LOGIC

CONCEPTS

Propositional Logic – Definition

A proposition is a collection of declarative statements that has either a truth value "true" or a truth value "false". A propositional consists of propositional variables and connectives. We denote the propositional variables by capital letters (A, B, etc). The connectives connect the propositional variables.

Some examples of Propositions are given below −

- "Man is Mortal", it returns truth value "TRUE"
- "12 + 9 = 3 − 2", it returns truth value "FALSE"

The following is not a Proposition −

- "A is less than 2". It is because unless we give a specific value of A, we cannot say whether the statement is true or false.

Connectives

In propositional logic generally we use five connectives which are − OR (∨), AND (∧), Negation/ NOT (¬), Implication / if-then (→), If and only if (⇔).

OR (∨) − The OR operation of two propositions A and B (written as A ∨ B) is true if at least any of the propositional variable A or B is true.

The truth table is as follows −

A	B	A ∨ B
True	True	True
True	False	True
False	True	True

False	False	False

AND (∧) − The AND operation of two propositions A and B (written as A ∧ B) is true if both the propositional variable A and B is true.

The truth table is as follows −

A	B	A ∧ B
True	True	False
True	False	False
False	True	False
False	False	True

Negation (¬) − The negation of a proposition A (written as ¬A) is false when A is true and is true when A is false.

The truth table is as follows –

A	¬A
True	False
False	True

Implication / if-then (→) − An implication A→B is False if A is true and B is false. The rest cases are true.

The truth table is as follows −

A	B	A → B
True	True	True
True	False	False
False	True	True
False	False	True

If and only if (⇔) − A⇔B is bi-conditional logical connective which is true when p and q are both false or both are true.

The truth table is as follows −

A	B	A ⇔ B
True	True	True
True	False	False
False	True	False
False	False	True

Tautologies

A Tautology is a formula which is always true for every value of its propositional variables.

Example − Prove [(A → B) ∧ A] → B is a tautology

The truth table is as follows −

A	B	A → B	(A → B) ∧ A	[(A → B) ∧ A] → B
True	True	True	True	True
True	False	False	False	True
False	True	True	False	True
False	False	True	False	True

As we can see every value of [(A → B) ∧ A] → B is "True", it is a tautology.

Contradictions

A Contradiction is a formula which is always false for every value of its propositional variables.

Example − Prove (A ∨ B) ∧ [(¬A) ∧ (¬B)] is a contradiction

The truth table is as follows −

A	B	A ∨ B	¬A	¬B	(¬A) ∧ (¬B)	(A ∨ B) ∧ [(¬A) ∧ (¬B)]
True	True	True	False	False	False	False
True	False	True	False	True	False	False
False	True	True	True	False	False	False
False	False	False	True	True	True	False

As we can see every value of (A ∨ B) ∧ [(¬A) ∧ (¬B)] is "False", it is a contradiction.

Contingency

A Contingency is a formula which has both some true and some false values for every value of its propositional variables.

Example − Prove (A ∨ B) ∧ (¬A) a contingency

The truth table is as follows −

A	B	A ∨ B	¬A	(A ∨ B) ∧ (¬A)
True	True	True	False	False
True	False	True	False	False
False	True	True	True	True
False	False	False	True	False

As we can see every value of (A ∨ B) ∧ (¬A) has both "True" and "False", it is a contingency.

Propositional Equivalences

Two statements X and Y are logically equivalent if any of the following two conditions −

- The truth tables of each statement have the same truth values.
- The bi-conditional statement X ⇔ Y is a tautology.

Example − Prove ¬(A ∨ B) and [(¬A) ∧ (¬B)] are equivalent

Testing by 1st method (Matching truth table)

A	B	A ∨ B	¬ (A ∨ B)	¬A	¬B	[(¬A) ∧ (¬B)]
True	True	True	False	False	False	False
True	False	True	False	False	True	False
False	True	True	False	True	False	False
False	False	False	True	True	True	True

Here, we can see the truth values of ¬ (A ∨ B) and [(¬A) ∧ (¬B)] are same, hence the statements are equivalent.

