INTRODUCTION TO MASS TRANSFER

Introduction of Mass Transfer

When a system contains two or more components whose concentrations vary from point to point, there is a natural tendency for mass to be transferred, minimizing the concentration differences within a system. The transport of one constituent from a region of higher concentration to that of a lower concentration is called mass transfer.

The transfer of mass within a fluid mixture or across a phase boundary is a process that plays a major role in many industrial processes. Examples of such processes are:

(i) Dispersion of gases from stacks
(ii) Removal of pollutants from plant discharge streams by absorption
(iii) Stripping of gases from waste water
(iv) Neutron diffusion within nuclear reactors
(v) Air conditioning

Many of air day-by-day experiences also involve mass transfer, for example:

(i) A lump of sugar added to a cup of coffee eventually dissolves and then eventually diffuses to make the concentration uniform.
(ii) Water evaporates from ponds to increase the humidity of passing-air-stream
(iii) Perfumes presents a pleasant fragrance which is imparted throughout the surrounding atmosphere.

The mechanism of mass transfer involves both molecular diffusion and convection.

Properties of Mixtures

Mass transfer always involves mixtures. Consequently, we must account for the variation of physical properties which normally exist in a given system. When a system contains three or more components, as many industrial fluid streams do, the problem becomes unwidely very quickly. The conventional engineering approach to problems of multicomponent system is to attempt to reduce them to representative binary (i.e., two component) systems.

In order to understand the future discussions, let us first consider definitions and relations which are often used to explain the role of components within a mixture.

Concentration of Species:

Concentration of species in multicomponent mixture can be expressed in many ways. For species A, mass concentration denoted by ρ_A is defined as the mass of A,m_A per unit volume of the mixture.

$$\rho_A = \frac{m_A}{V} \quad \text{------------------------------------} \quad (1)$$

The total mass concentration density ρ is the sum of the total mass of the mixture in unit volume:

$$\rho = \sum_i \rho_i$$

where ρ_i is the concentration of species i in the mixture.

Molar concentration of, A, C_A is defined as the number of moles of A present per unit volume of the mixture.

By definition,

$$\text{Number of moles} = \frac{\text{mass of A}}{\text{molecular weight of A}}$$

$$n_A = \frac{m_A}{M_A} \quad \text{----------------------------} \quad (2)$$

Therefore from (1) & (2)

$$C_A = \frac{n_A}{V} = \frac{\rho_A}{M_A}$$

For ideal gas mixtures,

$$n_A = \frac{p_A V}{RT} \quad \text{[from Ideal gas law PV = nRT]}$$

$$C_A = \frac{n_A}{V} = \frac{p_A}{RT}$$

where p_A is the partial pressure of species A in the mixture. V is the volume of gas, T is the absolute temperature, and R is the universal gas constant.

The total molar concentration or molar density of the mixture is given by

$$C = \sum_i C_i$$

Velocities

In a multicomponent system the various species will normally move at different velocities; and evaluation of velocity of mixture requires the averaging of the velocities of each species present.

If ν_I is the velocity of species i with respect to stationary fixed coordinates, then mass-average velocity for a multicomponent mixture defined in terms of mass concentration is,

$$\nu = \frac{\sum_i \rho_i \nu_i}{\sum_i \rho_i} = \frac{\sum_i \rho_i \nu_i}{\rho}$$

By similar way, molar-average velocity of the mixture ν^* is

$$\nu^* = \frac{\sum_i C_i V_i}{C}$$

For most engineering problems, there will be title difference in ν^* and ν and so the mass average velocity, ν, will be used in all further discussions.

The velocity of a particular species relative to the mass-average or molar average velocity is termed as diffusion velocity

(i.e.) Diffusion velocity = $\nu_i - \nu$

The mole fraction for liquid and solid mixture, x_A ,and for gaseous mixtures, y_A, are the molar concentration of species A divided by the molar density of the mixtures.

$$x_A = \frac{C_A}{C} \quad \text{(liquids and solids)}$$

$$y_A = \frac{C_A}{C} \quad \text{(gases).}$$

The sum of the mole fractions, by definition must equal 1;

(i.e.) $$\sum_i x_i = 1$$

$$\sum_i y_i = 1$$

by similar way, mass fraction of A in mixture is;

$$w_A = \frac{\rho_A}{\rho}$$

1. The molar composition of a gas mixture at 273 K and $1.5 * 10^5$ Pa is:

O_2	7%
CO	10%
CO_2	15%
N_2	68%

Determine

a) the composition in weight percent
b) average molecular weight of the gas mixture
c) density of gas mixture
d) partial pressure of O_2.

