ANALYSIS OF SHAPE, TEXTURE AND PATTERN

Representation of Shapes and Contours

The most general form of representation of a contour in discretized space is in terms of the (x, y) coordinates of the digitized points (pixels) along the contour. A contour with N points could be represented by the series of coordinates

$$\{x(n), y(n)\},\ n = 0, 1, 2, \ldots, N-1.$$

Observe that there is no gray level associated with the pixels along a contour. A contour may be depicted as a binary or bilevel image.

Signatures of contours

The dimensionality of representation of a contour may be reduced from two to one by converting from a coordinate-based representation to distances from each contour point to a reference point. A convenient reference is the centroid or center of mass of the contour, whose coordinates are given by

$$\overline{x} = \frac{1}{N}\sum_{n=0}^{N-1} x(n), \quad \text{and} \quad \overline{y} = \frac{1}{N}\sum_{n=0}^{N-1} y(n).$$

The signature of the contour is then defined as

$$d(n) = \sqrt{[x(n)-\overline{x}]^2 + [y(n)-\overline{y}]^2}, \qquad n = 0, 1, 2, \ldots, N-1;$$

Models for Generation of Texture

Texture: It is one of the important characteristics of images and texture analysis. They are divided majorly into two types. Periodic and random. Brick walls and floors with tiles are examples for periodic in texture. Texture may be related to visual or tactile sensations such as fineness, coarseness, smoothness, and periodicity.

Models for generation of texture: texture may be modeled as the convolution of an input impulse field. With a spot or a texton that would act as a filter. The spot noise model of van wijk for synthesizing random texture uses this model.

- Random texture may be modeled as a filtered version of a field of white noise, where the filter is represented by a spot of a certain shape and size. it will be usually of small spatial extent compared to size of image. it is readily seen that the spectra of the textured images are essentially those of corresponding spots.

- Ordered texture may be modeled as the placement of a basic pattern or texton at positions determined by a 2d field of periodic impulses. The separations between the impulses in x and y directions determine the periodicity or pitch in two directions.
- Oriented texture may be generated using the spot noise model by providing line segments. Examples of oriented texture can be seen in mammograms.

Statistical Analysis

Simple measures of texture may be derived based on the moments of the gray level pdf of given image. the k_{th} central moment of the pdf *p(l)* is defined as

$$m_k = \sum_{l=0}^{L-1} (l - \mu_f)^k \, p(l),$$

Where l =0,1,2,……,L-1 are the gray levels in the image *f*, $\mu 1$=*mean* gray level given by

$$\mu_f = \sum_{l=0}^{L-1} l \, p(l).$$

The second central moment which is the variance of gray levels and it is given by

$$\sigma_f^2 = m_2 = \sum_{l=0}^{L-1} (l - \mu_f)^2 \, p(l),$$

This can be served as a measure of inhomogenisity . The third , fourth moments are called as skewness and kurtosis.Its defined as

$$\text{skewness} = \frac{m_3}{m_2^{3/2}},$$

$$\text{kurtosis} = \frac{m_4}{m_2^2},$$

The gray level co- occurrence matrix:

This is the most commonly used texture in particular of random texture are the statistical measures proposed by Haralick et al. they are based upon the moments of joint PDF, known as gray level co-occurrence matrix (GCM). They are also known as spatial gray level dependence matrices used for computing various orientation and distances.

Haralick's Measures of texture

He proposed several quantities as measures of textures.

$$p(l_1,l_2) = \frac{P(l_1,l_2)}{\sum_{l_1=0}^{L-1} \sum_{l_2=0}^{L-1} P(l_1,l_2)}.$$

The maximum correlation coefficient feature F14 is defined as the square root of the second largest eigen value of Q.

$$\mathbf{Q}(l_1,l_2) = \sum_{k=0}^{L-1} \frac{p(l_1,k)\,p(l_2,k)}{p_x(k)\,p_y(k)}.$$

These measures have been applied for several types of images including medical images . The three features are correlation, difference entropy and entropy to perform better than other combinations. It is defined as rubber band straightening transform to map ribbons around breast masses in mammograms into rectangle arrays.

