Convective Mass Transfer Correlations

Extensive data have been obtained for the transfer of mass between a moving fluid and certain shapes, such as flat plates, spheres and cylinders. The techniques include sublimation of a solid, vapourization of a liquid into a moving stream of air and the dissolution of a solid into water.

These data have been correlated in terms of dimensionless parameters and the equations obtained are used to estimate the mass transfer coefficients in other moving fluids and geometrically similar surfaces.

Flat Plate

From the experimental measurements of rate of evaporation from a liquid surface or from the sublimation rate of a volatile solid surface into a controlled air-stream, several correlations are available. These correlation have been found to satisfy the equations obtained by theoretical analysis on boundary layers,

$$Sh = 0.664\ \mathrm{Re}_L^{1/2} Sc^{1/3}\ (\text{laminar})\ \mathrm{Re}_L < 3 * 10^5 \quad \text{------------- (4.39)}$$

$$Sh = 0.036\ \mathrm{Re}_L^{0.8}\ Sc^{1/3}\ (\text{turbulent})\ \mathrm{Re}_L > 3 * 10^5 \quad \text{----------- (4.40)}$$

Using the definition of j factor for mass transfer on equation (4.39) and (4.40) we obtain

$$j_D = 0.664\ \mathrm{Re}_L^{-1/2}\ (\text{laminar})\ \mathrm{Re}_L < 3 * 10^5 \quad \text{------------- (4.41)}$$

$$J_D = 0.037\ \mathrm{Re}_L^{-0.2}\ (\text{turbulent})\ \mathrm{Re}_L > 3 * 10^5 \quad \text{----------- (4.42)}$$

These equations may be used if the Schmidt number in the range $0.6 < \mathrm{Sc} < 2500$.

7. If the local Nusselt number for the laminar boundary layer that is formed over a flat plate is

$$Nu_x = 0.332\ \mathrm{Re}_x^{1/2}\ Sc^{1/3}$$

Obtain an expression for the average film-transfer coefficient $\bar{k}_c$, when the Reynolds number for the plate is

a) $\mathrm{Re}_L = 100\,000$
b) $\mathrm{Re}_L = 1500\,000$

The transition from laminar to turbulent flow occurs at $\mathrm{Re}_x = 3 * 10^5$.

Derivation:

By definition : $$\bar{k}_c = \frac{\int_o^L k_c \, dx}{\int_o^L dx}$$

and $$Nu_x = \frac{k_c \, x}{D_{AB}}; \qquad Re_x = \frac{x \, v \, \rho}{\mu}; \qquad Sc = \frac{\mu}{\rho D_{AB}};$$

For Re $_L$ = 100 000 ; (which is less than the Reynolds number corresponding to Transition value of 3 * 10 ^{5})

$$\bar{k}_c = \frac{\int_o^L 0.332 \left(\frac{x \, v \, \rho}{\mu} \right)^{\frac{1}{2}} (Sc)^{\frac{1}{3}} \frac{D_{AB}}{x} \, d\,x}{L}$$

$$= \frac{0.332 \, (Sc)^{1/3} \left(\frac{v \, \rho}{\mu} \right)^{1/2}}{L} D_{AB} \int_o^L \frac{d\,x}{x^{1/2}}$$

$$= \frac{0.332}{\frac{1}{2} L} Sc^{1/3} \left(\frac{v \, \rho}{\mu} \right)^{1/2} D_{AB} \left[x^{1/2} \right]_o^L$$

(i.e.) $$\frac{\bar{k}_c \, L}{D_{AB}} = 0.664 \; Re_L^{\;1/2} \; Sc^{1/3}$$ [answer (a)]

For Re $_L$ = 1500 000 (> 3 * 10 ^{5})

$$\bar{k}_c = D_{AB} \frac{\left(\int_o^{L_t} 0.332 \; Re_x^{\;1/2} \; Sc^{1/3} \; \frac{d\,x}{x} + \int_{L_t}^{L} 0.0292 \; Re_x^{\;4/5} \; Sc^{1/3} \; \frac{d\,x}{x} \right)}{L}$$

where L $_t$ is the distance from the leading edge of the plane to the transition point where Re $_x$ = 3 * 10 ^{5}.