Calculations:

Let the gas mixture constitutes 1 mole. Then

O_2 = 0.07 mol
CO = 0.10 mol
CO_2 = 0.15 mol
N_2 = 0.68 mol

Molecular weight of the constituents are:

O_2 = 2 * 16 = 32 g/mol
CO = 12 + 16 = 28 g/mol
CO_2 = 12 + 2 * 16 = 44 g/mol
N_2 = 2 * 14 = 28 g/mol

Weight of the constituents are: (1 mol of gas mixture)

O_2 = 0.07 * 32 = 2.24 g

CO = 0.10 * 28 = 2.80 g
CO_2 = 0.15 * 44 = 6.60 g
N_2 = 0.68 * 28 = 19.04 g

Total weight of gas mixture = 2.24 + 2.80 + 6.60 + 19.04
= 30.68 g

Composition in weight percent:

$$O_2 = \frac{2.24}{30.68} * 100 = 7.30\%$$

$$CO = \frac{2.80}{30.68} * 100 = 9.13\%$$

$$CO_2 = \frac{6.60}{30.68} * 100 = 21.51\%$$

$$N_2 = \frac{19.04}{30.68} * 100 = 62.06\%$$

Average molecular weight of the gas mixture $M = \frac{\text{Weight of gas mixture}}{\text{Number of moles}}$

$$M = \frac{30.68}{1} = 30.68\ g/mol$$

Assuming that the gas obeys ideal gas law,
PV = nRT

$$\frac{n}{V} = \frac{P}{RT}$$

$$\frac{n}{V} = \text{molar density} = \rho_m$$

Therefore, density (or mass density) = $\rho_m M$
Where M is the molecular weight of the gas.

$$Density = \rho_m M = \frac{PM}{RT} = \frac{1.5 * 10^5 * 30.68}{8314 * 273}\ kg/m^3$$

= 2.03 kg/m^3

Partial pressure of O_2 = [mole fraction of O_2] * total pressure

$$= \frac{7}{100} * \left(1.5 * 10^{5}\right)$$

$= 0.07 * 1.5 * 10^{5}$

$= 0.105 * 10^{5}$ Pa

Diffusion flux

Just as momentum and energy (heat) transfer have two mechanisms for transport- molecular and convective, so does mass transfer. However, there are convective fluxes in mass transfer, even on a molecular level. The reason for this is that in mass transfer, whenever there is a driving force, there is always a net movement of the mass of a particular species which results in a bulk motion of molecules. Of course, there can also be convective mass transport due to macroscopic fluid motion. In this chapter the focus is on molecular mass transfer.

The mass (or molar) flux of a given species is a vector quantity denoting the amount of the particular species, in either mass or molar units, that passes per given increment of time through a unit area normal to the vector. The flux of species defined with reference to fixed spatial coordinates, N_A is

$$N_A = C_A \nu_A \qquad \text{---------------------- (1)}$$

This could be written interms of diffusion velocity of A, (i.e., ν_A - ν) and average velocity of mixture, ν, as

$$N_A = C_A (\nu_A - \nu) + C_A \nu \qquad \text{--------------- (2)}$$

By definition

$$\nu = \nu^* = \frac{\sum_i C_i \nu_i}{C}$$

Therefore, equation (2) becomes

$$N_A = C_A (\nu_A - \nu) + \frac{C_A}{C} \sum_i C_i \nu_i$$

$$= C_A (\nu_A - \nu) + y_A \sum_i C_i \nu_i$$

For systems containing two components A and B,

$$N_A = C_A(\nu_A - \nu) + y_A(C_A \nu_A + C_B \nu_B)$$
$$= C_A(\nu_A - \nu) + y_A(N_A + N_B)$$
$$N_A = C_A(\nu_A - \nu) + y_A N \quad \text{----------- (3)}$$

The first term on the right hand side of this equation is diffusional molar flux of A, and the second term is flux due to bulk motion.

Fick's law:

An empirical relation for the diffusional molar flux, first postulated by Fick and, accordingly, often referred to as Fick's first law, defines the diffusion of component A in an isothermal, isobaric system. For diffusion in only the Z direction, the Fick's rate equation is

$$J_A = -D_{AB}\frac{dC_A}{dZ}$$

where D_{AB} is diffusivity or diffusion coefficient for component A diffusing through component B, and dC_A / dZ is the concentration gradient in the Z-direction.

A more general flux relation which is not restricted to isothermal, isobasic system could be written as

$$J_A = -C D_{AB}\frac{dy_A}{dZ} \quad \text{----------------- (4)}$$

using this expression, Equation (3) could be written as

$$N_A = -C D_{AB}\frac{dy_A}{dZ} + y_A N \quad \text{--------------- (5)}$$

Relation among molar fluxes:

For a binary system containing A and B, from Equation (5),

$$N_A = J_A + y_A N$$

or $$J_A = N_A + y_A N \quad \text{----------------------- (6)}$$

Similarly,

$$J_B = N_B + y_B N \quad \text{--------------------- (7)}$$

Addition of Equation (6) & (7) gives,

$$J_A + J_B = N_A + N_B - (y_A + y_B) N \quad \text{---------- (8)}$$

By definition $N = N_A + N_B$ and $y_A + y_B = 1$.
Therefore equation (8) becomes,

$$J_A + J_B = 0$$
$$J_A = -J_B$$

$$C D_{AB} \frac{d y_A}{d z} = - C D_{BA} \frac{d y_B}{d Z} \quad \text{--------------- (9)}$$

From $y_A + y_B = 1$
$dy_A = - dy_B$

Therefore Equation (9) becomes,

$$D_{AB} = D_{BA} \quad \text{----------------------------------- (10)}$$

This leads to the conclusion that diffusivity of A in B is equal to diffusivity of B in A.