Testing by 2nd method (Bi-conditionality)

A	B	¬ (A ∨ B)	[(¬A) ∧ (¬B)]	[¬ (A ∨ B)] ⇔ [(¬A) ∧ (¬B)]
True	True	False	False	True
True	False	False	False	True
False	True	False	False	True
False	False	True	True	True

As [¬ (A ∨ B)] ⇔ [(¬A) ∧ (¬B)] is a tautology, the statements are equivalent.

EQUIVALENT LAWS:

Domination laws:	$p \vee \mathbf{T} \equiv \mathbf{T}$, $p \wedge \mathbf{F} \equiv \mathbf{F}$
Identity laws:	$p \wedge \mathbf{T} \equiv p$, $p \vee \mathbf{F} \equiv p$
Idempotent laws:	$p \wedge p \equiv p$, $p \vee p \equiv p$
Double negation law:	$\neg(\neg p) \equiv p$
Negation laws:	$p \vee \neg p \equiv \mathbf{T}$, $p \wedge \neg p \equiv \mathbf{F}$

The first of the Negation laws is also called "law of excluded middle". Latin: "tertium non datur".

Commutative laws:	$p \wedge q \equiv q \wedge p$, $p \vee q \equiv q \vee p$
Associative laws:	$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$
	$(p \vee q) \vee r \equiv p \vee (q \vee r)$
Distributive laws:	$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$
	$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$
Absorption laws:	$p \vee (p \wedge q) \equiv p$, $p \wedge (p \vee q) \equiv p$

Inverse, Converse, and Contra-positive

A conditional statement has two parts − **Hypothesis** and **Conclusion**.

Example of Conditional Statement − "If you do your homework, you will not be punished." Here, "you do your homework" is the hypothesis and "you will not be punished" is the conclusion.

Inverse –An inverse of the conditional statement is the negation of both the hypothesis and the conclusion. If the statement is "If p, then q", the inverse will be "If not p, then not q". The inverse of "If you do your homework, you will not be punished" is "If you do not do your homework, you will be punished."

Converse −The converse of the conditional statement is computed by interchanging the hypothesis and the conclusion. If the statement is "If p, then q", the inverse will be "If q, then *p*". The converse of "If you do your homework, you will not be punished" is "If you will not be punished, you do not do your homework".

Contra-positive –The contra-positive of the conditional is computed by interchanging the hypothesis and the conclusion of the inverse statement. If the statement is "If p, then q", the inverse will be "If not *q*, then not *p*". The Contra-positive of "If you do your homework, you will not be punished" is "If you will be punished, you do your homework".

Duality Principle

Duality principle set states that for any true statement, the dual statement obtained by interchanging unions into intersections (and vice versa) and interchanging Universal set into Null set (and vice versa) is also true. If dual of any statement is the statement itself, it is said **self-dual** statement.

Example −The dual of (A ∩ B) ∪ C is (A ∪ B) ∩ C

Elementary Product: A product of the variables and their negations in a formula is called an elementary product. If P and Q are any two atomic variables, then P, ¬ P ∧ q , ¬ Q ∧ P ∧ ¬ P are some examples of elementary products.

Elementary Sum: A sum of the variables and their negations in a formula is called an elementary sum. If P and Q are any two atomic variables, then P, ¬ p ∨ q, ¬ q ∨ p are some examples of elementary sums.

Normal Forms

We can convert any proposition in two normal forms −

1. Conjunctive normal form 2.Disjunctive normal form

Conjunctive Normal Form

A compound statement is in conjunctive normal form if it is obtained by operating AND among variables (negation of variables included) connected with ORs.

Examples

- (P ∪ Q) ∩ (Q ∪ R)
- (¬P ∪ Q ∪ S ∪¬T)

Disjunctive Normal Form

A compound statement is in disjunctive normal form if it is obtained by operating OR among variables (negation of variables included) connected with ANDs.

Examples

- $(P \cap Q) \cup (Q \cap R)$
- $(\neg P \cap Q \cap S \cap \neg T)$

Predicate Logic deals with predicates, which are propositions containing variables.

Functionally Complete set

A set of logical operators is called functionally complete if every compound proposition is logically equivalent to a compound proposition involving only this set of logical operators. $\wedge$, $\vee$, and $\neg$ form a functionally complete set of operators.

