Lecture 10

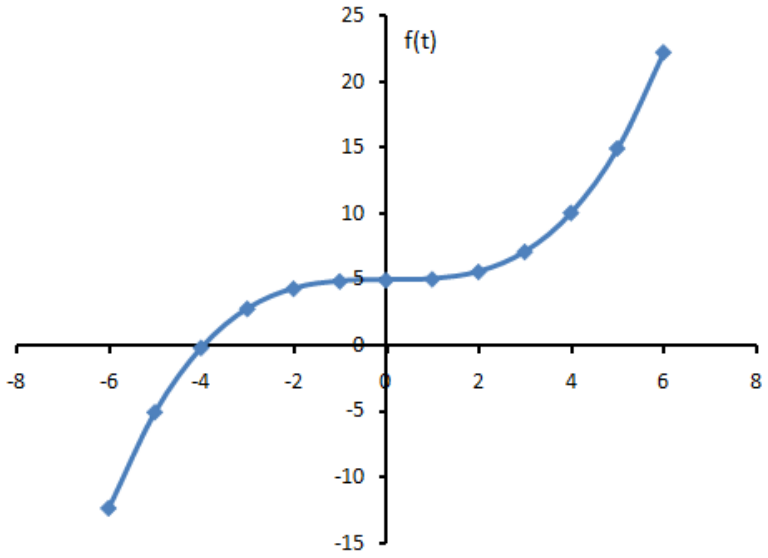
Learning Objectives:
At the end of this class, students should be able to:

- familiar with cubic functions
- familiar with polynomial functions
- familiar with rational functions
- identify composite functions
- solve related problems

10.1 Cubic Function

A cubic function has the general form $y = f(x) = a_3x^3 + a_2x^2 + a_1x + a_o$ Where a_3, a_2, a_1 and a_o are real numbers, and $a_3 \neq 0$. The domain of this function is the set of all real numbers.

The number of cattle expected to be afflicted by the disease, under the situation of epidemic, is estimated by the function $n = f(t) = 0.08t^3 + 5$. Where *n* equals the number of cattle afflicted and *t* equals the number of days since the disease was first detected. This is an example of cubic function. Using excel, we get the following graph of the function $f(t) = 0.08t^3 + 5$.



10.2 Polynomial Functions

Each of the previous function is an example of a polynomial function. A polynomial function of degree n has the general form

$$y = f(x) = a_nx^n + a_{n-1}x^{n-1} + \ldots + a_2x^2 + a_1x + a_o$$

Where a_n, $a_{n-1}, \ldots, a_2$, a_1 and a_o are real numbers, and $a_n \neq 0$.

The exponent on each x must be a non-negative integer and the degree of the polynomial is the highest power (exponent) in the function.

For example, $f(x) = x^8 + 5$ is a polynomial function of degree 8.

10.3 Rational Functions

A rational function has the general form $y = f(x) = \frac{g(x)}{h(x)}$, $h(x) \neq 0$. Where $g(x)$ and $h(x)$ are both polynomial functions.

For example, $f(x) = \frac{2x+3}{x^2 - 5x + 6}$ is a rational function.

10.4 Composite Functions

A composite function exists when one function can be viewed as a function of the values of another function. If $y = g(u)$ and $u = h(x)$, the composite function $y = f(x) = g(h(x))$ is created by substituting $u = h(x)$ in to the function $y = g(u)$ wherever *u* appears.

Illustration 1

If $y = g(u) = u^2 + 5u + 10$ and $u = h(x) = x + 2$, find the composite function $y = f(x) = g(h(x))$.

Solution

We know that $y = f(x) = g(h(x))$

$$= g(x+2)$$
$$= (x+2)^2 + 5(x+2) + 10$$
$$= x^2 + 4x + 4 + 5x + 10 + 10$$
$$= x^2 + 9x + 24$$

10.5 Piecewise Defined Functions

The functions which are defined by two or more than two equations are called piecewise defined functions. For example, the absolute value function is a piecewise defined function. We know that

$$f(x) = |x| = \begin{cases} x, & \text{if } x \geq 0 \\ -x, & \text{if } x < 0 \end{cases}$$

The graph of this function is as follows.

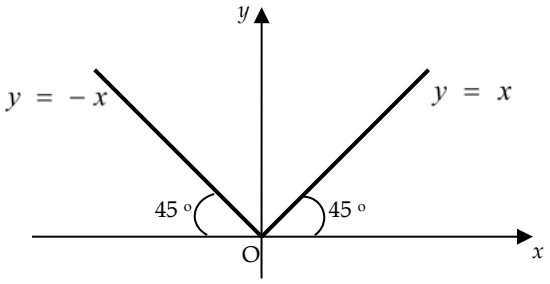


Illustration 2

The selling price *S* of a product to be defined as a piecewise function of the cost *C* of the product, as follows:

$$S = f(C) = \begin{cases} 3C & \text{if } 0 \leq C \leq 20 \\ 1.5C + 30 & \text{if } C > 20 \end{cases}$$

a) Find the selling price of a product that costs \$15.
b) Find the selling price of a product that costs \$25.

Solution

a) As per the definition of the function
$S = f(15) = 3 \times 15 = \$45$
b) Here, $S = f(25) = 1.5 \times 25 + 30 = \$67.50$

Exercise for Reader

1. Find $f(g(x))$ and $g(f(x))$ for the following functions.
   a) $f(x) = \sqrt{x} + 1$ and $g(x) = \dfrac{x}{1+x}$
   b) $f(x) = 2\sqrt{x} + 3$ and $g(x) = x^2 + 1$
   c) $f(x) = \dfrac{x}{x^2+1}$ and $g(x) = \dfrac{1}{x}$
   d) $f(x) = \sqrt{x+1}$ and $g(x) = \dfrac{1}{x-1}$
2. The 2012 monthly charge in dollars for *x* kilowatt hours (kWh) of electricity used by a residential customer of Excelsior Electric Membership Corporation during the months of November through June is given by the function

$$C(x) = \begin{cases} 10 + 0.094x & \text{if } 0 \le x \le 100 \\ 19.40 + 0.075(x - 100) & \text{if } 100 \le x \le 500 \\ 49.40 + 0.05(x - 500) & \text{if } x > 500 \end{cases}$$

   i) What is the monthly charge if 1100 kWh of electricity are consumed in a month?
   ii) What is the monthly charge if 450 kWh are consumed in a month?