FINAL EXAM

TIME: 3 HOURS
INSTRUCTION: ATTEMPT ALL QUESTIONS.

Question 1: (50 marks a=10 marks, b - f= 20 marks each)

(a) Give at least least 3 (a list is fine) experimental or observational confirmations of general relativity.

(b) One of the great triumphs of modern cosmology is the determination that the universe is comprised of approximately 30% ordinary and dark matter (with $w \simeq 0$), and 70% dark energy (with $w = -1$). Assume for simplicity that the density of matter and dark energy are currently equal (they're not, see above).

What happens to the ratio of densities of DE/Matter once the universe increases in scale (a) by a factor of 2?

(c) You are dangled at radial coordinate, r above a black hole of mass, M. How far above the black hole (in units of M) must you be held so that your clock runs exactly half as fast as a starship some distance away?

(d) In a sentence or two, describe the Equivalence Principle (any of them). For a point of extra credit, explain the distinction between the weak equivalence principle, and what your book describes as the Einstein Equivalence Principle.

(e) Using only a pencil, straight-edge and a protractor (and not even necessarily any equations), describe how you might determine whether you are currently on a curved surface, and if so, how you might figure out the radius of curvature.

(f) Consider a planet in a circular orbit in the equatorial plane of a central star (the Schwarzschild metric). Compute a relationship between the reduced energy, $\tilde{E} = -U_0$ as a function of the reduced angular momentum, $\tilde{L} = -p_\phi$, as a function of r.

What is the limit for $r \gg 2M$, including only the most significant term that include $\tilde{L}$?

Question 2: (40 Marks)

Be aware that though this appears to be a single *very* long problem, it's actually 2 separate parts in disguise. In other words, if you have a tough time on parts a,b,c, and h, the others can be done independently. In other words, don't lose heart!

Consider a metric of the form:

$$g_{\mu\nu} = \begin{pmatrix} -f(y) & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

where "y" in this context is specifically the y-coordinate, and $f(y)$ is some as-yet unknown function. At the origin:

$$f(0) = 1$$

I'd also like you (for clarity) to use $f' \equiv df/dy$, $f'' \equiv d^2f/dy^2$ in your answers below.

(a) What are the conserved dynamic quantities for the metric?

(b) A particle moves in the metric starting *at the origin* (where the metric is locally Minkowskian), but with an initial 4-momentum in the **x direction** only (also, in time, if you need that reminder).

Some later time, it's at the coordinates $\mathcal{P} = (x, y)$. In terms of the final position and the initial 4-momentum (above) what is the p^2 component of the 4-momentum at $\mathcal{P}$?

(c) Using the result from the previous part, compute $\frac{dp^2}{d\tau}$:

(d) Compute all non-zero components of the Christoffel symbols of the metric.

(e) Compute all non-zero components of the Riemann tensor.

(f) Compute all non-zero components of the Einstein tensor (you may need to do a bit of work with the Ricci tensor and scalar first).

(g) As it happens, the actual functional form of f is:

$$f(y) = (1 + ky)^2$$

where k is some constant. This solution clearly satisfies the boundary condition($f(0) = 1$) of the metric.

What is the stress-energy tensor which gives rise to this solution?

(h) Using this form of the metric, describe the initial (non-relativistic) behavior of a particle *placed* near the origin.