Minterms: For two variables p and q there are 4 possible formulas which consist of conjunctions of p,q or its negation given by $p \wedge q$, $p \wedge \neg q$, $\neg p \wedge q$ and $\neg p \wedge \neg q$

Maxterms: For two variables p and q there are 4 possible formulas which consist of disjunctions of p,q or its negation given by $p \vee q$, $p \vee \neg q$, $\neg p \vee q$ and $\neg p \vee \neg q$

Principal Disjunctive Normal Form: For a given formula an equivalent formula consisting of disjunctions of minterms only is known as principal disjunctive normal form(PDNF)

Principal Conjunctive Normal Form: For a given formula an equivalent formula consisting of conjunctions of maxterms only is known as principal conjunctive normal form(PCNF)

Predicate Logic – Definition

A predicate is an expression of one or more variables defined on some specific domain. A predicate with variables can be made a proposition by either assigning a value to the variable or by quantifying the variable.

The following are some examples of predicates −

- Let E(x, y) denote "x = y"
- Let X(a, b, c) denote "a + b + c = 0"
- Let M(x, y) denote "x is married to y"

Well Formed Formula

Well Formed Formula (Wff) is a predicate holding any of the following −

- All propositional constants and propositional variables are wffs
- If x is a variable and Y is a wff, ∀x Y and ∃x Y are also wff
- Truth value and false values are wffs
- Each atomic formula is a wff
- All connectives connecting wffs are wffs

Quantifiers

The variable of predicates is quantified by quantifiers. There are two types of quantifier in predicate logic − Universal Quantifier and Existential Quantifier.

Universal Quantifier

Universal quantifier states that the statements within its scope are true for every value of the specific variable. It is denoted by the symbol ∀.

∀x P(x) is read as for every value of x, P(x) is true.

Example − "Man is mortal" can be transformed into the propositional form ∀x P(x) where P(x) is the predicate which denotes x is mortal and the universe of discourse is all men.

Existential Quantifier

Existential quantifier states that the statements within its scope are true for some values of the specific variable. It is denoted by the symbol ∃.

∃x P(x) is read as for some values of x, P(x) is true.

Example − "Some people are dishonest" can be transformed into the propositional form ∃x P(x) where P(x) is the predicate which denotes x is dishonest and the universe of discourse is some people.

Nested Quantifiers

If we use a quantifier that appears within the scope of another quantifier, it is called nested quantifier.

Example

- $\forall a\ \exists b\ P(x, y)$ where $P(a, b)$ denotes $a + b = 0$
- $\forall a\ \forall b\ \forall c\ P(a, b, c)$ where $P(a, b)$ denotes $a + (b + c) = (a + b) + c$

Note − $\forall a\ \exists b\ P(x, y) \neq \exists a\ \forall b\ P(x, y)$

To deduce new statements from the statements whose truth that we already know, **Rules of Inference** are used.

What are Rules of Inference for?

Mathematical logic is often used for logical proofs. Proofs are valid arguments that determine the truth values of mathematical statements.

An argument is a sequence of statements. The last statement is the conclusion and all its preceding statements are called premises (or hypothesis). The symbol "∴", (read therefore) is placed before the conclusion. A valid argument is one where the conclusion follows from the truth values of the premises.

Rules of Inference provide the templates or guidelines for constructing valid arguments from the statements that we already have.

Addition

If P is a premise, we can use Addiction rule to derive $P \lor Q$.

```
 P
 Q
----------
∴ P ∨ Q
```

Example

Let P be the proposition, "He studies very hard" is true

Therefore − "Either he studies very hard Or he is very bad student." Here Q is the proposition "he is a very bad student".

Conjunction

If P and Q are two premises, we can use Conjunction rule to derive P ∧ Q.

```
P
Q
----------
∴ P ∧ Q
```

Example

Let P − "He studies very hard"

Let Q − "He is the best boy in the class"

Therefore − "He studies very hard and he is the best boy in the class"

Simplification

If P ∧ Q is a premise, we can use Simplification rule to derive P.

```
P ∧ Q
----------
∴ P
```

Example

"He studies very hard and he is the best boy in the class"

Therefore − "He studies very hard"

Modus Ponens

If P and P→Q are two premises, we can use Modus Ponens to derive Q.

```
P→Q
P
----------
∴ Q
```

Example

"If you have a password, then you can log on to face book"

