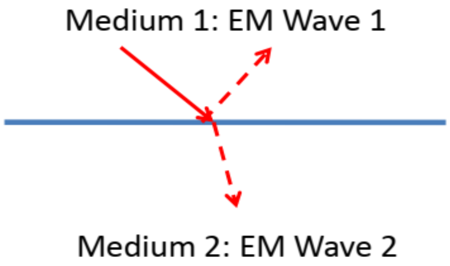
Reflection and Transmission

- When a light wave hits an interface as shown in the diagram, there is reflection and transmission. If we denote r as the reflection coefficient, i.e. fraction of E field reflected, and t as the transmission coefficient, the field in medium 1 can be written as $E_1 = E_{inc} + rE_{inc}$ and the field in medium 2 is simply $E_2 = tE_{inc}$ By applying the boundary conditions, we can find r and t as a function of refractive index and the angle of incidence



Reflection and Transmission

- If incident light is TE polarized we only apply the tangential boundary condition because there is no normal component. If it is TM polarized we need to apply the tangential as well as normal boundary condition. In this manner, we see that r and t will be different for different states.

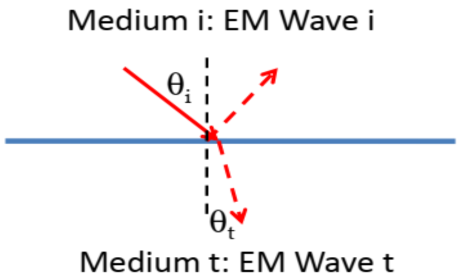


Fresnel Reflection Coefficients

- Application of the appropriate conditions yield the following relationships for the reflection (R) and transmission (T) coefficients for TE and TM polarizations; 'i' and 't' subscripts refer to incident and transmitted media

$$R_{TE} = \left(\frac{n_i \cos\theta_i - n_t \cos\theta_t}{n_i \cos\theta_i + n_t \cos\theta_t} \right)^2 \qquad R_{TM} = \left(\frac{n_i \cos\theta_t - n_t \cos\theta_i}{n_i \cos\theta_t + n_t \cos\theta_i} \right)^2$$

$$T_{TE} = \frac{n_t \cos\theta_t}{n_i \cos\theta_i} \left(\frac{2 n_i \cos\theta_i}{n_i \cos\theta_i + n_t \cos\theta_t} \right)^2 \qquad T_{TM} = \frac{n_t \cos\theta_t}{n_i \cos\theta_i} \left(\frac{2 n_i \cos\theta_i}{n_i \cos\theta_t + n_t \cos\theta_i} \right)^2$$



Fresnel Reflection Coefficients

- As one can verify T = 1 – R irrespective of polarization as there is no absorption of light which we have considered. The figures below show the behavior of R for both polarizations under two circumstances, when ni < nt (left figure) and when ni > nt (right figure). In the