$$\bar{k}_c = D_{AB} \frac{\left(0.332 \, Sc^{1/3} \left(\frac{v \, \rho}{\mu} \right)^{1/2} \int_o^{L_t} \frac{d\,x}{x^{1/2}} + 0.0292 \; Sc^{1/3} \left(\frac{v \, \rho}{\mu} \right)^{4/5} \int_{L_t}^{L} \frac{d\,x}{x^{1/5}} \right)}{L}$$

$$\frac{\bar{k}_c \, L}{D_{AB}} = 0.664 \; Re_t^{1/2} \; Sx^{1/3} + \frac{0.0292}{4/5} \; Sc^{1/3} \left[x^{4/5} \right]_{L_t}^{L} \left(\frac{V \, \rho}{\mu} \right)^{4/5}$$

$$= 0.664 \; Re_t^{\;1/2} \; Sc^{1/3} + 0.0365 \; Sc^{1/3} \left(Re_L^{\;4/5} - Re_t^{\;4/5} \right)$$

$$\frac{\bar{k}_c L}{D_{AB}} = 0.664\,\mathrm{Re}_t^{1/2}\,Sc^{1/3} + 0.0365\,\mathrm{Re}_L^{4/5}\,Sc^{1/3} - 0.0365\,\mathrm{Re}_t^{4/5}\,Sc^{1/3}\,\mathrm{w}$$

here $\mathrm{Re}_t = 3 * 10^5$

Single Sphere

Correlations for mass transfer from single spheres are represented as addition of terms representing transfer by purely molecular diffusion and transfer by forced convection, in the form

$Sh = Sh_o + C\,\mathrm{Re}^m\,Sc^n$ ---------------------- (4.43)

Where C, m and n are constants, the value of n is normally taken as 1/3
For very low Reynold's number, the Sherwood number should approach a value of 2. This value has been derived in earlier sections by theoretical consideration of molecular diffusion from a sphere into a large volume of stagnant fluid. Therefore the generalized equation becomes

$Sh = 2 + C\,\mathrm{Re}^m\,Sc^{1/3}$ -------------------------- (4.44)

For mass transfer into liquid streams, the equation given by Brain and Hales

$Sh = \left(4 + 1.21\;Pe_{AB}^{2/3}\right)^{1/2}$ -------------------------- (4.45)

correlates the data that are obtained when the mass transfer Peclet number, Pe_{AB} is less than 10,000. This Peclet number is equal to the product of Reynolds and Schmidt numbers (i.e.)

$Pe_{AB} = \mathrm{Re}\;Sc$ --------------------------------- (4.46)

For Peclet numbers greater than 10,000, the relation given by Levich is useful

$Sh = 1.01\,Pe_{AB}^{1/3}$ -------------------------- (4.47)

The relation given by Froessling

$Sh = 2 + 0.552\,\mathrm{Re}^{1/2}\;Sc^{1/3}$ ---------------------- (4.48)

correlates the data for mass transfer into gases for at Reynold's numbers ranging from 2 to 800 and Schmidt number ranging 0.6 to 2.7.

For natural convection mass transfer the relation given by Schutz

$$Sh = 2 + 0.59\left(Gr_{AB}\, Sc\right)^{1/4} \text{ ----------------------- } (4.49)$$

is useful over the range

$2 * 10^{8} < Gr_{AB}\, Sc < 1.5 * 10^{10}$

6. The mass flux from a 5 cm diameter naphthalene ball placed in stagnant air at 40°C and atmospheric pressure, is $1.47 * 10^{-3}$ mol/m^{2}. sec. Assume the vapor pressure of naphthalene to be 0.15 atm at 40°C and negligible bulk concentration of naphthalene in air. If air starts blowing across the surface of naphthalene ball at 3 m/s by what factor will the mass transfer rate increase, all other conditions remaining the same?