"You have a password"

Therefore − "You can log on to face book"

Modus Tollens

If P→Q and ¬Q are two premises, we can use Modus Tollens to derive ¬P.

```
P→Q
¬Q
----------
∴ ¬P
```

Example

"If you have a password, then you can log on to face book"

"You cannot log on to face book"

Therefore − "You do not have a password"

Disjunctive Syllogism

If ¬P and P ∨ Q are two premises, we can use Disjunctive Syllogism to derive Q.

```
¬P
P ∨ Q
----------
∴ Q
```

Example

"The ice cream is not vanilla flavored"

"The ice cream is either vanilla flavored or chocolate flavored"

Therefore − "The ice cream is chocolate flavored"

Hypothetical Syllogism

If P→Q and Q→R are two premises, we can use Hypothetical Syllogism to derive P→R

```
P → Q
Q → R
----------
∴ P → R
```

Example

"If it rains, I shall not go to school"

"If I don't go to school, I won't need to do homework"

Therefore − "If it rains, I won't need to do homework"

Constructive Dilemma

If (P→Q) ∧ (R→S) and P ∨ R are two premises, we can use constructive dilemma to derive Q ∨ S.

```
(P → Q) ∧ (R → S)
      P ∨ R
   ----------
    ∴ Q ∨ S
```

Example

"If it rains, I will take a leave"

"If it is hot outside, I will go for a shower"

"Either it will rain or it is hot outside"

Therefore − "I will take a leave or I will go for a shower"

Destructive Dilemma

If (P→Q) ∧ (R→S) and ¬Q ∨ ¬S are two premises, we can use destructive dilemma to derive P ∨ R.

```
(P → Q) ∧ (R → S)
     ¬Q ∨ ¬S
   ----------
    ∴ P ∨ R
```

Example "If it rains, I will take a leave"

"If it is hot outside, I will go for a shower"

"Either I will not take a leave or I will not go for a shower"

Therefore − "It rains or it is hot outside"

Solved Problems

Write the truth table for the formula $(p \wedge q) \vee (\neg p \wedge \neg q)$

Ans:

P	q	$\neg p$	$\neg q$	$p \wedge q$	$\neg p \wedge \neg q$	$(p \wedge q) \vee (\neg p \wedge \neg q)$
T	T	F	F	T	F	T
T	F	F	T	F	F	F
F	T	T	F	F	F	F
F	F	T	T	F	T	T

Check whether $((p \rightarrow q) \rightarrow r) \vee \neg p$ is a tautology.

Ans:

$$((p \rightarrow q) \rightarrow r) \vee \neg p \Leftrightarrow ((\neg p \vee q) \rightarrow r) \vee \neg p \Leftrightarrow (\neg(\neg p \vee q) \vee r) \vee \neg p \Leftrightarrow (p \wedge \neg q) \vee (r \vee \neg p)$$

$$\Leftrightarrow (r \vee \neg p \vee p) \wedge (r \vee \neg p \vee \neg q) \Leftrightarrow T \wedge (r \vee \neg p \vee \neg q) \Leftrightarrow (r \vee \neg p \vee \neg q)$$

The given statement is not a tautology

What is meant by Tautology? Without using truth table, show that

$((P \vee Q) \wedge \neg(\neg P \wedge (\neg Q \vee \neg R))) \vee (\neg P \wedge \neg Q) \vee (\neg P \wedge \neg R)$ is a tautology.

Solution: A Statement formula which is true always irrespective of the truth values of the individual variables is called a tautology.

Consider $\neg(\neg P \wedge (\neg Q \vee \neg R) \Rightarrow \neg(\neg P \wedge \neg(Q \wedge R) \Rightarrow P \vee (Q \wedge R) \Rightarrow (P \vee Q) \wedge (P \vee R)$ (1)

Consider $(\neg P \wedge \neg Q) \vee (\neg P \wedge \neg R) \Rightarrow \neg(P \vee Q) \vee \neg(P \vee R) \Rightarrow \neg((P \vee Q) \wedge (P \vee R))$ (2)

Using (1) and (2)

$$((P \vee Q) \wedge (P \vee Q) \wedge (P \vee R)) \vee \neg((P \vee Q) \wedge (P \vee R))$$

$$\Rightarrow [(P \vee Q) \wedge (P \vee R)] \vee \neg[(P \vee Q) \wedge (P \vee R)] \Rightarrow T$$