Fractal analysis

Fractal analysis are defined in several different ways, the most common of which is that of a pattern composed of different occurrences of a basic unit. It includes the notion of self similarity or nested reccurence.Fractal patterns occur abundantly in biological and physiological systems. They are used in several biomedical and signal analyses.

Fractal dimension

It's not obvious in other patterns such as clouds coarse lines mammograms to have fractal like characteristics. In such cases the fractal nature perceived is more easily related to the notion of complexity in the dimension of the object leading to the concept of fractal dimension. If larger ruler is used to measure the length of a coast line the minor details would be skipped if smaller ruler is used to measure smaller details.

$$l(\eta) = l_0\, \eta^{1-d_f},$$

Fractional Brownian motion model

Fractal signals would be modelled in terms of fractal brownian motion model. The expectation of the difference between the values of such a signal at a position η and another at

$$E[|f(\eta + \Delta\eta) - f(\eta)|] \propto |\Delta\eta|^H.$$

The slope of the plot is used to estimate H and fractal dimension.

Applications

It is used for detection of tumor.also to differente benign or malignant tumor.

Segmentation And Structural Analysis Of Texture

Most methods of texture analysis are based on statistial or strucxtural methods.they are based on statistical characterization.such as GCM.They are used for analysis of random or fine texture.it requires some type of segmentation. These elements play an important role In preattentive vision and texture perception.this model does not permit scale and orientation.ordered texture may be modelled as being composed of repeated placement of a basic motif or texton.they are used to classify types of texture including grating,raffia,brick,straw,wool.the analysis of a raffia pattern for example resulted in the extraction of three primitives.we defined texture units in terms of 8 connected neighbours of each pixel.the main steps of the system include the generation of 1D descriptors of texture elements from edge repitition data,extraction of elements corresponds to preceeding description.the generation of 2Ddescriptor of each texture are primitive typeand computation of spatial arrangement or placement rules.the texture spectrum was defined as the histogram or spectrum of the frequency of occurrence of all possible texture units in the image.it should be noted that the texture spectrum as above is not based upon a linear orthogonal transform,such as the fourier transform.The primitives were characterized in terms of the ststistics of their GCMs and shape attributes;The textured image could then be described in terms of its primitives and placement rules.

Gabor functions may be used to design filters with tunable orientation,radial frequency bandwidth,and centre frequewncies that can achieve jointly optimal resolution in the space and frequency domains.Gabbar filters are efficient in detecting discontinuities in texture phase ,and are useful in texture segmentation.The other related methods for texture segmentation include wavelet frames for the characterization of texture properties at multiple scales and circular mellin features for rotation-invariant and scale-invariant texture analysis.If the texton and the placement rule(impulse field) can can be obtained from a given image with ordered texture,the most important characteristics of the image will have been determined.

Homomorphic deconvulution of periodic patterns:

Linear filters may be applied to the complex cepstrum for the homomorphic deconvolution of signals that contain convolved components.basically it should not overlap.an image with visual echoes for exraction .it is used successfully extract the basic wavelets or motifs .

Audification And Sonification Of Texture In Images

Meijer proposed a sonification procedure to present image data to the blind. In this method , the frequency of an oscillator is associated with the position of each pixel in the image, and the amplitude is made proportional to ther pixel intensity.Texture analysis is often confounded by other neighbouring or surrounding features. In order to verify the potential of the proposed methods for aural analysis of texture, aset of pilot experiments was designed and presented to 10 subjects. The results indicated that the methods would facilitate qualitataive and compararative analysis of texture.

Martins conducted preliminary tests on the audification ofMR images using selected areas corresponding to the gray and white matter of the brain and to normal and infarcted tissues. By using the audification method, differences between the various tissue types were easily perceived by two radiologists:visual discrimination of same areas while remainingwere difficult.

Applications And Analysis Of Breast Masses:

It is used for differentiating masses from normal breast tissues in digitized mammograms.Highnam et al proposed fuzzy region growing methods for segmenting breast masses,furether proposed classification of segmented masses as benign or malignant based on transistion information present around the segmented regions.Rangayan proposed a region based edge profile accutance measure for sharpness of mass bundaries.measures of texture and gradsien ribbons of pixels around mass boundaries ,hypothesis that transistional information from the inside of the mass to surrounding tissues is important in distuguishing benign or malignant tumours.

