FINAL EXAMINATION

INSTRUCTION: ATTEMPT ALL QUESTIONS IN **SECTION A** AND **ANY OTHER TWO** IN **SECTION B**

USE A SCIENTIFIC CALCULATOR

TIME: 3 HOURS

SECTION A (50 MARKS)

QUESTION 1. (8 Marks)

Derive the following results from the 28 basic laws.

(a) $A = (A \cap B) \cup (A \cap \tilde{B})$.

(b) $A \cup B = (A \cap B) \cup (A \cap \tilde{B}) \cup (\tilde{A} \cap B)$.

(c) $A \cap (A \cup B) = A$.

(d) $A \cup (\tilde{A} \cap B) = A \cup B$.

QUESTION 2. (4 Marks)

Show that

$$A \times (B + C) = (A \times B) + (A \times C).$$

QUESTION 3. (4 Marks)

Show that the following equation is not always true.

$$A + (B \times C) = (A + B) \times (A + C).$$

QUESTION 4. (4 Marks)

Complete the interpretations of the addition and multiplication tables for the number systems representing

(a) parity,

(b) the sets $\mathcal{U}$ and $\emptyset$.

QUESTION 5. (6 Marks)

The first number system above (about parity) can be interpreted to deal with the remainders of integers when divided by 2. An even number leaves 0, an odd number leaves 1. Construct tables of addition and multiplication for remainders of integers when divided by 3. [Hint: These will be 3 by 3 tables.]

QUESTION 6. (8 Marks)

Let $[A_1, A_2, A_3]$ and $[B_1, B_2]$ be two partitions. Prove that the cross-partition of the two given partitions really is a partition, that is, it satisfies requirements (1) and (2) for partitions.

QUESTION 7. (8 Marks)

Let p and q be statements with truth sets P and Q. Form the partition $[P \cap Q, P \cap \tilde{Q}, \tilde{P} \cap Q, \tilde{P} \cap \tilde{Q}]$. State in each case below which of the cells must be empty in order to make the given statement a logically true statement.

(a) $p \rightarrow q$

(b) $p \leftrightarrow q$

(c) $p \vee \neg p$

(d) p

QUESTION 8. (6 Marks)

A subject can be completely classified by introducing several simple subdivisions and taking their cross-partition. Thus, courses

in college may be partitioned according to subject, level of advancement, number of students, hours per week, interests, etc. For each of the following subjects, introduce five or more partitions. How many cells are there in the complete classification (cross-partition) in each case?

(a) Detective stories.

(b) Diseases.

SECTION B (30 MARKS)

QUESTION 9. (15 Marks)

Let A, B, and C be any three sets of a universal set $\mathcal{U}$. Draw a Venn diagram and show that

$$\begin{aligned} n(A \cup B \cup C) &= n(A) + n(B) + n(C) \\ &\quad - n(A \cap B) - n(A \cap C) - n(B \cap C) \\ &\quad + n(A \cap B \cap C). \end{aligned}$$

QUESTION 10. (15 Marks)

PART A.

If p and q are equivalent statements and $n(P) = 10$, what is $n(P \cup Q)$?

PART B.

If p implies q, prove that $n(P \cup \tilde{Q}) = n(P) + n(\tilde{Q})$.

PART C.

Show that $n(\tilde{A} \cap \tilde{B}) = n(A \,\tilde{\cup}\, B) = n(\mathcal{U}) - n(A \cup B)$.

QUESTION 11. (15 Marks)

PART A.

A group of seven boys and ten girls attends a dance. If all the boys dance in a particular dance, how many choices are there for the girls who dance? For the girls who do not dance? How many choices are there for the girls who do not dance, if three of the girls are sure to be asked to dance?

PART B.

How many ways can you answer a ten-question true-false exam, marking the same number of answers true as you do false? How many if it is desired to have no two consecutive answers the same?

QUESTION 12. (15 Marks)

Verify that the following formula gives the number of elements in the intersection of three sets.

$$\begin{aligned} n(A_1 \cap A_2 \cap A_3) &= n(A_1) + n(A_2) + n(A_3) \\ &\quad - n(A_1 \cup A_2) - n(A_1 \cup A_3) - n(A_2 \cup A_3) \\ &\quad + n(A_1 \cup A_2 \cup A_3). \end{aligned}$$