Prove the following equivalences by proving the equivalences of the dual

$\neg((\neg P \wedge Q) \vee (\neg P \wedge \neg Q)) \vee (P \wedge Q) \equiv P$

Solution: It's dual is

$\neg((\neg P \vee Q) \wedge (\neg P \vee \neg Q)) \wedge (P \vee Q) \equiv P$

Consider,

$\neg((\neg P \vee Q) \wedge (\neg P \vee \neg Q)) \wedge (P \vee Q) \equiv P$	**Reasons**
$\Rightarrow ((P \wedge \neg Q) \vee (P \wedge Q)) \wedge (P \vee Q)$	(Demorgan's law)
$\Rightarrow ((Q \wedge P) \vee (\neg Q \wedge P)) \wedge (P \vee Q)$	(Commutative law)
$\Rightarrow ((Q \vee \neg Q) \wedge P) \wedge (P \vee Q)$	(Distributive law)
$\Rightarrow (T \wedge P) \wedge (P \vee Q)$	$(P \vee \neg P \Rightarrow T)$
$\Rightarrow P \wedge (P \vee Q)$	$(P \wedge T = P)$
$\Rightarrow P$	(Absorption law)

Obtain DNF of $Q \vee (P \wedge R) \wedge \neg((P \vee R) \wedge Q)$.

Solution:

$Q \vee (P \wedge R) \wedge \neg((P \vee R) \wedge Q)$

$\Leftrightarrow (Q \vee (P \wedge R)) \wedge (\neg((P \vee R) \wedge Q)$ (Demorgan law)

$\Leftrightarrow (Q \vee (P \wedge R)) \wedge ((\neg P \wedge \neg R) \vee \neg Q)$ (Demorgan law)

$\Leftrightarrow (Q \wedge (\neg P \wedge \neg R)) \vee (Q \wedge \neg Q) \vee ((P \wedge R) \wedge \neg P \wedge \neg R) \vee ((P \wedge R) \wedge \neg Q)$

(Extended distributed law)

$\Leftrightarrow (\neg P \wedge Q \wedge \neg R) \vee F \vee (F \wedge R \wedge \neg R) \vee (P \wedge \neg Q \wedge R)$ (Negation law)

$\Leftrightarrow (\neg P \wedge Q \wedge \neg R) \vee (P \wedge \neg Q \wedge R)$ (Negation law)

Obtain the conjunctive normal form of $P \leftrightarrow Q$

$P \leftrightarrow Q \quad \Leftrightarrow (P \rightarrow Q) \wedge (Q \rightarrow P)$

$\Leftrightarrow (\neg P \vee Q) \wedge (\neg Q \vee P)$

Obtain Pcnf and Pdnf of the formula $(\neg P \vee \neg Q) \rightarrow (P \leftrightarrow \neg Q)$

Solution:

Let S = $(\neg P \vee \neg Q) \rightarrow (P \leftrightarrow \neg Q)$

P	Q	¬ P	¬ Q	$\neg P \vee \neg Q$	$P \leftrightarrow \neg Q$	S	Minterm	Maxterm
T	T	F	F	F	F	T	$P \wedge Q$	
T	F	F	T	T	T	T	$P \wedge \neg Q$	
F	T	T	F	T	T	T	$\neg P \wedge Q$	
F	F	T	T	T	F	F		$P \vee Q$

PCNF: $P \vee Q$ and PDNF: $(P \wedge Q) \vee (P \wedge \neg Q) \vee (\neg P \wedge Q)$

Obtain the PDNF and PCNF of $P \vee (\neg P \rightarrow (Q \vee (\neg Q \rightarrow R)))$.