IMAGE REPRESENTATION, RECONSTRUCTION, RECOGNITION

Representation: The result of segmentation is a set of regions. Regions have then to be represented and described.
Two main ways of representing a region:
- external characteristics (its boundary): focus on shape
- internal characteristics (its internal pixels): focus on color, textures…

Description

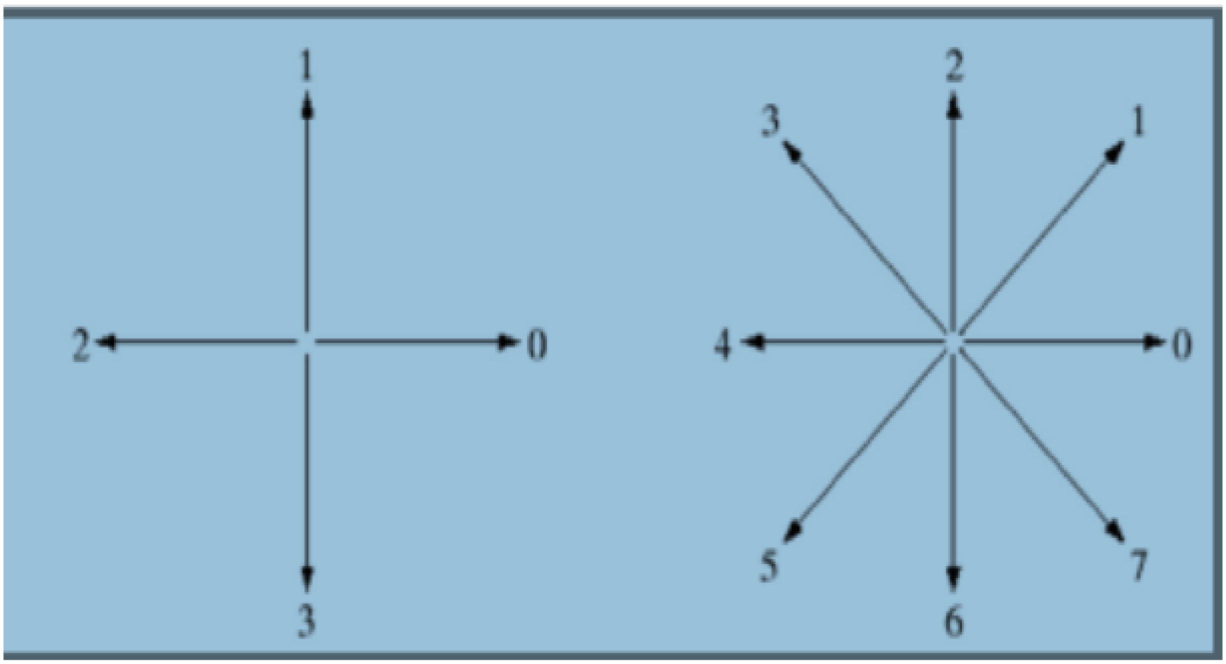
E.g.: a region may be represented by its boundary, and its boundary described by some features such as length, regularity…Features should be insensitive to translation, rotation, and scaling. Both boundary and regional descriptors are often used together.

Representation of boundaries

In order to represent a boundary, it is useful to compact the raw data (list of boundary pixels)

Chain codes: list of segments with defined length and direction

- 4-directional chain codes
- 8-directional chain codes



It may be useful to down sample the data before computing the

Chain code- to reduce the code dimension- to remove small detail along the boundary

To remove the dependence from the starting point: the code is a circular sequence; the new starting point is the one who gives a sequence of numbers giving the smallest integer

• To normalize wrt rotation: first differences can be used

E.g., 10103322 -> 3133030 (counting ccw) and adding the last transition (circular sequence: 2 -> 1) -> 31330303

A signature is a 1-D representation of a boundary (which is a 2-D thing): it should be easier to describe.e.g. Distance from the centroid vs angle.