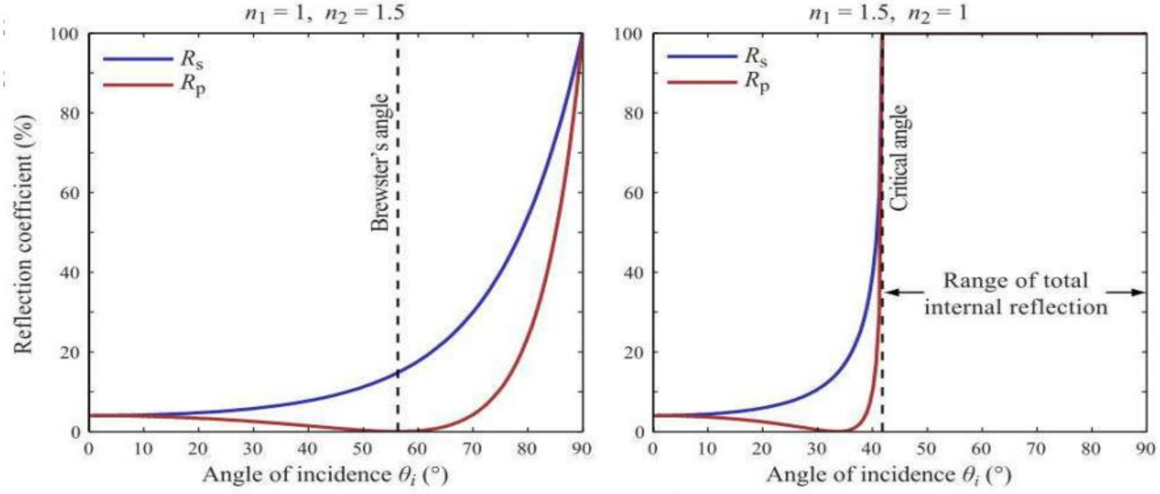


Image courtesy: Wikipedia Commons

Brewster Angle

- As shown in the plot of reflection from an interface, we see that at a certain angle, reflection for TM polarization goes to zero. This angle is called Brewster angle. It can be shown that Brewster angel θ_b can be derived as, $\tan\theta_b = n_t/n_i$

- Brewster angle can be used to create polarized light through reflection. If an unpolarized light source is reflected at Brewster angle, the reflected light will only consist of TE polarization as R for TM = 0.

- By stacking several glass plates at Brewster angle, one can obtain very pure polarization state in the reflected light.

Optical Filters

- We looked at reflection at a single interface. When there are multiple interfaces there are interference effects from the multiple interfaces which diminishes the intensity of some wavelengths and enhance the other wavelengths. These devices can be used as optical filters which are very useful in fluorescence microscopy as we will see in later sections.

Anisotropic Materials

- So far we have treated materials as if they possess a refractive index which is independent of the angle of incidence of impinging light. There are materials where this is not true. These materials are called optically anisotropic materials. In this situation we talk about an axis or axes of anisotropy.

Anisotropic Materials

- In the case of uniaxial materials, there exists an optical axis such that light polarized perpendicular to the optic axis has a refractive index called the ordinary refractive index ‘no’ and light polarized parallel to the optic axis has a different value of refractive index called extraordinary refractive index ‘ne’. These two values, no and ne, characterize a uniaxially anisotropic material. Similarly there exists bi-axially anisotropic materials which are characterized by nx, ny and nz.
- A light wave polarized in an arbitrary direction with respect to the optical axis will be split into two rays, the ordinary, corresponding to no and the extraordinary ray, corresponding to ne.

Polarizers

- As described above Brewster angle reflectance is one method to produce polarized light. However, most commercial polarizers make use of birefringence by creating prism structures where the ordinary and the extraordinary rays (which are orthogonally polarized) are separated using total internal reflection. Examples of such devices are Glan-Thompson and Glan-Taylor polarizers.

- Transmission of polarized light through an analyzer (which is another polarizer at an offset angle to the first polarizer) is maximum when the axes of both polarizers are aligned. When the axis of the polarizers are orthogonal to each other transmission drops to 0. The dependence of intensity on the angle between the axes can be described by a $\cos^2\theta$ function.

Phase Retarders

- Another application of optically anisotropic materials is in the construction of phase retarders, also called wave plates. We have already seen that an arbitrarily polarized wave is split into extraordinary and ordinary wave which have different refractive indices which means they travel at different speeds.
- The difference in speed implies that there is a phase difference between the two waves as they exit from the birefringent material. When this phase difference is π, such a device is called a half-wave plate, when it is $\pi/2$, it is called a quarter-wave plate.
- It can be shown that a quarter-wave plate will convert a linearly polarized wave to a circularly polarized wave and vice versa when its axis is at 45 degrees to the direction of incident polarization .

Summary

- From the discussion of EM waves it is clear that light is a transverse EM wave.

- The state of polarization is important to analyze situations involving reflection, propagation through anisotropic materials etc.