Lecture 11

Learning Objectives:

At the end of this class, students should be able to:

- familiar with exponential functions
- understand the concept of exponential growth and decay

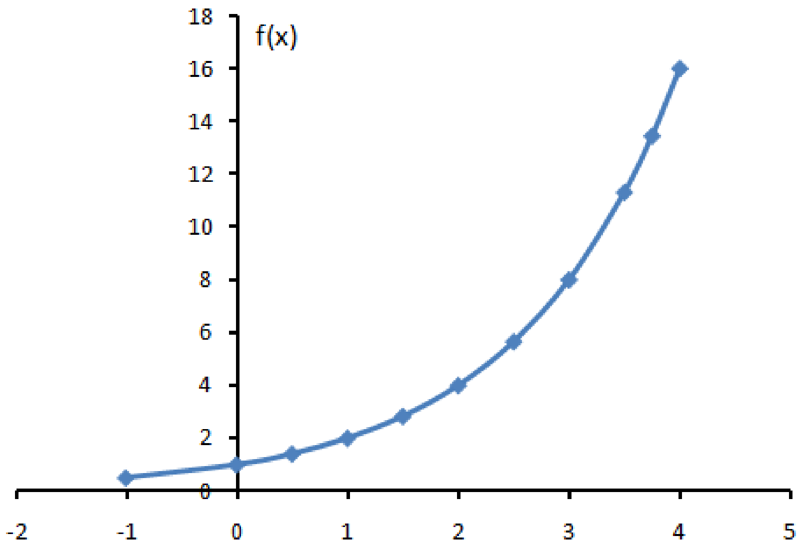
11.1 Exponential Functions

The function defined by $f(x) = b^x$ where $b > 0$ and $b \neq 1$, is called an exponential function with base *b* and exponent *x*. The domain of this function is the set of all real numbers.

For example, $f(x) = 2^x$ is an exponential function with base 2. Let us list the coordinates (*x*, *f*(*x*)) of some points on the graph of $f(x) = 2^x$ in tabular form, as follows.

x	-1	0	0.5	1	1.5	2	2.5	3	3.5	3.75	4
f(*x*)	0.5	1	1.41	2	2.83	4	5.66	8	11.31	13.45	16

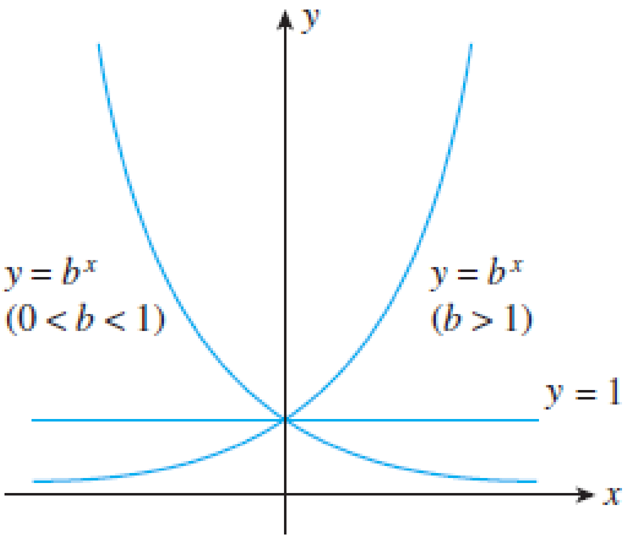
The graph of the function $f(x) = 2^x$ is as follows:



From the above figure, we observe that the range of the function $f(x) = 2^x$ is the interval $(0, \infty)$.

The general shape for the exponential function $f(x) = b^x$ where $b > 0$ and $b \neq 1$, depends on the value of *b*.

The functions: $y = f(x) = b^x$ with b > 1, $y = f(x) = b^x$ with 0 < b < 1, and $y = f(x) = b^x$ with b = 1, i.e., $y = f(x) = 1$ are plotted in the same graph as shown in the following figure.



Basic Properties of the Exponential Function

The exponential function $f(x) = b^x$ ($b > 0$ and $b \neq 1$) has the following properties.

1. The domain of the function is (-∞, ∞).
2. The range of the function is (0, ∞).
3. The graph passes through the point (0, 1).
4. The graph is a continuous curve, with no holes or jumps.
5. If $b > 1$, then $f(x) = b^x$ increases as x increases.
6. If $0 < b < 1$, then $f(x) = b^x$ decreases as x increases.

11.2 A Model for Proportional Change

The exponential function is used to model a relationship in which a constant change in the independent variable gives the same proportional change (i.e., percentage increase or decrease) in the dependent variable.

If the change is positive, this is called exponential growth and if it is negative, it is called exponential decay.

For example, because a radioactive substance decays at a rate proportional to the amount of the substance present, the amount of the substance present at a given time can be modeled with an exponential function. Also, because the growth rate of a population of bacteria in a petri dish is proportional to its size, the number of bacteria in the dish at a given time can be modeled by an exponential function such as $P = P_o e^{kt}$ where P_o is the number of bacteria present initially (at time $t = 0$) and k is a constant called the growth constant.

11.3 Exponential Growth

Exponential growth process may be described by the general function $V = f(t) = V_0 e^{kt}$ where V equals the value of the function at time t, V_o equals the value of the function at $t = 0$, k is the percent rate of growth, and t is the time measured in the appropriate units (hours, days, weeks, months, etc.).

Illustration

Employers are increasingly turning to GPS (global positioning system) technology to keep track of their fleet vehicles. The estimated number of automatic vehicle trackers installed on fleet vehicles in the United States is approximated by $N(t) = 0.6e^{0.17t}$ $(0 \le t \le 5)$ where $N(t)$ is measured in millions and t is measured in years, with corresponding to 2010.

a) What was the number of automatic vehicle trackers installed in the year 2010? How many were projected to be installed in 2015?
b) Sketch the graph of N.

Solution

a) We have $N(t) = 0.6e^{0.17t}$. The number of automatic vehicle trackers installed in the year 2010 is $N(0) = 0.6e^{0.17\times 0} = 0.6$ millions, i.e., 600,000.
   The number of automatic vehicle trackers installed in the year 2015 is calculated by $N(5) = 0.6e^{0.17\times 5} = 1.403778811$ millions, i.e., 1,403,778.
b) This is left for reader.

11.4 Exponential Decay

Exponential decay process may be described by the general function $V = f(t) = V_0 e^{-kt}$ where V equals the value of the function at time t, V_o equals the value of the function at t = 0, k is the percent rate of decay, and t is the time measured in the appropriate units (hours, days, weeks, months, etc.).

Exercise for Reader

1. According to a study conducted in 2000, the projected number of Web addresses (in billions) is approximated by the function $N(t) = 0.45e^{0.5696t}$ $(0 \le t \le 5)$ where t is measured in years, with t = 0 corresponding to the beginning of 1997.
   a) Complete the following table by finding the number of Web addresses in each year:

Year	0	1	2	3	4	5
Number of Web Addresses (billions)						

   b) Sketch the graph of N.
2. The number of Internet users in China is projected to be $N(t) = 94.5e^{0.2t}$ $(1 \le t \le 6)$ where $N(t)$ is measured in millions and t is measured in years, with t = 1 corresponding to the beginning of 2005.
   a) How many Internet users were there at the beginning of 2005?, at the beginning of 2006?
   b) How many Internet users were there expected to be at the beginning of 2010?

   the cost of making the diaries?