Solution:

$P \vee (\neg P \rightarrow (Q \vee (\neg Q \rightarrow R)))$

$\Rightarrow P \vee (P \vee (Q \vee (Q \vee R)))$

$\Rightarrow (P \vee Q \vee R)$

$$S = (P \vee Q \vee R)$$

$$\neg S = (\neg P \vee Q \vee R) \wedge (\neg P \vee \neg Q \vee R) \wedge (\neg P \vee \neg Q \vee \neg R)$$

$$\wedge (P \vee \neg Q \vee \neg R) \wedge (\neg P \vee Q \vee \neg R) \wedge (P \vee Q \vee \neg R) \wedge (P \vee \neg Q \vee R)$$

$$\neg\neg S = \neg((\neg P \vee Q \vee R) \wedge (\neg P \vee \neg Q \vee R) \wedge (\neg P \vee \neg Q \vee \neg R)$$

$$\wedge (P \vee \neg Q \vee \neg R) \wedge (\neg P \vee Q \vee \neg R) \wedge (P \vee Q \vee \neg R) \wedge (P \vee \neg Q \vee R))$$

$$= (P \wedge \neg Q \wedge \neg R) \vee (P \wedge Q \wedge \neg R) \vee (P \wedge Q \wedge R)$$

$$\vee (\neg P \wedge Q \wedge R) \vee (P \wedge \neg Q \wedge R) \vee (\neg P \wedge \neg Q \wedge R) \vee (\neg P \wedge Q \wedge \neg R)$$

Show that $R \to S$ can be derived from the premises $P \to (Q \to S), \neg R \vee P$ & Q

Solution:

R	Assumed premises
$\neg R \vee P$	Rule P
$R \to P$	Rule T
P	Rule T
$P \to (Q \to S)$	Rule P
$Q \to S$	Rule P
Q	Rule P
S	Rule T
$R \to S$	Rule CP

Show that the following statements constitute a valid argument.

If there was rain, then traveling was difficult. If they had umbrella, then traveling was not difficul

They had umbrella. Therefore there was no rain.

Solution:

Let P : There was rain Q : Traveling was difficult R : They had umbrella

Then, the given statements are symbolized as

(1) $P \to Q$ (2) $R \to \sim Q$ (3) R

Conclusion : $\sim P$

1) R	Rule P
2) $R \to \sim Q$	Rule P
3) $\sim Q$	Rule T,1,2
4) $P \to Q$	Rule P
5) $\sim P$	Rule T,3,4

Therefore, it is a valid conclusion.

Symbolize the following statements:

(a) All men are mortal

(b) All the world loves a lover

(c) X is the father of mother of Y

(d) No cats has a tail

(e) Some people who trust others are rewarded

Solution:

(a) Let M(x): x is a man H(x): x is Mortal

$(\forall x)\ (M(x) \rightarrow H(x))$

(b) Let P(x): x is a person L(x): x is a lover R(x,y): x loves y

$(x)\ (P(x) \rightarrow (y)\ (P(y) \wedge L(y) \rightarrow R(x,y)))$

(c) Let P(x): x is a person F(x,y): x is the father of y

M(x,y): x is the mother of y $(\exists z)\ (P(z) \wedge F(x,z) \wedge M(z,y))$

(d) Let C(x): x is a cat T(x): x has a tail

$(\forall x)\ (C(x) \rightarrow \neg T(x))$

(e) Let P(x): x is a person T(x): x trust others R(x): x is rewarded

$(\exists x)\ (P(x) \wedge T(x) \wedge R(x))$

Show that the following premises are inconsistent.

(1) If Nirmala misses many classes through illness then he fails high school.

(2) If Nirmala fails high school, then he is uneducated.

(3) If Nirmala reads a lot of books then he is not uneducated.

(4) Nirmala misses many classes through illness and reads a lot of books.

Solution:

E : Nirmala misses many classes

S: Nirmala fails high school

A: Nirmala reads lot of books

H: Nirmala is uneducated

Statement:

(1) $E \to S$

(2) $S \to H$

(3) $A \to \sim H$

(4) $E \wedge A$

Premises are : $E \to S$, $S \to H$, $A \to \sim H$, $E \wedge A$

1) $E \to S$	Rule P
2) $S \to H$	Rule P
3) $E \to H$	Rule T, 1,2
4) $A \to \sim H$	Rule P
5) $H \to \sim A$	Rule T,4
6) $E \to \sim A$	Rule T,3,5
7) $\sim E \vee \sim A$	Rule T,6
8) $\sim(E \wedge A)$	Rule T,7
9) $E \wedge A$	Rule P
10) $(E \wedge A) \wedge \sim(E \wedge A)$	Rule T,8,9

