

## Lecture 15

### Learning Objectives

At the end of this class, students should be able to:

- Check whether the given function has discontinuities
- understand the concept of differentiation
- use differentiation rules

### 15.1 Point of Discontinuity

A discontinuous function is a function that we cannot draw on a graph without lifting our pencil. A function which is not continuous is termed as discontinuous.

A function is said to have a point of discontinuity at  $x = a$  if the function is undefined for  $x = a$ .

#### *Illustration 1*

Find the discontinuities of the function  $f(x) = 5x^4 + 3x^2 + 2x + 7$ .

#### *Solution*

The given function is a polynomial function. Since the polynomial function is continuous at every value of  $x$ . Thus, the given function is continuous in the entire domain.

#### *Illustration 2*

Find the discontinuities of the function  $f(x) = \frac{5x}{x^2 + 1}$ .

#### *Solution*

The given function is a rational function. The denominator of the given function is  $x^2 + 1$  which is never equal to zero. Therefore, we conclude that the given function is continuous for all values of  $x$ .

#### *Illustration 3*

Find the discontinuities of the function  $f(x) = \frac{x + 2}{x^2 - 3x + 2}$ .

#### *Solution*

The given function is not defined if the denominator becomes zero.

i.e.  $x^2 - 3x + 2 = 0$

or  $(x - 1)(x - 2) = 0$

Thus, the denominator of the given function becomes zero when  $x = 1$  or  $x = 2$ . The given function is discontinuous at these two points.

#### **Illustration 4**

Find the intervals on which the function  $f(x) = \frac{x}{9x^2 - 3x}$  is continuous.

#### **Solution**

The given function is discontinuous if the denominator becomes zero,

$$\text{i.e. } 9x^2 - 3x = 0$$

$$\text{or } 3x(3x - 1) = 0$$

Thus, the denominator becomes zero when  $x = 0$  or  $x = 1/3$ . This means the given function is continuous except at  $x = 0$  and  $x = 1/3$ .

Hence, the given function is continuous on the intervals  $(-\infty, 0)$ ,  $(0, 1/3)$ , and  $(1/3, \infty)$ .

### **15.2 Derivatives**

Let  $y = f(x)$  be a given function then the symbol  $\frac{dy}{dx}$  is used to express the derivative of  $y$  with respect to  $x$ . The interpretation  $\frac{dy}{dx}$  is the change in the dependent variable  $y$  due to one unit change in independent variable  $x$ . This type of change is also known as *instantaneous rate of change* or just *rate of change*. The process of obtaining the derivative is called *differentiation* of the function  $f(x)$ .

The other symbols that are used to represent derivative are  $\frac{df(x)}{dx}$ , or  $f'(x)$ , or  $y'$ , or  $y_1$  or simply by  $f'$ .

The derivative  $\frac{dy}{dx}$  provides the general expression for the slope of the graph of a function  $y = f(x)$  at any point  $x$  in the domain.

### **5.3 Rules of Differentiation**

We apply various rules that simplify the process of finding the derivative of a function.

- 1. Derivative of a Constant Function:** Derivative of a constant function is always zero. If  $f(x) = c$ , where  $c$  is any constant, then  $f'(x) = 0$ .

#### **Illustration 5**

Let  $f(x) = 5$ , then find  $f'(x)$ .

#### **Solution**

We have  $f(x) = 5$ , then

$$\begin{aligned} f'(x) &= \frac{d}{dx}(5) \\ &= 0 \end{aligned}$$

**2. Power Rule:** If  $f(x) = x^n$ , where  $n$  is a real number, then  $f'(x) = nx^{n-1}$ .

**Illustration 6**

Let  $f(x) = x^5$ , then find  $f'(x)$ .

**Solution**

We have  $f(x) = x^5$ , then

$$\begin{aligned} f'(x) &= \frac{d}{dx}(x^5) \\ &= 5x^4 \end{aligned}$$

**3. Derivative of a Constant Multiple of a Function:** If  $y = c f(x)$ , where  $c$  is a real number, then  $y' = c f'(x)$ .

**Illustration 7**

Let  $f(x) = 3x^5$ , then find  $f'(x)$ .

**Solution**

We have  $f(x) = 3x^5$ , then

$$\begin{aligned} f'(x) &= \frac{d}{dx}(3x^5) \\ &= 3 \frac{d}{dx}(x^5) \\ &= 3 \times 5x^4 \\ &= 15x^4 \end{aligned}$$

**4. Sum or Difference Rule:** If  $f(x) = u(x) \pm v(x)$ , where  $u$  and  $v$  are differentiable functions, then  $f'(x) = u'(x) \pm v'(x)$

**Illustration 8**

Let  $f(x) = 4x^3 - 2x + 15$ , then find  $f'(x)$ .

**Solution**

We have  $f(x) = 4x^3 - 2x + 15$ , then

$$\begin{aligned}f'(x) &= \frac{d}{dx}(4x^3 - 2x + 15) \\&= 4 \times 3x^2 - 2 \times 1 \\&= 12x^2 - 2\end{aligned}$$

### Exercise for Reader

1. For the following functions, determine whether there are any discontinuities and, if so, where they occur.

a)  $f(x) = 2 + 5x$

b)  $f(x) = \frac{x^3}{7}$

c)  $f(x) = \frac{1}{x+4}$

d)  $f(x) = \frac{2x+3}{x^2+4x-21}$

e)  $f(x) = \frac{2+5x}{x^2-4x}$

f)  $f(x) = \frac{2}{x^3-x}$

2. Find the derivative of the following functions with respect to  $x$ .

a)  $f(x) = -10x + 7$

b)  $f(x) = 5\sqrt{x}$

c)  $f(x) = x^3 - 6x^2 + 4x - 25$

d)  $f(x) = -2x^{-3} + 3x^{-1} + 2$