For spheres :

$$Sh = 2.0 + 0.6\,(Re)^{0.5}\,(Sc)^{0.33}$$

Where Sh is the Sherwood number and Sc is the Schmids number. The viscosity and density of air are $1.8 * 10^{-5}$ kg/m.s and 1.123 kg/m^{3}, respectively and the gas constant is 82.06 cm^{3} . atm/mol.K.

Calculations:

$$Sh = \frac{k_c L}{D_{AB}}$$ where L is the characteristic dimension for sphere L = Diameter.

$$Sc = \frac{\mu}{\rho D_{AB}}$$

$$R_c = \frac{D v \rho}{\mu}$$

$$Mass\ flux,\quad N_A = K_c\,\Delta c \text{ -----------------------------} (1)$$

$$Sh = 2.0 + 0.6\,(Re)^{0.5}\,(Sc)^{0.33}$$

$$\frac{k_c D}{D_{AB}} = 2.0 + 0.6\left(\frac{DV\rho}{\mu}\right)^{0.5}\left(\frac{\mu}{\rho D_{AB}}\right)^{0.33} \text{ ----------------------- } (2)$$

also $N = K_G\,\Delta\,\bar{p}_A$

Therefore $\dfrac{k_c}{RT} = K_G$

Given:

$$N = 1.47 * 10^{-3} \frac{mol}{m^2.\sec} = \frac{K_c}{RT} \Delta \bar{p}_A$$

$$\frac{k_c}{RT}\left(\frac{0.15}{1} - 0\right) = 1.47 * 10^{-3} * 10^{-4} \frac{mol}{cm^2.\sec}$$

$$k_c = \frac{1.47 * 10^{-7}}{0.15} * 82.06 * (273 + 40)$$

= 0.0252 cm/sec

$k_c = 2.517 * 10^{-4}$ m/sec ------------------------------(3)

Estimation of D $_{AB}$:

From (2),

$$\frac{2.517 * 10^{-4} * 5 * 10^{-2}}{D_{AB}} = 2 \quad \text{(since v = 0)}$$

Therefore D $_{AB}$ = 6.2925 * 10^{-6} m^2/sec.

And

$$\frac{k_c * 5 * 10^{-2}}{6.2925 * 10^{-6}} = 2 + 0.6\left(\frac{5 * 10^{-2} * 3 * 1.123}{1.8 * 10^{-5}}\right)^{0.5}\left(\frac{1.8 * 10^{-5}}{1.123 * 6.2925 * 10^{-6}}\right)^{0.33}$$

7946 k_c = 2 + 0.6 * (96.74) * (1.361)

k_c = 0.0102 m/sec. -- (4)

$$\frac{(4)}{(3)} \Rightarrow \frac{N_{A2}}{N_{A1}} = \frac{0.0102}{2.517 * 10^{-4}} = 40.5$$

Therefore, rate of mass transfer increases by 40.5 times the initial conditions.

Single Cylinder

Several investigators have studied the rate of sublimation from a solid cylinder into air flowing normal to its axis. Bedingfield and Drew correlated the available data in the form

$$\frac{k_G \, P \, Sc^{0.56}}{G_m} = 0.281 \left(\mathrm{Re}^{/}\right)^{-0.4} \text{ ------------------------ (4.50)}$$

which is valid for $400 < \mathrm{Re}^{/} < 25000$

and $0.6 < \mathrm{Sc} < 2.6$

Where $\mathrm{Re}^{/}$ is the Reynold's number in terms of the diameter of the cylinder, G_m is the molar mass velocity of gas and P is the pressure.