Which is nothing but false

Therefore given set of premises are inconsistent

Let p, q, r be the following statements:
p: I will study discrete mathematics
q: I will watch T.V.
r: I am in a good mood.
Write the following statements in terms of p, q, r and logical connectives.
(1) If I do not study and I watch T.V., then I am in good mood.
(2) If I am in good mood, then I will study or I will watch T.V.
(3) If I am not in good mood, then I will not watch T.V. or I will study.
(4) I will watch T.V. and I will not study if and only if I am in good mood.
Solution:

(1) $(\neg p \wedge q) \rightarrow r$
(2) $r \rightarrow (p \vee q)$
(3) $\neg r \rightarrow (\neg q \vee p)$
(4) $(q \wedge \neg p) \leftrightarrow r$

Use the indirect method to prove that the conclusion $\exists z Q(z)$ follows from the premises $\forall x(P(x) \rightarrow Q(x))$ and $\exists y P(y)$

Solution:

1	$\neg \exists z Q(z)$	P(assumed)
2	$\forall z \neg Q(z)$	T, (1)
3	$\exists y P(y)$	P
4	$P(a)$	ES, (3)
5	$\neg Q(a)$	US, (2)
6	$P(a) \wedge \neg Q(a)$	T, (4),(5)
7	$\neg(P(a) \rightarrow Q(a))$	T, (6)
8	$\forall x(P(x) \rightarrow Q(x))$	P
9	$P(a) \rightarrow Q(a)$	US, (8)
10	$P(a) \rightarrow Q(a) \wedge \neg(P(a) \rightarrow Q(a))$	T,(7),(9) contradiction

Show that $(\exists x)(P(x) \wedge Q(x)) \Rightarrow (\exists x)P(x) \wedge (\exists x)Q(x)$

Solution:

1) $(\exists x)(P(x) \wedge Q(x))$	Rule P
2) $P(a) \wedge Q(a)$	ES, 1
3) $P(a)$	Rule T, 2
4) $Q(a)$	Rule T, 2
5) $(\exists x)P(x)$	EG, 3
6) $(\exists x)Q(x)$	EG, 4
7) $(\exists x)P(x) \wedge (\exists x)Q(x)$	Rule T, 5, 6

ASSIGNMENT PROBLEMS

1. Write the statement in symbolic form "Some real numbers are rational".
2. Symbolize the expression "x is the father of the mother of y"
3. Symbolize the expression "All the world loves a lover"
4. Write the negation of the statement "If there is a will, then there is a way".
5. Construct the truth table for $\neg(p \wedge q)$
6. Find the CNF and DNF of $\neg(p \vee q) \leftrightarrow (p \wedge q)$
7. Show that $P \to Q, Q \to \neg R, R, P \vee (J \wedge S)$ imply $J \wedge S$
8. Show that $P \to Q, P \to R, Q \to R, P$ are inconsistent.
9. Prove that $(\exists x)(P(x) \wedge Q(x) \Rightarrow (\exists x)P(x) \wedge (\exists x)Q(x)$
10. Show that $\neg P(a,b)$ follows logically from $(x)(y)(P(x,y) \to W(x,y)$ and $\neg W(a,b)$
11. Show that $\neg P \vee Q, \neg Q \vee R, R \to S \Rightarrow P \to S$
12. Show that $\neg(P \wedge \neg Q) \wedge \neg Q \vee R \wedge \neg R \Rightarrow \neg P$

13. Show that P is equivalent to $\neg\neg P, P \wedge P, P \vee P, P \wedge (P \vee Q), (P \wedge Q) \vee (P \wedge \neg Q)$

14. Indicate which one are tautologies (or) contradictions

(a) $(P \wedge Q) \Leftrightarrow P$ (b) $P \rightarrow P \vee Q$

15. If R:Ram is rich, H:Ram is happy ,Write in symbolic form

(a) Ram is poor but happy (b) Ram is poor or unhappy

(c) Ram is neither rich nor happy

16. Show that the hypothesis, “It is not sunny this afternoon and it is colder than yesterday,” ” We will go swimming only if it is sunny,” “If we do not go swimming then we will take a canoe trip,” and “If we take a canoe trip, then we will be home by sunset “lead to the conclusion “we will be home by sunset”.