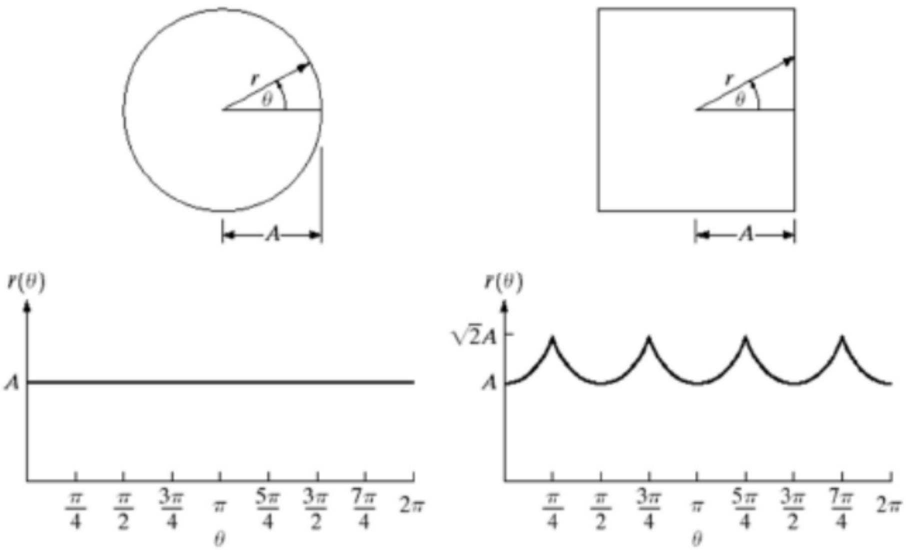
Signatures are invariant to translation

• Invariance to rotation: depends on the starting point

- the starting point could e.g. be the one farthest from the centroid

• Scaling varies the amplitude of the signature

- invariance can be obtained by normalizing between 0 and 1, or
- by dividing by the variance of the signature

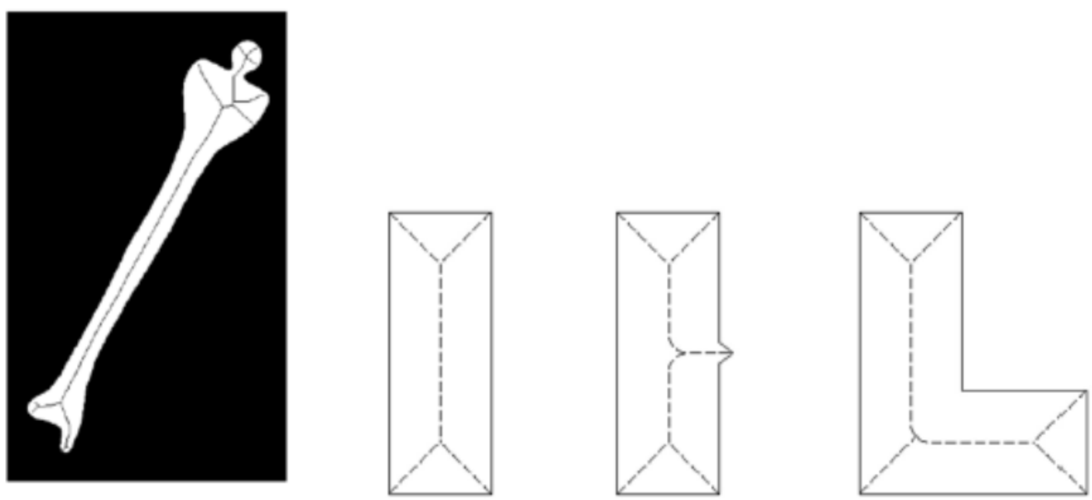


Distance-versus-angle signatures. In (a) $r(\theta)$ is constant. In (b), the signature consists of repetitions of the pattern $r(\theta) = A\sec\theta$ for $0 \le \theta \le \pi/4$ and $r(\theta) = A\csc\theta$ for $\pi/4 < \theta \le \pi/2$.

Representation of a shape

One way to represent a shape is to reduce it to a graph, by obtaining its skeleton via thinning (skeletonization).MAT (medial axis transformation) algorithm

- MAT is composed by all the points which have more than one closest boundary points ("prairie fire concept")



Boundary descriptors:

Simple descriptors

- Length (e.g., for chain code: hor+vert+21/2*diagonal) - diameter (length of the major axis)
- Basic rectangle (formed by the major and the minor axis; encloses the boundary) and its eccentricity (major/minor axis).

