Lecture 9

Learning Objectives:
At the end of this class, students should be able to:

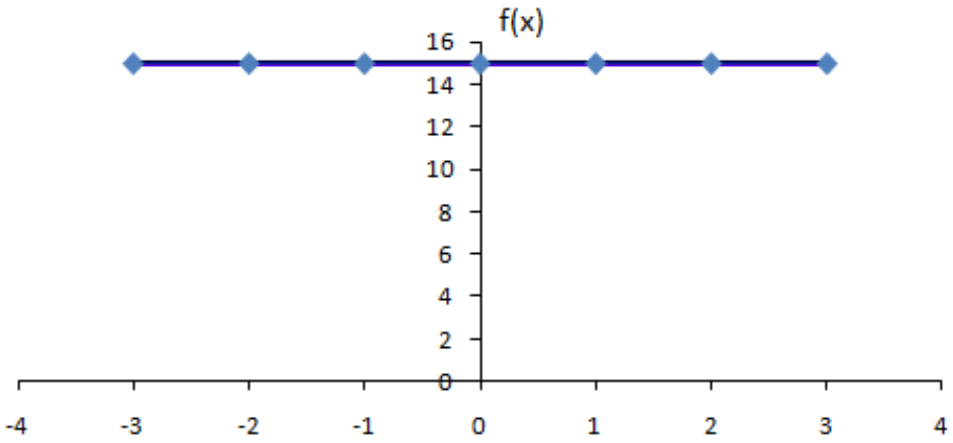
- understand the concept of constant function
- understand the concept of linear function
- understand the concept of quadratic function
- solve related problems

9.1 Constant Functions

A constant function has the general form $y=f(x)=a_o$, where a_o is a real number. The domain of this function is the set of all real numbers and range is the set consisting of only one number a_o.

For example, $y=f(x)=10$ is a constant function. Similarly, if a product is sold for \$15 per unit, the marginal revenue function can be stated as the constant function. Mathematically, marginal revenue $(MR)=f(x)=15$, where *x* equals the number of units sold of the product. The domain of the constant function is the set of all real numbers whereas the range consists of a single value a_o.

The graph of a constant function is a straight-line parallel to *x*-axis. The following figure represents the graph of the function $f(x)=15$.



9.2 Linear Functions

A linear function has the general form $y=f(x)=a_1x+a_o$; where a_o and a_1 are real numbers, and $a_1 \neq 0$. The domain and range of this function are the set of real numbers.

For example, the total cost of owning and operating a patrol car is given by $C(x) = 0.80x + 20{,}000$. This is an example of linear function.

This total cost function $C(x) = 0.80x + 20{,}000$ has two components: variable cost and fixed cost. The fixed cost is given by the term 20,000 and the variable cost is given by the term $0.80x$.

Illustration 1

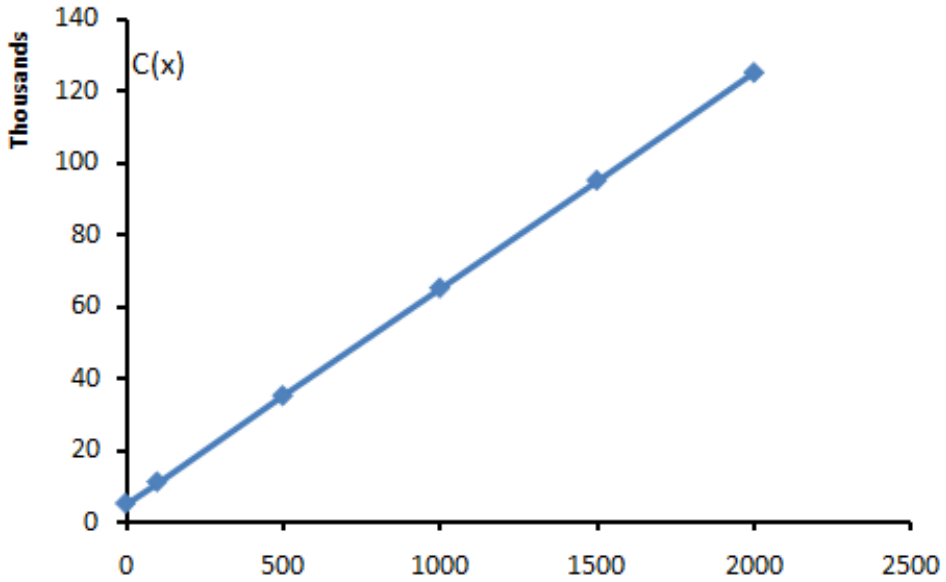
A manufacturer's total cost consists of a fixed overhead of \$5,000 plus production costs of \$60 per unit. Express the total cost as a function of the number of units produced and draw the graph.

Solution

The total cost function is given by

$$C(x) = 60x + 5{,}000$$

Where, x represents the number of units produced. The graph of this function is as follows.