Flow Through Pipes

Mass transfer from the inner wall of a tube to a moving fluid has been studied extensively. Gilliland and Sherwood, based on the study of rate of vapourization of nine different liquids into air given the correlation

$$Sh \, \frac{p_{B,lm}}{P} = 0.023 \, \mathrm{Re}^{0.83} \, Sc^{0.44} \text{ -------------- (4.51)}$$

Where $p_{B,\,lm}$ is the log mean composition of the carrier gas, evaluated between the surface and bulk stream composition. P is the total pressure. This expression has been found to be valid over the range

$2000 < \mathrm{Re} < 35000$
$0.6 < \mathrm{Sc} < 2.5$

Linton and Sherwood modified the above relation making it suitable for large ranges of Schmidt number. Their relation is given as

$$Sh = 0.023 \, \mathrm{Re}^{0.83} \, Sc^{1/3} \text{ --------------------- (4.52)}$$

and found to be valid for

$2000 < \mathrm{Re} < 70000$
and $1000 < \mathrm{Sc} < 2260$

8. A solid disc of benzoic acid 3 cm in diameter is spin at 20 rpm and 25°C. Calculate the rate of dissolution in a large volume of water. Diffusivity of benzoic acid in water is $1.0 * 10^{-5}$ cm ^{2}/sec, and solubility is 0.003 g/cc. The following mass transfer correlation is applicable:

$\mathrm{Sh} = 0.62 \, \mathrm{Re}^{1/2} \, \mathrm{Sc}^{1/3}$

Where $\mathrm{Re} = \frac{D^2 \omega \rho}{\mu}$ and ω is the angular speed in radians/time.

Calculations:

Dissolution rate = $N_A S$ ----------------------------(1)

Where N_A = mass flux, and
S = surface area for mass transfer

$N_A = k_c (C_{As} - C_{A\infty})$ -------------------------------- (2)

Where C_{As} is the concentration of benzoic and at in water at the surface of the dose.
$C_{A\infty}$ is the concentration benzoic acid in wate for an from the surface of the disc.

Given:

$Sh = 0.62\, Re^{½}\, Sc^{1/3}$

(i.e.) $$\frac{k_c D}{D_{AB}} = 0.62 \left(\frac{D^2 \omega \rho}{\mu} \right)^{\frac{1}{2}} \left(\frac{\mu}{\rho D_{AB}} \right)^{\frac{1}{3}} \text{ ----------------- (3)}$$

1 rotation = 2 π radian

Therefore 20 rotation per minute = 20 * 2 π radian/min

$$= \frac{20}{60} * 2\pi \ radian/\mathrm{sec}$$

For water $\rho = 1$ g/cm^3 μ = 1 centipoise = 0.01 g/cm.sec.

From (3),

$$k_c = 0.62\ D_{AB} \left(\frac{\omega \rho}{\mu} \right)^{\frac{1}{2}} \left(\frac{\mu}{\rho D_{AB}} \right)^{\frac{1}{3}}$$

$$= 0.62 * 1.0 * 10^5 * \left(\frac{(40\pi/60) * 1}{0.01} \right)^{\frac{1}{2}} \left(\frac{0.01}{1 * 1.0 * 10^{-5}} \right)^{\frac{1}{3}}$$

$= 8.973 * 10^{-4}$ cm/sec.

From (2),

$N_A = 8.973 * 10^{-4} (0.003 - 0)$
$= 2.692 * 10^{-6}$ g/cm^2.sec

From (1),

$$\begin{aligned} N_A S &= N_A * (2\pi r^2) \\ &= 2.692 * 10^{-6} * (2\pi * 1.5^2) \\ &= 3.805 * 10^{-5} \text{ g/sec} \\ &= 0.137 \text{ g/hr.} \end{aligned}$$

Mass transfer between phases:

Instead of a fluid in contact with a solid, suppose we now consider two immiscible fluids, designated 1 and 2, in contact with each other. If fluid 1 has dissolved in it a substance A that is also soluble in fluid 2, then as soon as the two fluids are brought together, substance A will begin to diffuse into fluid 2. As long as the two phases remain in contact, the transport of A will continue until a condition of equilibrium is reached.