Order of a shape: the number of digits

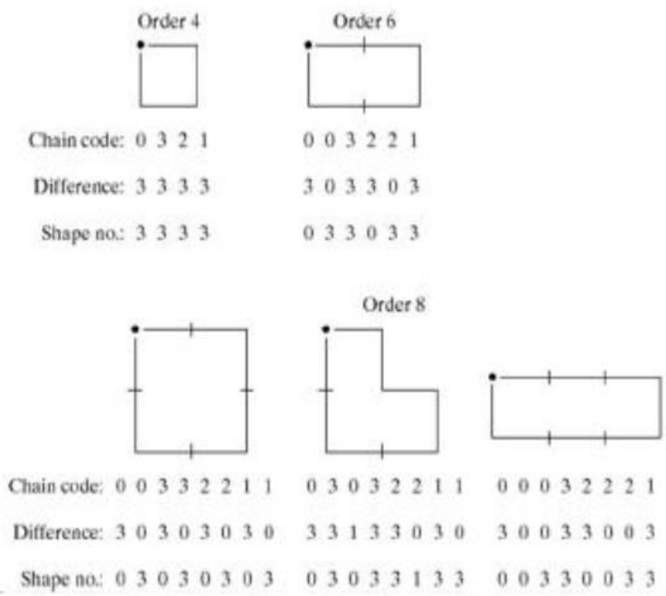
Shape number: the first difference of smallest magnitude

(treating the chain code as a circular sequence)

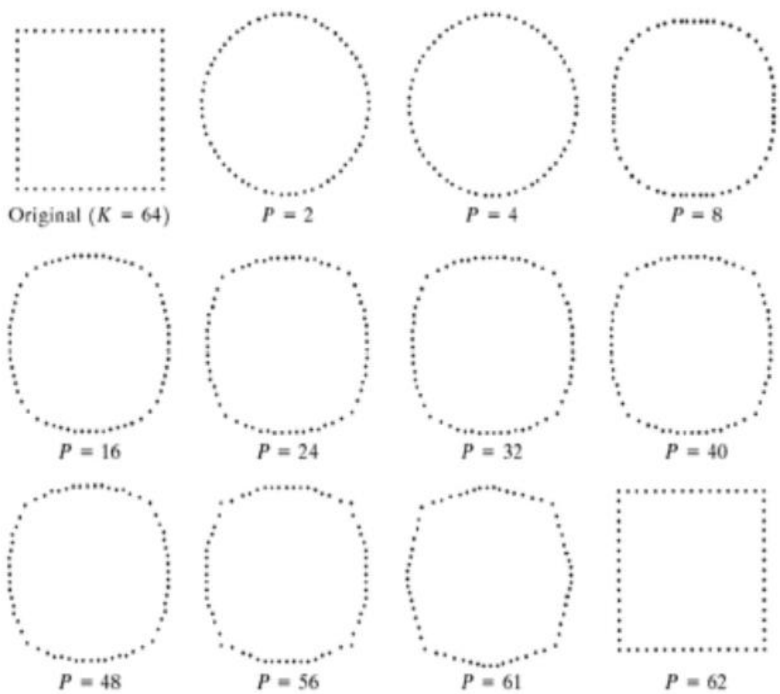
It is advisable to normalize the grid orientation by aligning the chain code grid to the basic rectangle.

The sequence of boundary points can be treated as a sequence of complex points in the complex plane

- It becomes a 1-D descriptor
- Can be DFT-transformed



The boundary can be approximated (by dropping DFT coefficients)



Fourier descriptors are not insensitive to translation..., but effects on the transform coefficients are known

Transformation	Boundary	Fourier Descriptor
Identity	$s(k)$	$a(u)$
Rotation	$s_r(k) = s(k)e^{j\theta}$	$a_r(u) = a(u)e^{j\theta}$
Translation	$s_t(k) = s(k) + \Delta_{xy}$	$a_t(u) = a(u) + \Delta_{xy}\delta(u)$
Scaling	$s_s(k) = \alpha s(k)$	$a_s(u) = \alpha a(u)$
Starting point	$s_p(k) = s(k - k_0)$	$a_p(u) = a(u)e^{-j2\pi k_0 u/K}$

