INSTRUCTION: ATTEMPT ALL QUESTIONS

TIME: 3 HOURS

1. Use Venn diagrams to test which of the following statements are logically true or logically false.

   (a) $p \vee \neg p$.

   (b) $p \wedge \neg p$.

   (c) $p \vee (\neg p \wedge q)$.

   (d) $p \rightarrow (q \rightarrow p)$.

   (e) $p \wedge \neg(q \rightarrow p)$.

2. Use Venn diagrams to test the following statements for equivalences.

   (a) $p \vee \neg q$.

   (b) $\neg(p \wedge q)$.

   (c) $\neg(q \wedge \neg q)$.

   (d) $p \rightarrow \neg q$.

   (e) $\neg p \vee \neg q$.

3. Use Venn diagrams for the following pairs of statements to test whether one implies the other.

   (a) $p; p \wedge q$.

   (b) $p \wedge \neg q; \neg p \rightarrow \neg q$.

   (c) $p \rightarrow q; q \rightarrow p$.

   (d) $p \wedge q; p \wedge \neg q$.

4. Devise a test for inconsistency of p and q, using Venn diagrams.

5. Three or more statements are said to be inconsistent if they cannot all be true. What does this state about their truth sets?

6. Consider these three statements.

   If this is a good course, then I will work hard in it.

   If this is not a good course, then I shall get a bad grade in it.

I will not work hard, but I will get a good grade in this course.

(a) Assign variables to the components of each of these statements.

(b) Bring the statements into symbolic form.

(c) Find the truth sets of the statements.

(d) Rest for consistency.

7. Use truth tables to find which of the following sets are empty.

   (a) $(P \cup Q) \cap (\tilde{P} \cup \tilde{Q})$.

   (b) $(P \cap Q) \cap (\tilde{Q} \cap R)$.

   (c) $(P \cap Q) - P$.

   (d) $(P \cup R) \cap (\tilde{P} \cup \tilde{Q})$

8. Use truth tables to find out whether the following sets are all different.

   (a) $P \cap (Q \cup R)$.

   (b) $(R - Q) \cup (Q - R)$.

   (c) $(R \cup Q) \cap (R \widetilde{\cap} Q)$.

   (d) $(P \cap Q) \cup (P \cap R)$.

   (e) $(P \cap Q \cap \tilde{R}) \cup (P \cap \tilde{Q} \cap R) \cup (\tilde{P} \cap Q \cap \tilde{R}) \cup (\tilde{P} \cap \tilde{Q} \cap R)$.

9. Use truth tables for the following pairs of sets to test whether one is a subset of the other.

   (a) $P; P \cap Q$.

   (b) $P \cap \tilde{Q}; Q \cap \tilde{P}$.

   (c) $P - Q; Q - P$.

   (d) $P \cap \tilde{Q}; P \cup Q$.

10. Show, both by the use of truth tables and by the use of Venn diagrams, that $p \wedge (q \vee r)$ is equivalent to $(p \wedge q) \vee (p \wedge r)$.

11. The *symmetric difference* of P and Q is defined to be $(P - Q) \cup (Q - P)$. What connective corresponds to this set operation?