Quadratic Functions

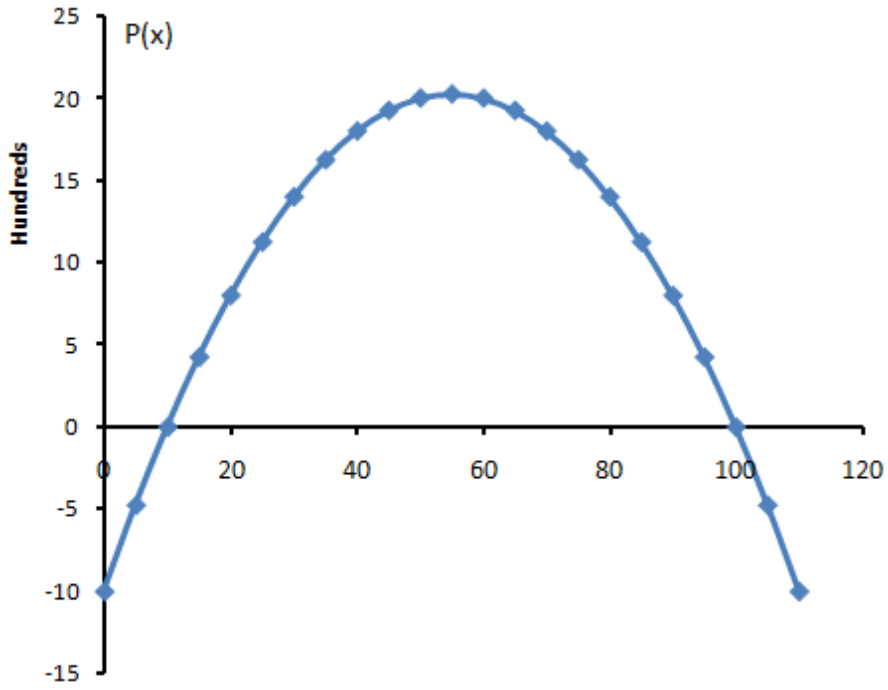
A quadratic function has the general form $y = f(x) = a_2x^2 + a_1x + a_o$. Where a_2, a_1 and a_o are real numbers, and $a_2 \neq 0$. The domain and range of this function are the set of real numbers. For example, the total revenue from the selling a particular product is given by $R = f(p) = 1{,}500 - 50p^2$, where p is the price stated in dollars.

Illustration 2

The profit function for a certain commodity is $P(x) = 110x - x^2 - 1000$. Sketch the graph of function. Find the level of production that yields maximum profit, and find the maximum profit.

Solution

The following figure represents the graph of $P(x) = 110x - x^2 - 1000$.



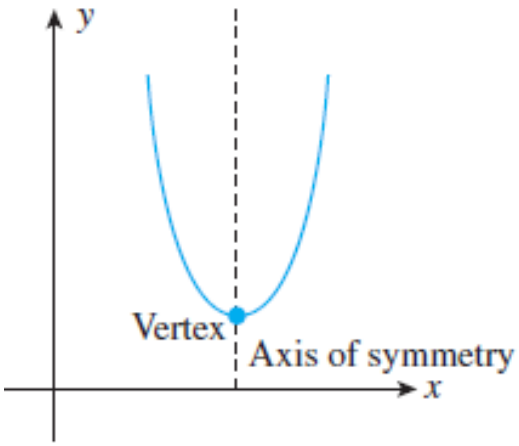
The maximum profit will occur at the value of x that corresponds to the highest point on the profit graph. This is the vertex of the parabola which lies at $x = -b/2a$.

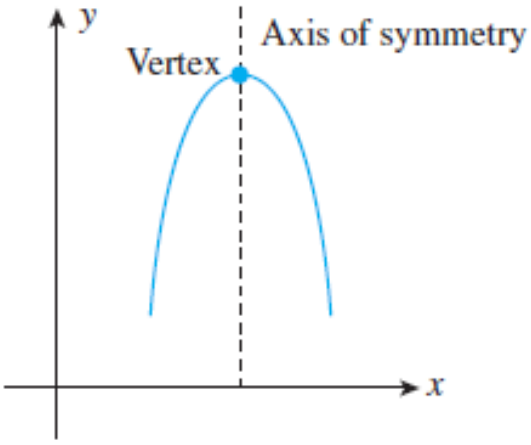
i.e., $x = -110/(2\times -1) = 55$

Thus, profit is maximized when 55 units are produced and sold.
the maximum profit is

$$P(55) = 110 \times 55 - (55)^2 - 1000 = 2025$$

In general, the graph of a quadratic function is a parabola that opens upward or downward. Furthermore, the parabola is symmetric with respect to a vertical line called the *axis of symmetry*. This line also passes through the lowest point or the highest point of the parabola. The point of intersection of the parabola with its axis of symmetry is known as the *vertex* of the parabola. The following figures represent these characteristics of the parabola.





Exercise for Reader

1. Puritron, a manufacturer of water filters, has a monthly fixed cost of $20,000, a production cost of $20 per unit, and a selling price of $30 per unit. Find the cost function, the revenue function, and the profit function for Puritron.
2. A manufacturer estimates that it costs $14 to produce each unit of a particular commodity that sells for $23 per unit. There is also a fixed cost of $1,200.
   a) Express the cost $C(x)$ and revenue $R(x)$ as functions of the number of units x that are produced and sold.
   b) What is the profit function for this commodity?
   c) How much profit is generated when 2,000 units of the commodity are produced?
3. The quarterly profit (in thousands of dollars) of Cunningham Realty is given by $P(x) = -(1/3)x^2 + 7x + 30;\ (0 \le x \le 50)$ where x (in thousands of dollars) is the amount of money Cunningham spends on advertising per quarter. Find the amount of money Cunningham should spend on advertising in order to realize a maximum quarterly profit. What is the maximum quarterly profit realizable by Cunningham?
4. The demand function for a certain brand of Bluetooth wireless headsets is given by $p = d(x) = -0.025x^2 - 0.5x + 60$ where p is the wholesale unit price in dollars and x is the quantity demanded each month, measured in units of a thousand. Sketch the corresponding demand curve. Above what price will there be no demand? What is the maximum quantity demanded per month?