Statistical moments

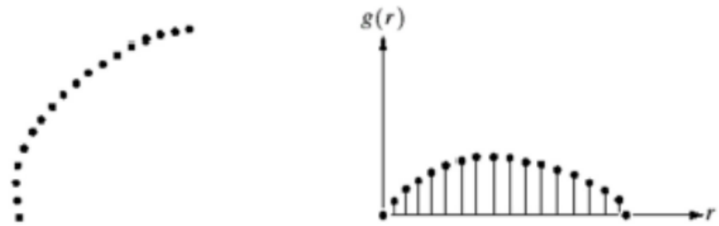
Once a boundary is described as a 1-D function, statistical moments (mean, variance, and a few higher-order central moments) can be used to describe it:

$$\mu_n(v)=\Sigma_i(v_i-m)^n p(v_i)$$

with

$$m=\Sigma_i v_i p(v_i)$$

(a) Boundary segment.
(b) Representation as a 1-D function.



Regional descriptors: Some simple descriptors

- Area
- Compactness =perimeter2/area

Topological Descriptors

If we stretch a shape like we would a cutout from a rubber sheet, there are certain shapes we can make and others we can't. Topology refers to properties of the shape that don't change, so long as you aren't allowed to tear or join parts of the shape.A useful topological descriptor is the Euler number E: the number of connected components C minus the number of holes H:

$$E = C - H$$

(You should know how to use connected-component labeling to determine the Euler number.) While the Euler number may seem like a simple descriptor (it is), it can be useful for separating simple shapes.

Topology is the study of the properties which are unaffected by any deformation (rubber-sheet distortion)- number of holes H- number of connected components C- Euler number, E=C-H

A region with three connected components.

A region with two holes.

Regions with Euler number equal to 0 and −1, respectively.

Texture descriptors are related to smoothness, coarseness, regularity…

- Statistical approaches give indications such as smooth, coarse, grainy…- Spectral techniques are related to DFT and can detect global periodicities

Some statistical descriptors

- *Statistical moments* of the gray-level histogram $p(z_i)$ — $\mu_n(z)=\Sigma(z_i-m)^n p(z_i)$
- *R* (R=0 in flat areas, -> 1 in "active" ones) — $R=1-[1+\sigma^2]^{-1}$
- *Uniformity* (maximum if all gray levels are equal) — $U=\Sigma p^2(z_i)$
- *Entropy* (is 0 for a constant image) — $e=-\Sigma p(z_i)\log_2 p(z_i)$

Fourier spectrum features:

- peaks give principal directions of the patterns - location of the peaks gives the fundamental period(s) - periodic components can be removed via filtering; the remaining non periodic image can be analyzed using statistical techniques

The spectrum can be also studied in polar coordinates

$S(r,\theta)$

For each pair r,θ, we can have two descriptors

$S(r)=\Sigma_\theta S_\theta(r)$

$S(\theta)=\Sigma_r S_r(\theta)$

Moments of 2-D functions

Moment: $m_{pq}=\Sigma\Sigma x^p y^q f(x,y)$

Central moment: $\mu_{pq}=\Sigma\Sigma(x-x')^p(y-y')^q f(x,y)$

with $x'=m_{10}/m_{00}$ and $y'=m_{01}/m_{00}$

It may be found that, e.g.

$\mu_{11}=m_{11}-y'm_{10}$
$\mu_{30}=m_{30}-3x'm_{20}+2x'^2m_{10}$ ...
If we define the *normalized central moments*

$$\eta_{pq}=\mu_{pq}/\mu_{00}^{\gamma} \quad \text{with} \quad \gamma=(p+q)/2+1$$

some *invariant moments* can be defined, e.g.