The situation discussed here occurs in a variety of engineering processes such as gas absorption, stripping, and in liquid – liquid extraction. In all these separation processes, two immiscible fluids are brought into contact and one or more components are transferred from one fluid phase to the other.

In the system of fluids 1 and 2 with A, the transported component, the concentration gradients in the region of the interface between the two fluids are illustrated in figure. Concentration C_{A1} and C_{A2} are the bulk phase concentrations

in fluids 1 and 2, respectively, C_{Ai} is the concentration of A at the interface, and N_A is the molar flux of A. For steady state conditions, we can define the flux of A as

$$N_A = k_{c1}\left(C_{A1} - C_{Ai}\right) = k_{c_2}\left(C_{Ai} - C_{A2}\right) = K_c\left(C_{A1} - C_{A2}\right) \qquad \text{\_\_\_\_\_\_\_\_\_\_\_\_ (1)}$$

where k_c = individual mass transfer coefficient defined in terms of the concentration difference in a single phase.

K_c = overall mass transfer coefficient defined in terms of the overall difference in composition.

Equation (1) is analogous to that in heat transfer, where the individual coefficients h are related to the overall coefficient U.

From equation (1),

$$\frac{1}{k_{c_1}} + \frac{1}{k_{c_2}} = \frac{1}{K_c} \qquad \text{\_\_\_\_\_\_\_\_\_\_\_\_ (2)}$$

In equation (1), the potential for mass transfer is exposed in terms of composition. However, this is not always the most convenient way to express it. For example, if fluid 1 is a gas and fluid 2 a liquid, as in gas absorption, the potential in gas phase is often expressed in terms of partial pressures, while that in the liquid phase may be expressed in terms of concentrations. The expression for the molar flux is then written for the individual phases as:

$$N_A = K_p\left(P_{AG} - P_{Ai}\right) = K_c\left(C_{Ai} - C_{AL}\right) \qquad \text{\_\_\_\_\_\_\_\_\_\_ (3)}$$

where

k_p = individual mass transfer coefficient for the gas phase with the potential defined in terms of partial pressures.

P_{AG}, C_{AL} = partial pressure and concentration of A in the bulk gas and liquid phases, respectively.

P_{Ai}, C_{Ai} = partial pressure and concentration of A, respectively, at the interface.

At the interface, it is usually assumed the two phases are in equilibrium. This means that P_{Ai} and C_{Ai} are related by an equilibrium relationship such as 'Henry's law :

$$P_{Ai} = HC_{Ai} \qquad \text{\_\_\_\_\_\_\_\_\_\_} \quad (4)$$

where H is Henry's law constant.

The flux N_A can also be expressed in terms of overall mass transfer coefficients as,

$$N_A = K_p\left(P_{AG} - P_{AE}\right) = K_c\left(C_{AE} - C_{AL}\right) \qquad \text{\_\_\_\_\_\_\_\_\_\_} \quad (5)$$

where

K_p = overall mass transfer coefficient with the overall potential defined in terms of partial pressures.

K_c = overall mass transfer coefficient with the overall potential defined in terms of concentrations.

P_{AE}, C_{AE} = equilibrium composition.

P_{AE} is related to the bulk liquid composition C_{AL} AS

$$P_{AE} = HC_{AL} \qquad \text{\_\_\_\_\_\_\_\_\_\_} \quad (6)$$

similarly, $C_{AE} = \dfrac{P_{AG}}{H}$ __________ (7)

The relationship between the individual and overall coefficients is readily obtained through the use of equations (3) to (7) as

$$\frac{1}{K_p} = \frac{1}{k_p} + \frac{H}{k_c} = \frac{H}{K_c} \qquad \text{\_\_\_\_\_\_\_\_\_\_} \quad (8)$$

In many system, mass transfer resistance is mainly in one phase. For example, gases such as nitrogen and oxygen do not dissolve much in liquids. Their Henry's law constant H is very large, thus $K_c \approx k_c$ is a good approximation. In this case, the liquid phase controls the mass transfer press since mass transfer is slowest there.