$\phi_1=\eta_{20}\eta_{02}$
$\phi_2=(\eta_{20}-\eta_{02})^2+4\eta_{11}^2$...
which are invariant to translation, rotation, and scaling

Descriptors for video sequences:

MPEG-1/2/4 make content available, whereas MPEG-7 allows you to find the content you need.
– A content description standard
» Video/images: Shape, size, texture, color, movements, positions, etc…
» Audio: Key, mood, tempo, changes, position in sound space, etc…
– Applications:
» Digital Libraries
» Multimedia Directory Services
» Broadcast Media Selection
» Editing, etc…

Projection Geometry

Let us consider the problem of reconstructing a 2D image given parallel-ray projections of the image measured at different angles. Let f(x,y) represent the density distribution within the image, Although discrete images are used in practice, the initial presentation here will be in continuous-space notations for easier comprehension. Consider the ray AB represented by the equation

$$x\cos\theta + y\sin\theta = t_1.$$

The integral of f(x, y) along the ray path AB is given by

$$p_\theta(t_1) = \int_{\mathrm{AB}} f(x,y)\,ds = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\,\delta(x\cos\theta + y\sin\theta - t_1)\,dx\,dy,$$

The mutually parallel rays within the imaging plane are represented by the coordinates (t, s) that are rotated by angle θ with respect to the (x,y) coordinates .Theoretically, we would need an infinite number of projections for all θ to be able to reconstruct the image. Before we consider reconstruction techniques, let us take a look at the projection or Fourier slice theorem.

The Fourier slice theorem

The projection or Fourier slice theorem relates the three spaces we encounter in image reconstruction from projections: the image, Fourier, and projection (Radon) spaces. Considering a 2D image, the theorem states that the 1D Fourier transform of a 1D projection of the 2D image is equal to the radial section (slice or profile) of the 2D Fourier transform of the 2D image at the angle of the projection. This is illustrated graphically in Figure 9.2, and may be derived as follows.

Let $F(u, v)$ represent the 2D Fourier transform of $f(x, y)$, given by

$$F(u,v) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x,y) \exp[-j2\pi(ux + vy)]\, dx\, dy.$$

Let $P_\theta(w)$ represent the 1D Fourier transform of the projection $p_\theta(t)$,

$$P_\theta(w) = \int_{-\infty}^{\infty} p_\theta(t) \exp(-j2\pi wt)\; dt,$$

where w represents the frequency variable corresponding to t. (*Note:* If x, y, s, and t are in mm, the units for u, v, and w will be $cycles/mm$ or mm^{-1}.) Let

$f(t, s)$ represent the image $f(x, y)$ rotated by angle θ, with the transformation given by

$$\begin{bmatrix} t \\ s \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}.$$

Then,

$$p_\theta(t) = \int_{-\infty}^{\infty} f(t,s)\, ds.$$

$$\begin{aligned} P_\theta(w) &= \int_{-\infty}^{\infty} p_\theta(t) \exp(-j2\pi wt)\, dt \\ &= \int_{-\infty}^{\infty} \left[\int_{-\infty}^{\infty} f(t,s)\, ds \right] \exp(-j2\pi wt)\, dt\,. \end{aligned}$$

Transforming from (t, s) to (x, y), we get

$$\begin{aligned} P_\theta(w) &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\exp[-j2\pi w(x\cos\theta + y\sin\theta)]\,dx\,dy \\ &= F(u,v) \text{ for } u = w\cos\theta,\ v = w\sin\theta \\ &= F(w,\theta), \end{aligned}$$

which expresses the projection theorem. Observe that $t = x\cos\theta + y\sin\theta$ and $dx\,dy = ds\,dt$.

A practical limitation of the Fourier method of reconstruction is that interpolation errors are larger for higher frequencies due to the increased spacing between the samples available on a discrete grid. Samples of $P_\theta(w)$ computed from $p_\theta(t)$ will be available on a polar grid, whereas the 2D Fourier transform $F(u, v)$ and/or the inverse-transformed image will be required on a Cartesian (rectangular grid). This limitation could cause poor reconstruction of high-frequency (sharp) details.

Back projection:

Let us now consider the simplest reconstruction procedure: backprojection (BP). Assuming the rays to be ideal straight lines, rather than strips of finite width, and the image to be made of dimensionless points rather than pixels or voxels of finite size, it can be seen that each point in the image $f(x, y)$ contributes to only one ray integral per parallel-ray projection $p_\theta(t)$, with $t = x\cos\theta + y\sin\theta$. We may obtain an estimate of the density at a point by

simply summing (integrating) all rays that pass through it at various angles, that is, by backprojecting the individual rays. In doing so, however, the contributions to the various rays of all of the other points along their paths are also added up, causing smearing or blurring; yet this method produces a reasonable estimate of the image. Mathematically, simple BP can be expressed

$$f(x,y) \simeq \int_0^{\pi} p_\theta(t)\,d\theta, \quad \text{where } t = x\cos\theta + y\sin\theta.$$

This is a sinusoidal path of integration in the (θ, t) Radon space. In practice, only a finite number of projections and a finite number of rays per projection will be available, that is, the (θ, t) space will be discretized; hence, interpolation will be required.

Filtered back projection:

Consider the inverse Fourier transform relationship

$$f(x,y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} F(u,v)\exp[j2\pi(ux+vy)]\,du\,dv.$$

Changing from the Cartesian coordinates (u,v) to the polar coordinates (w,θ), where $w = \sqrt{(u^2+v^2)}$ and $\theta = \tan^{-1}(v/u)$, we get

$$\begin{aligned} f(x,y) &= \int_0^{2\pi}\int_0^{\infty} F(w,\theta)\exp[j2\pi w(x\cos\theta + y\sin\theta)]\,w\,dw\,d\theta \\ &= \int_0^{\pi}\int_0^{\infty} F(w,\theta)\exp[j2\pi w(x\cos\theta + y\sin\theta)]\,w\,dw\,d\theta \\ &+ \int_0^{\pi}\int_0^{\infty} F(w,\theta+\pi) \\ &\times \exp\{j2\pi w[x\cos(\theta+\pi) + y\sin(\theta+\pi)]\}\,w\,dw\,d\theta. \end{aligned}$$

Here, $u = w\cos\theta$, $v = w\sin\theta$, and $du\,dv = w\,dw\,d\theta$. Because $F(w,\theta+\pi) = F(-w,\theta)$, we get

$$\begin{aligned} f(x,y) &= \int_0^{\pi}\left[\int_{-\infty}^{\infty} F(w,\theta)\,|w|\,\exp(j2\pi wt)\,dw\right]d\theta \\ &= \int_0^{\pi}\left[\int_{-\infty}^{\infty} P_\theta(w)\,|w|\,\exp(j2\pi wt)\,dw\right]d\theta, \end{aligned}$$

with $t = x\cos\theta + y\sin\theta$ as before. If we define

$$q_\theta(t) = \int_{-\infty}^{\infty} P_\theta(w)\,|w|\,\exp(j2\pi wt)\,dw,$$

we get

$$f(x,y) = \int_0^{\pi} q_\theta(t)\,d\theta = \int_0^{\pi} q_\theta(x\cos\theta + y\sin\theta)\,d\theta.$$

An important feature of the FBP technique is that each projection may be filtered and backprojected while further projection data are being acquired, which was of help in on-line processing with the first-generation CT scanners Furthermore, the inverse Fourier transform of the filter $|w|$ (with modifications to account for the discrete nature of measurements, smoothing window, etc.; could be used to convolve the projections directly in the t space using fast array processors. FBP is the most widely used procedure for image reconstruction from projections; however, the procedure provides good reconstructed images only when a large number of projections spanning the full angular range of 0^o to 180^o are available.

Algebraic Reconstruction Techniques:

The following important characteristics of ART are worth observing, ART proceeds ray by ray and is iterative.

1) If the hyper planes of the entire given ray sums are mutually orthogonal, we may start with any initial guess and reach the solution in only one cycle. On the other hand, if the hyper planes subtend small angles with one another, a large number of iterations will be required. The number of iterations may be reduced by using optimized ray, access schemes.

2) If the number of ray sums is greater than the number of pixels that is $M \geq N$ but the measurements are noisy, no unique solution exists -the procedure will oscillate in the neighborhood of the intersections of the hyper planes.

3) If $M > N$ the system is under-determined and an indefinite or infinite number of partial solutions exist.It has been shown that unconstrained. ART converges to the minimum-variance estimate.

4) The major advantage of ART is that any a priori information available about the image may be introduced easily into the iterative procedure.(for example, upper and/or lower limits on pixel values and the spatial boundaries of the image).This may help in obtaining a useful 'solution' even if the system is under determined.